جامـــــعـة المـــــسـتـقـبـل
AL MUSTAQBAL UNIVERSITY

قســـم الامـــــن الـــــــــسيبرانـــــــــي
DEPARTMENT OF CYBER SECURITY

SUBJECT:

SETS AND ELEMENTS

CLASS:

FIRST

LECTURER:

ASST. LECT. MUSTAFA AMEER AWADH

LECTURE: (2)

Power set

The power set of some set S, denoted P(S), is the set of all subsets of S (including S itself and the empty set)

$P(S) = \{e : e \subseteq S\}$

Example 1:

Let A = { 1,2 3}

Power set of set A = P(A) ={1},{2},{3},{1,2},{1,3},{2,3},{},A

Example 2:

P({0,1})={{},{0},{1},{0,1}}

Classes of sets:

Collection of subset of a set with some properties

Example:

Suppose A = { 1,2 3} ,

let X2 be the class of subsets of A which contain exactly two

elements of A. Then

class X0 = [{}]

class X1 = [{1},{2},{3}]

class X2 = [{1,2},{1,3},{2,3}]

class X3 = [{1,2,3}]

Cardinality

The cardinality of a set S, denoted **|S|**, is simply the number of elements a set has, so

|{a,b,c,d}| = 4,

The cardinality of the power set

Theorem:

If |A| = n then |P(A)| = 2^n

(Every set with n elements has 2^n subsets)

Problem set

Write the answers to the following questions.

1. |{1,2,3,4,5,6,7,8,9,0}|
2. |P({1,2,3})|
3. P({0,1,2})
4. P({1})

Answers

1. 10
2. 2^3=8
3. {{},{0},{1},{2},{0,1},0,2},{1,2},{0,1,2}}
4. {{},{1}}

The Cartesian product

The Cartesian Product of two sets is the set of all tuples made

from elements of two sets.

We write the Cartesian Product of two sets A and B as A × B. It is defined as:

$$A \times B = \{(a,b) | a \in A \text{ and } b \in B\}$$

It may be clearer to understand from examples;

$$\{0,1\} \times \{2,3\} = \{(0,2),(0,3),(1,2),(1,3)\}$$

$$\{a,b\} \times \{c,d\} = \{(a,c),(a,d),(b,c),(b,d)\}$$

$$\{0,1,2\} \times \{4,6\} = \{(0,4),(0,6),(1,4),(1,6),(2,4),(2,6)\}$$

Example:

If A = {1, 2, 3} and B = {x, y} then

A . B = {(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)}

B . A = {(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)}

It is clear that, the cardinality of the Cartesian product of two sets A and B is:

$$|A \times B| = |A|\,|B|$$

A Cartesian Product of two sets A and B can be produced by making tuples of each element of A with each element of B; this can be visualized as a grid (which *Cartesian* implies) or table: if, *e.g.*, A = { 0, 1 } and B = { 2, 3 }, the grid is

×		A	
		0	**1**
B	**2**	(0,2)	(1,2)
	3	(0,3)	(1,3)

Problem set

Answer the following questions:

1. $\{2,3,4\}\times\{1,3,4\}$
2. $\{0,1\}\times\{0,1\}$
3. $|\{1,2,3\}\times\{0\}|$
4. $|\{1,1\}\times\{2,3,4\}|$

Answers

1. {(2,1),(2,3),(2,4),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)}
2. {(0,0),(0,1),(1,0),(1,1)}
3. 3
4. 6

EXAMPLE

What is the Cartesian product $A \times B \times C$, where

$A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$?

Solution:

The Cartesian product $A \times B \times C$ consists of all ordered triples

(a, b, c), where $a \in A$, $b \in B$, and $c \in C$. Hence,

$A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0),$

$(1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}$.

EXAMPLE

Suppose that $A = \{1, 2\}$. It follows that

$A2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$ and

$A3 = \{(1,1,1), (1,1,2), (1,2,1), (1,2, 2), (2,1,1), (2,1,2), (2, 2,1), (2, 2, 2)\}$.

Computer Representation of Sets

There are various ways to represent sets using a computer. One method is to store the elements of the set in an unordered fashion. However, if this is done, the operations of computing the union, intersection, or difference of two sets would be time- consuming, because each of these operations would require a large amount of searching for elements. We will present a method for storing elements using an arbitrary ordering of the elements of the universal set. This method of representing sets makes computing combinations of sets easy. Assume that the universal set U is finite (and of reasonable size so that the number of elements of U is not larger than the memory size of the computer being used). First, specify an arbitrary ordering of the elements of U, for instance: $a1, a2, \ldots, an$. Represent a subset A of U with the bit string of length n, where the ith bit in this string is 1 if ai belongs to A and is 0 if ai does not belong to A.

Example

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and the ordering of elements of U has the elements in increasing order; that is,

$a_i = i$. What bit strings represent

1- the subset of all odd integers in *U,*

2- The subset of all even integers in *U,* and

3- the subset of integers not exceeding 5 in *U*?

Solution:

1- The bit string that represents the set of odd integers in *U,* namely, $\{1, 3, 5, 7, 9\}$, has a one bit in the first, third, fifth, seventh, and ninth positions, and a zero elsewhere.
It is: 10 1010 1010.

2- we represent the subset of all even integers in *U,* namely,

$\{2, 4, 6, 8, 10\}$, by the string 01 0101 0101.

3- The set of all integers in *U* that do not exceed 5, namely,

$\{1, 2, 3, 4, 5\}$, is represented by the String 11 1110 0000.

Using bit strings to represent sets, it is easy to find complements of sets and unions, intersections, and differences of sets. To find the bit string for the complement of a set from the bit string for that set, we simply change each 1 to a 0 and each 0 to 1, because

$x \in A$ if and only if $x \notin \bar{A}$. Note that this operation corresponds to taking the negation of each bit when we associate a bit with a truth value—with 1 representing true and 0 representing false.

Example

We have seen that the bit string for the set $\{1, 3, 5, 7, 9\}$ (with universal set $\{1, 2, 3, 4,5, 6, 7, 8, 9, 10\}$) is 10 1010 1010.What is the bit string for the complement of this set?

Solution:

The bit string for the complement of this set is obtained by replacing 0s with 1s and vice versa. This yields the string 010101 0101, which corresponds to the set {2, 4, 6, 8, 10}.

To obtain the bit string for the union and intersection of two sets we perform bitwise Boolean operations on the bit strings representing the two sets. The bit in the ith position of the bit string of the **union** is 1 if either of the bits in the ith position in the two strings is 1 (or both are 1), and is 0 when both bits are 0. Hence, the bit string for the union is the bitwise *OR* of the bit strings for the two sets. The bit in the ith position of the bit string of the **intersection** is 1 when the bits in the corresponding position in the two strings are both 1, and is 0 when either of the two bits is 0 (or both are). Hence, the bit string for the intersection is the bitwise *AND* of the bit strings for the two sets.

EXAMPLE

The bit strings for the sets {1, 2, 3, 4, 5} and {1, 3, 5, 7, 9} are 11 1110 0000 and 10 1010 1010, respectively. Use bit strings to find the union and intersection of these sets.

Solution:

The bit string for the **union** of these sets is:

11 1110 0000 ∨ 10 1010 1010 = 11 1110 1010,

Which corresponds to the set {1, 2, 3, 4, 5, 7, 9}.

The bit string for the **intersection** of these sets is

11 1110 0000 ∧ 10 1010 1010 = 10 1010 0000, which

corresponds to the set {1, 3, 5}.