## 1. Sensor Calibration:

- In language;

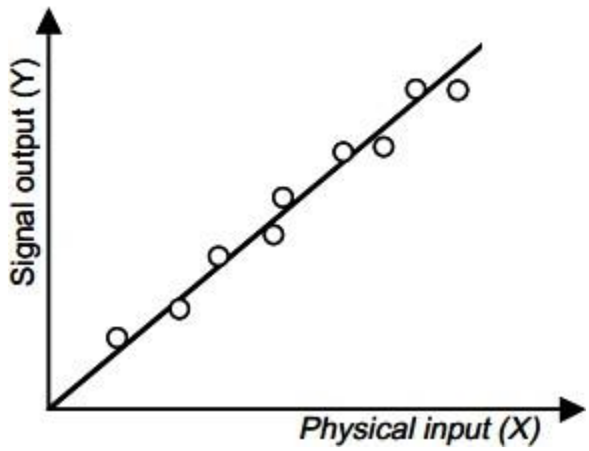
**Calibrate** means "to check, adjust, or determine by comparison with a standard".

**Calibration** is a "comparison between measurements".

**Sensor Calibration** is the relationship between the physical measurement variable (X) and the signal variable (S)

- A sensor or instrument is calibrated by applying a number of KNOWN physical inputs and recording the response of the system.

- The purpose of the calibration is to find the unknown coefficients (parameters) of the sensor transfer function so that the fully defined function can be employed during the measurement process to compute any stimulus in the desirable range, not only at the points used during the calibration.




## Calibration Methods:

Calibration of a sensor can be done in several possible ways, some of which are;

1. Calculation of the transfer function or its approximation to fit the selected calibration points (curve fitting by computing coefficients of a selected approximation).

2. Adjustment of the data acquisition system to modify the measured data by making them to fit into a normalized or "ideal" transfer function. An example is scaling of the acquired data.

3. Modification of the sensor' properties to fit the predetermined transfer function.

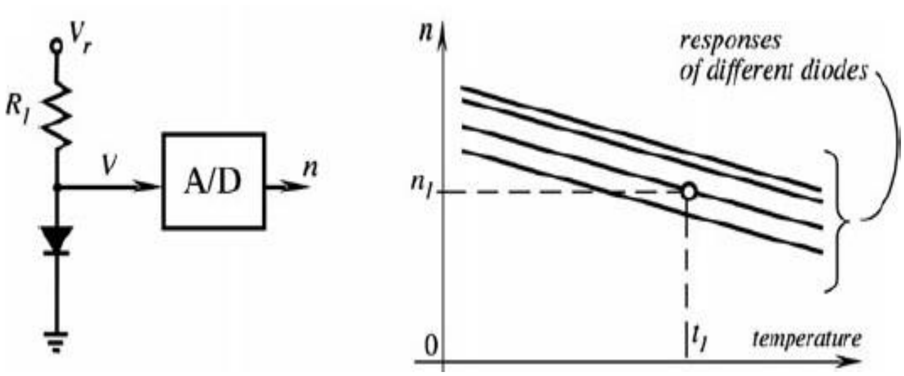
4. Creating a sensor-specific reference device with matching properties at particular calibrating points.

**Example:** Three methods of calibrating a thermistor (temperature sensitive resistor) is given in Page 22 of Reference 1.

Computation of Transfer Function Parameters:

**Example:** A forward-biased semiconductor p–n junction is used as a temperature sensor, its transfer function is linear;

$$n = n_1 + B(t - t_1)$$




At first calibrating temperature $t_1$, the output is $n_1$. After subjecting the sensor to the second calibrating temperature $t_2$, the output is $n_2$

$$n_2 = n_1 + B(t_2 - t_1)$$

$$\text{the sensitivity (slope)} = B = \frac{n_2 - n_1}{t_2 - t_1}$$

After calibration is done, temperature can be computed from

$$t = t_1 + \frac{(n - n_1)}{B}$$

Computation of Transfer Function Parameters:

**Example:** For nonlinear transfer functions, calibration at one data point may be sufficient only in some rare cases, but often two and more input–output pairs would be required. When a 2nd or a 3rd degree polynomial transfer functions are employed, respectively 3 and 4 calibrating pairs are required.

