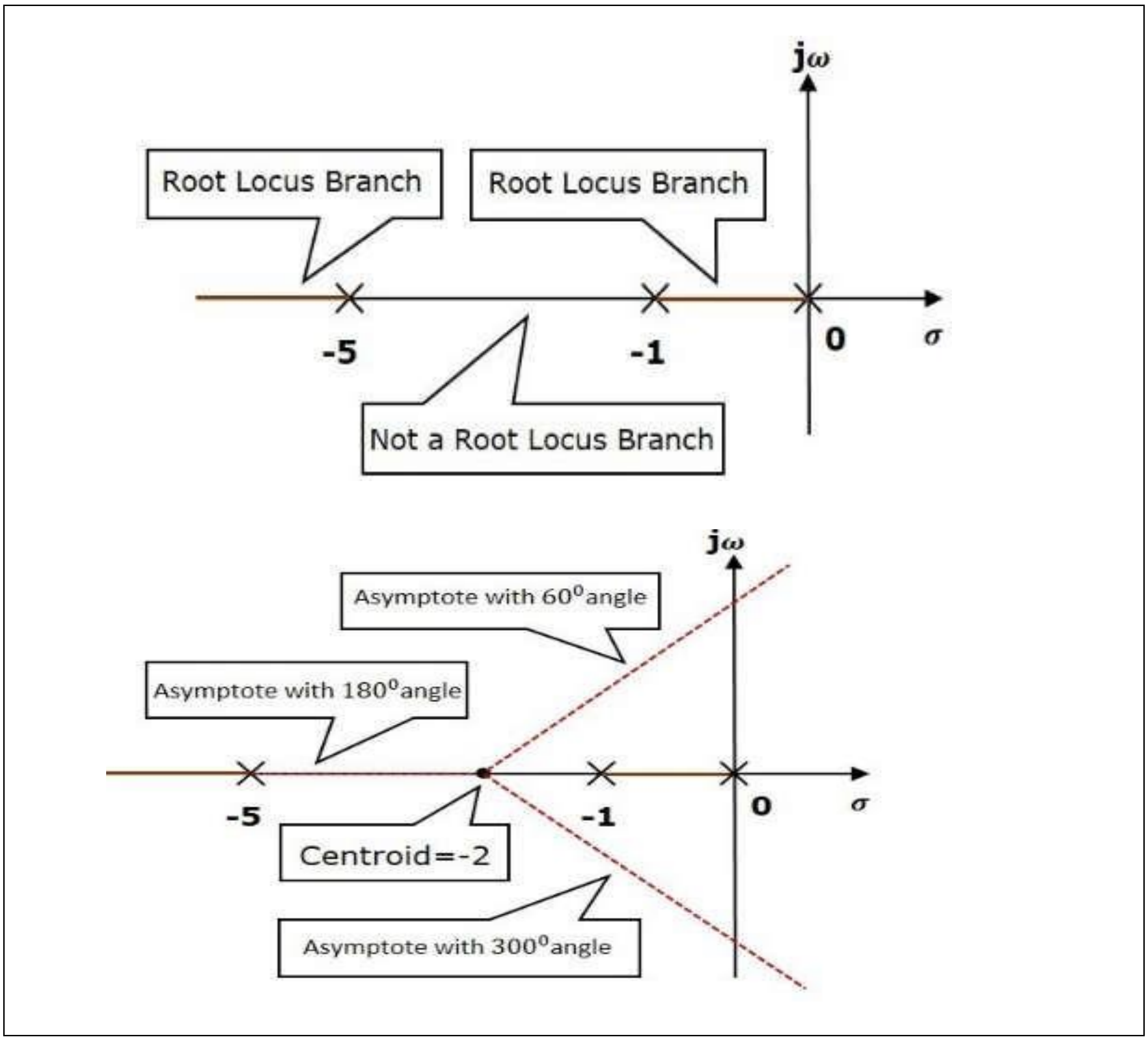
Root Locus

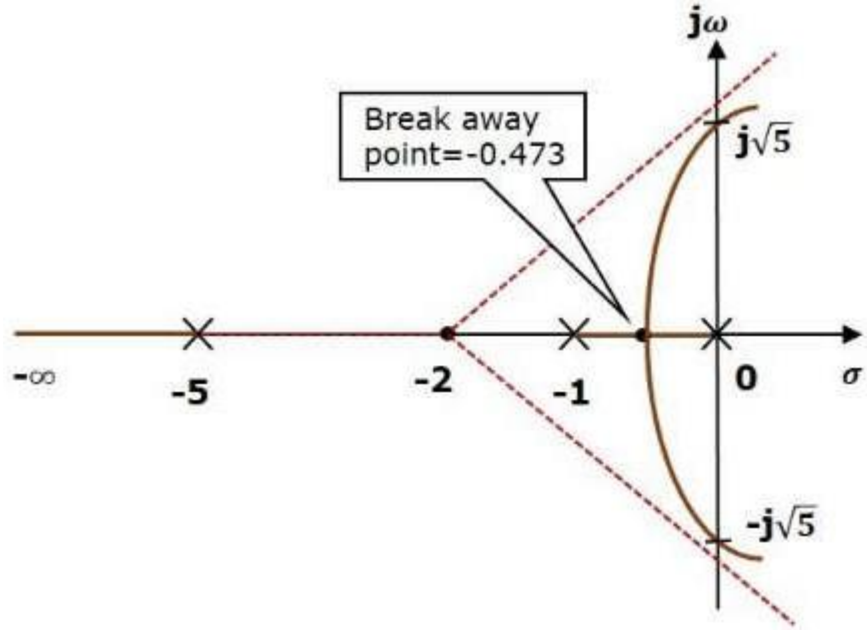
4. Self- Test

1. Draw the root locus of the control system having open loop transfer function,

$$G(s)H(s) = \frac{K}{s(s+1)(s+5)}$$



Email: hasan.hamad.ali@uomus.edu.iq



Post- Test

5. Post- Test

1. Sketch the root loci for the system shown in Figure 6-42(a).

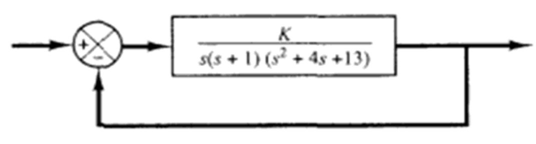


Figure 6–42
(a) Control system;

Email: hasan.hamad.ali@uomus.edu.iq

Solution. The open-loop poles are located at $s = 0, s = -1, s = -2 + j3$, and $s = -2 - j3$. A root locus exists on the real axis between points $s = 0$ and $s = -1$. The angles of the asymptotes are found as follows:

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ(2k+1)}{4} = 45^\circ, -45^\circ, 135^\circ, -135^\circ$$

The intersection of the asymptotes and the real axis is found from

$$s = -\frac{0+1+2+2}{4} = -1.25$$

The breakaway and break-in points are found from $dK/ds = 0$. Noting that

$$K = -s(s+1)(s^2+4s+13) = -(s^4+5s^3+17s^2+13s)$$

we have

$$\frac{dK}{ds} = -(4s^3+15s^2+34s+13) = 0$$

from which we get

$$s = -0.467, \qquad s = -1.642 + j2.067, \qquad s = -1.642 - j2.067$$

Point $s = -0.467$ is on a root locus. Therefore, it is an actual breakaway point. The gain values K corresponding to points $s = -1.642 \pm j2.067$ are complex quantities. Since the gain values are not real positive, these points are neither breakaway nor break-in points.

The angle of departure from the complex pole in the upper half s plane is

$$\theta = 180^\circ - 123.69^\circ - 108.44^\circ - 90^\circ$$

or

$$\theta = -142.13^\circ$$

Next we shall find the points where root loci may cross the $j\omega$ axis. Since the characteristic equation is

Email: hasan.hamad.ali@uomus.edu.iq

$$s^4 + 5s^3 + 17s^2 + 13s + K = 0$$

by substituting $s = j\omega$ into it we obtain

$$(j\omega)^4 + 5(j\omega)^3 + 17(j\omega)^2 + 13(j\omega) + K = 0$$

or

$$(K + \omega^4 - 17\omega^2) + j\omega(13 - 5\omega^2) = 0$$

from which we obtain

$$\omega = \pm 1.6125, \qquad K = 37.44 \qquad \text{or} \qquad \omega = 0, \qquad K = 0$$

The root-locus branches that extend to the right-half s plane cross the imaginary axis at $\omega = \pm 1.6125$. Also, the root-locus branch on the real axis touches the imaginary axis at $\omega = 0$. Figure 6–42(b) shows a sketch of the root loci for the system. Notice that each root-locus branch that extends to the right half s plane crosses its own asymptote.

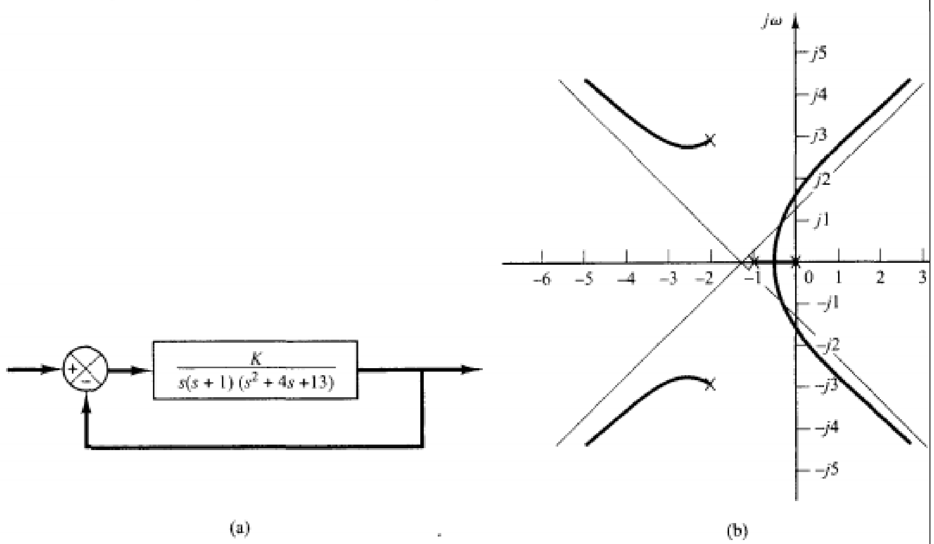


Figure 6–42
(a) Control system; (b) root-locus plot.

Effects of Adding Open Loop Poles and Zeros on Root Locus

The root locus can be shifted in 's' plane by adding the open loop poles and the open loop zeros. If we include a pole in the open loop transfer function, then some of root locus branches will move towards right half of 's' plane. Because of this, the damping ratio δ decreases. Which implies, damped frequency ω_d increases and the time domain specifications like delay time t_d, rise time t_r and peak time t_p decrease. But, it affects the system stability.

If we include a zero in the open loop transfer function, then some of root locus branches will move towards left half of 's' plane. So, it will increase the control system stability. In this case, the damping ratio δ increases. Which implies, damped frequency ω_d decreases and the time domain specifications like delay time t_d, rise time t_r and peak time t_p increase.
So, based on the requirement, we can include (add) the open loop poles or zeros to the transfer function.

Uses of Root Locus

In addition, in determining the stability of the system, root locus also helps to determine:

Damping ratio

The damping ratio is a dimensionless unit that describes how the system decay affects the oscillations of the system.

Natural frequency

It is represented by wn. The value of the system gain K at the location of poles helps in computing the natural frequency and the damping ratio of the system. P, PI, and PID controllers P (proportional), PI (Proportional Integral), and PID (Proportional Integral Derivative) controllers can be designed with the help of root locus technique. Here, the input of the system to be controlled is made proportional to the system gain K.

Lag and lead compensators

The compensators are the additional components in the system added to compensate for deficient performance. The phase lead compensator helps to shift the root locus towards the left in the complex s-plane, and it further increases the system's stability. Similarly, lag and lead compensators can be designed in various ways with the help of the root locus.

Advantages of Root locus

The advantages of root locus are as follows:

We can analyze the absolute stability of the system with the help of a root locus plot.

Using the magnitude and angle conditions, we can find the limiting value of the system gain K for any point on the root locus.

Enhances system designing with better accuracy.

It helps in analyzing the stability of the system with time delay.

Root locus plots help us determine the gain margin, relative stability, phase margin, and the system's settling time.

The root locus technique is easy to implement as compared to other techniques in the control system.

It helps in analyzing the performance of the control system.

Email: hasan.hamad.ali@uomus.edu.iq