For a 3rd order polynomial;

$$S = as^3 + bs^2 + cs + d$$

- To find four parameters (a, b, c, and d), four calibrating i/o pairs are required:

$$S_1 = as_1^3 + bs_1^2 + cs_1 + d$$
$$S_2 = as_2^3 + bs_2^2 + cs_2 + d$$
$$S_3 = as_3^3 + bs_3^2 + cs_3 + d$$
$$S_4 = as_4^3 + bs_4^2 + cs_4 + d$$

- To solve this system, first one computes the determinants of the systems:

$$\Delta = \left(\frac{s_1^2 - s_2^2}{s_1 - s_2} - \frac{s_1^2 - s_4^2}{s_1 - s_4}\right)\left(\frac{s_1^3 - s_2^3}{s_1 - s_2} - \frac{s_1^3 - s_3^3}{s_1 - s_3}\right) - \left(\frac{s_1^2 - s_2^2}{s_1 - s_2} - \frac{s_1^2 - s_3^2}{s_1 - s_3}\right)\left(\frac{s_1^3 - s_2^3}{s_1 - s_2} - \frac{s_1^3 - s_4^3}{s_1 - s_4}\right)$$

$$\Delta_a = \left(\frac{s_1^2 - s_2^2}{s_1 - s_2} - \frac{s_1^2 - s_4^2}{s_1 - s_4}\right)\left(\frac{S_1 - S_2}{s_1 - s_2} - \frac{S_1 - S_3}{s_1 - s_3}\right) - \left(\frac{s_1^2 - s_2^2}{s_1 - s_2} - \frac{s_1^2 - s_3^2}{s_1 - s_3}\right)\left(\frac{S_1 - S_2}{s_1 - s_2} - \frac{S_1 - S_4}{s_1 - s_4}\right)$$

$$\Delta_b = \left(\frac{s_1^3 - s_2^3}{s_1 - s_2} - \frac{s_1^3 - s_3^3}{s_1 - s_3}\right)\left(\frac{S_1 - S_2}{s_1 - s_2} - \frac{S_1 - S_4}{s_1 - s_4}\right) - \left(\frac{s_1^3 - s_2^3}{s_1 - s_2} - \frac{s_1^3 - s_4^3}{s_1 - s_4}\right)\left(\frac{S_1 - S_2}{s_1 - s_2} - \frac{S_1 - S_3}{s_1 - s_3}\right)$$

- Then, the polynomial coefficients are calculated as;

$$a = \frac{\Delta_a}{\Delta}; \quad b = \frac{\Delta_b}{\Delta};$$

$$c = \frac{1}{s_1 - s_4}\left[S_1 - S_4 - a\left(s_1^3 - s_4^3\right) - b\left(s_1^2 - s_4^2\right)\right];$$

$$d = S_1 - as_1^3 - bs_1^2 - cs_1$$

$$y = A + Bx$$

where

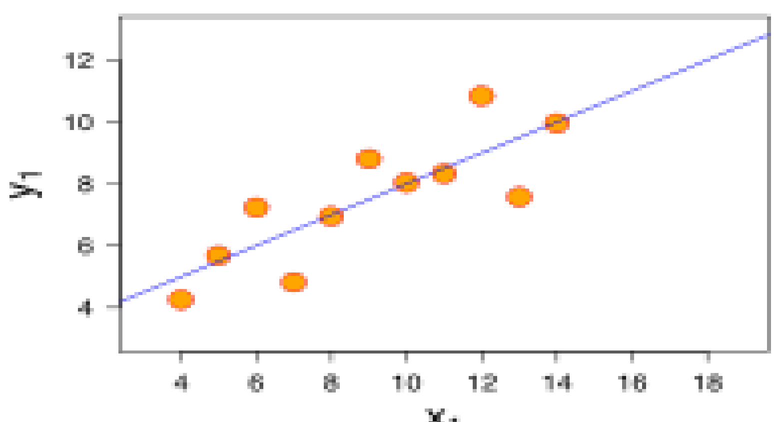
$$B = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2} \quad \text{and} \quad A = \bar{y} - B\bar{x}$$

## Linear Regression Formula:

Simple linear regression is a way to describe a relationship between two variables through an equation of a straight line, called line of best fit, that most closely models this relationship.

The following formula is used:

Example: consider the given experimental data. The regression equation is a linear equation, we need to solve for A and B. Computations are shown below;




| | |
|---|---|
| $B = \Sigma [ (x_i - \bar{x})(y_i - \bar{y}) ] / \Sigma [ (x_i - \bar{x})^2 ]$ | $A = \bar{y} - B * \bar{x}$ |
| $B = 470/730 = 0.644$ | $A = 77 - (0.644)(78) = 26.768$ |

| No. | $x_i$ | $y_i$ | $(x_i - \bar{x})$ | $(y_i - \bar{y})$ | $(x_i - \bar{x})^2$ | $(y_i - \bar{y})^2$ | $(x_i - \bar{x})(y_i - \bar{y})$ |
|---|---|---|---|---|---|---|---|
| 1 | 95 | 85 | 17 | 8 | 289 | 64 | 136 |
| 2 | 85 | 95 | 7 | 18 | 49 | 324 | 126 |
| 3 | 80 | 70 | 2 | -7 | 4 | 49 | -14 |
| 4 | 70 | 65 | -8 | -12 | 64 | 144 | 96 |
| 5 | 60 | 70 | -18 | -7 | 324 | 49 | 126 |
| **Sum** | 390 | 385 | | | 730 | 630 | 470 |
| **Mean** | 78 | 77 | | | | | |

## References: