جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

كلية العلوم
قسم الادلة الجنائية

REAL AND COMPLEX NUMBERS

Mathematics

المرحلة الاولى

م.م ريام ثائر احمد

المحاضرة الاولى

REAL AND COMPLEX NUMBERS

Real number representation

Real number represent graphically in one dimension (either horizontal or vertical) as shown.

Due to *Quadratic* Algebric Equation ;

$$ax^2 + bx + C = 0$$

The solution will be ${}_1$ and x_2. The square root of $(\sqrt{b^2 - 4ac})$ may be (posative ,negative or zero).

The *negative* value will be expressed as ***(complex number)***

Complex Numbers represent by:

1. ***Rectangular*** **coordinate representation**

$$z = x + iy \implies \quad as\ a\ point\ with\ (x\ ,y)$$

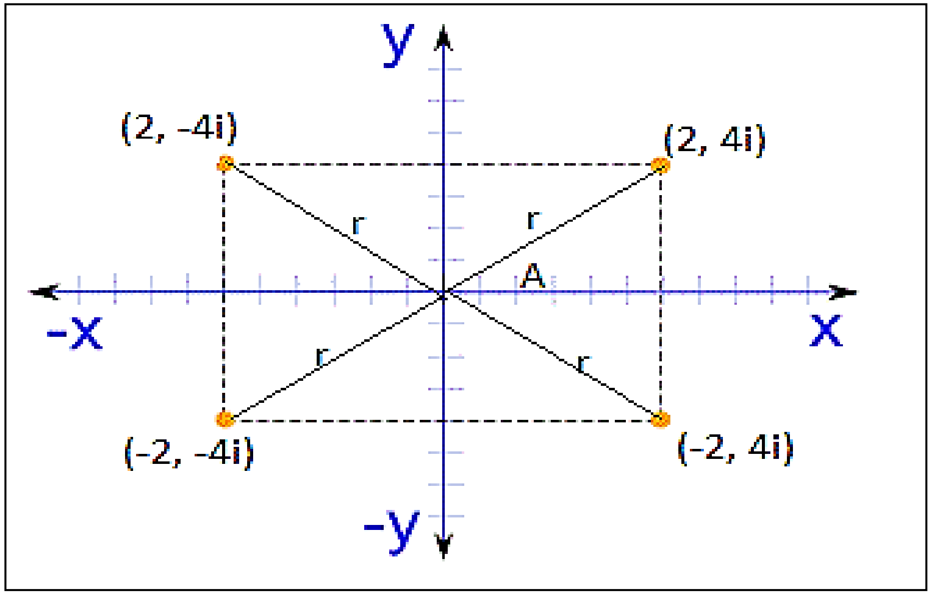
x and y are real numbers.

x are the real part of $\implies x = Re(z)$

y are the imaginary part of $z \implies y = Im(z)$

$$z = Re(z) + Im(z).\boldsymbol{i}$$

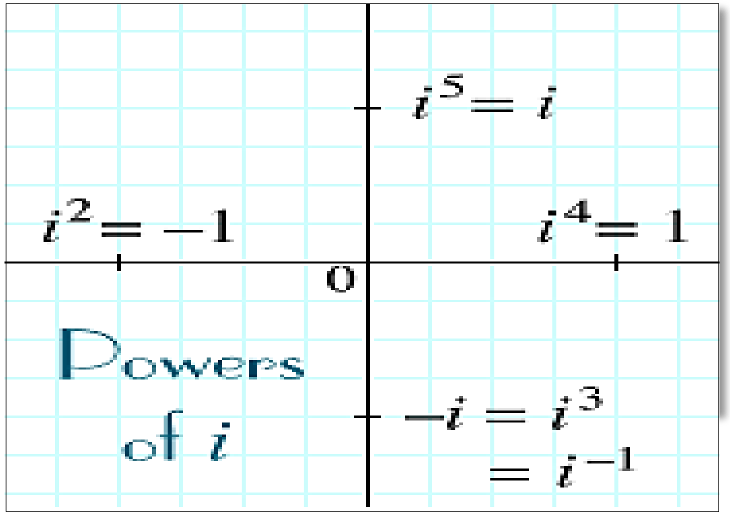
Argand Plane **or** ***Complex Plane***



Complex unit

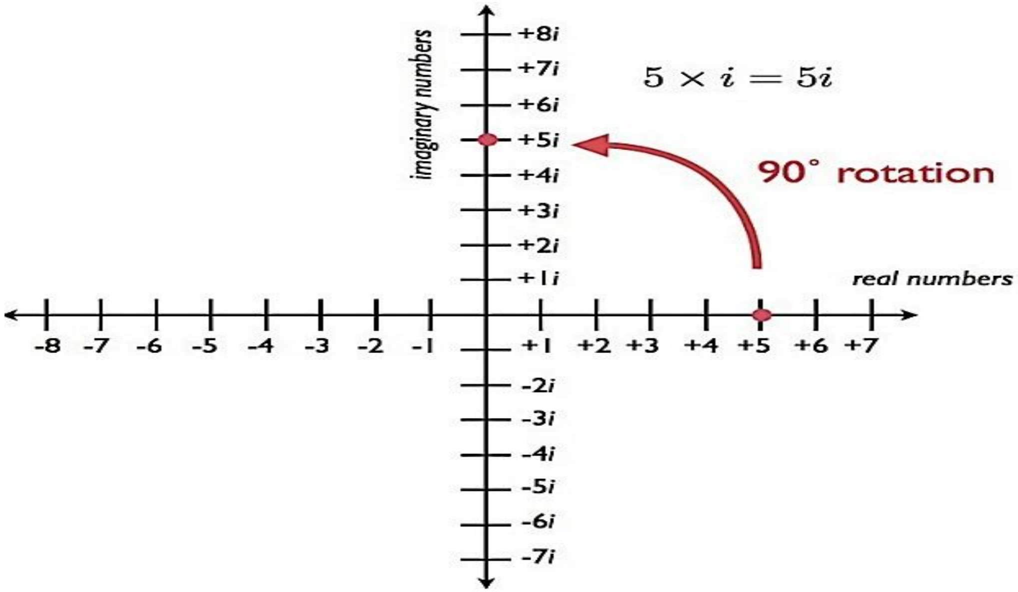
Now,

$$i = \sqrt{-1}$$
$$i^2 = -1$$



$$i^5 = i^2 . i^2 . i = (i^2)^2 . i = (-1)^2 . i = +i$$

$$i^{101} = (i^2)^{50} . i = (-1)^{50} . i = +i$$



2. Polar coordinate representation

$z = re^{i\theta}$ specified by (r, θ)

$z = [\cos\theta + i\sin\theta]$ specified by (r, θ) *known as Euler's formula*

$z = r\angle\theta$

$z = r\angle\theta$

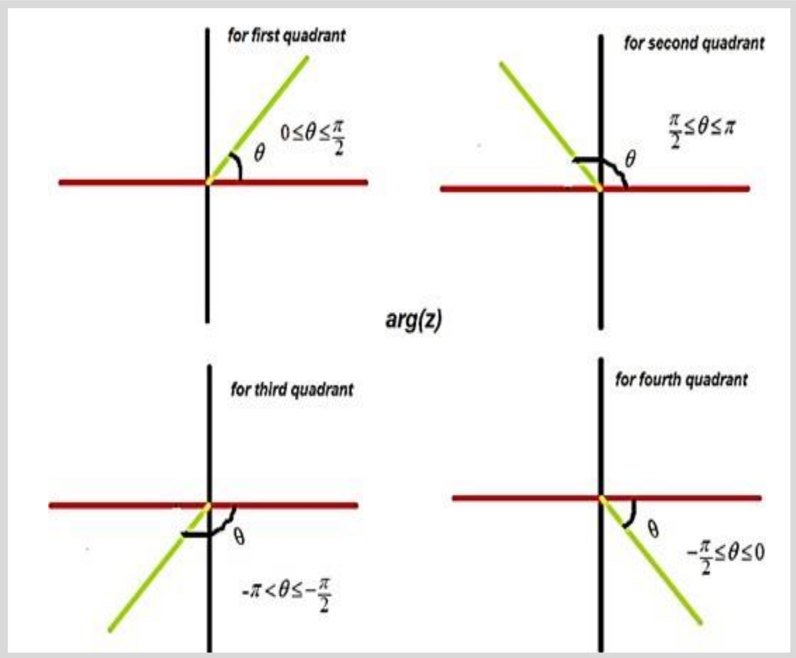
- **Length** of z
- **Amplitude** z
- **Modulus** z
- **Absolute value**

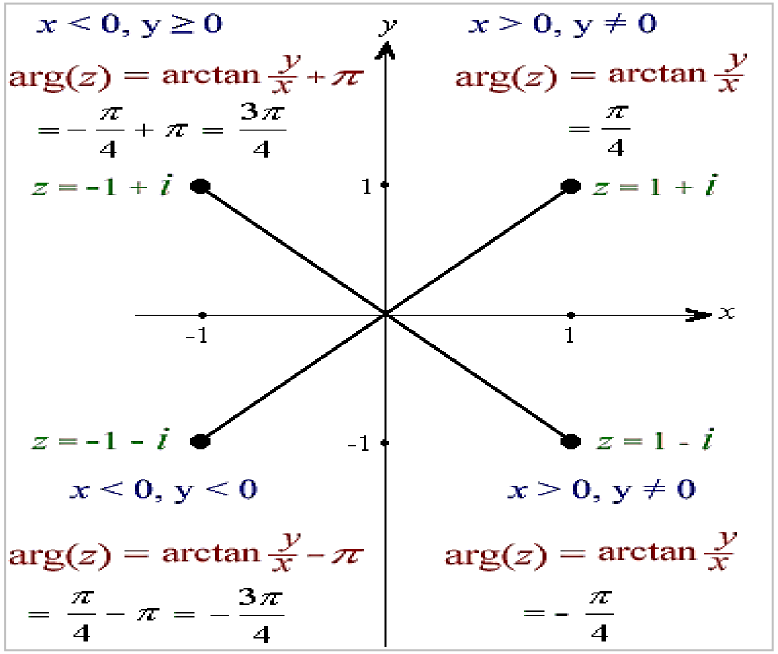
$|z| = \sqrt{x^2 + y^2} = r$

r is the " ***radius of circle*** " **centered** at orign.

θ is called '***angle*** " or , " ***argument***' or '***phase*** " represent the direction of Z and can be evaluated by :

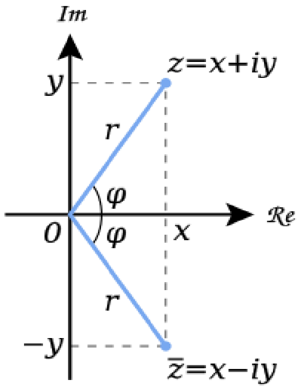
$$\theta = tan^{-1}(\frac{y}{x})$$

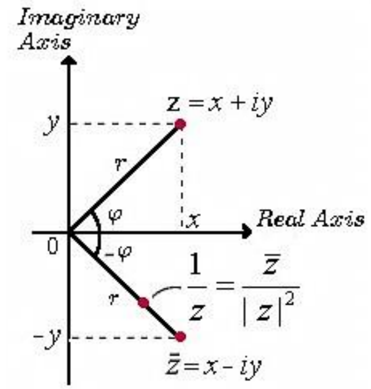


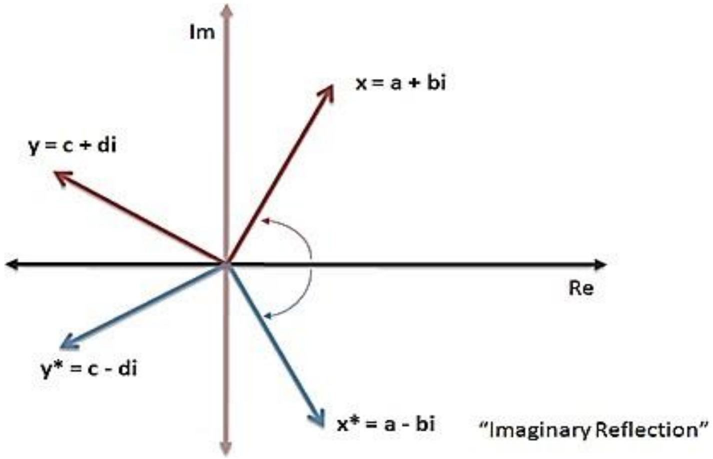


Complex conjugate of z Represented by $\bar{z}$ or z^*

$$\bar{z} = x - iy \implies \quad as\ a\ point\ with\ (x\ ,-y)$$







Inflection of z

$$-z = -x - iy \implies \quad as\ a\ point\ with\ (-x\ ,-y)$$

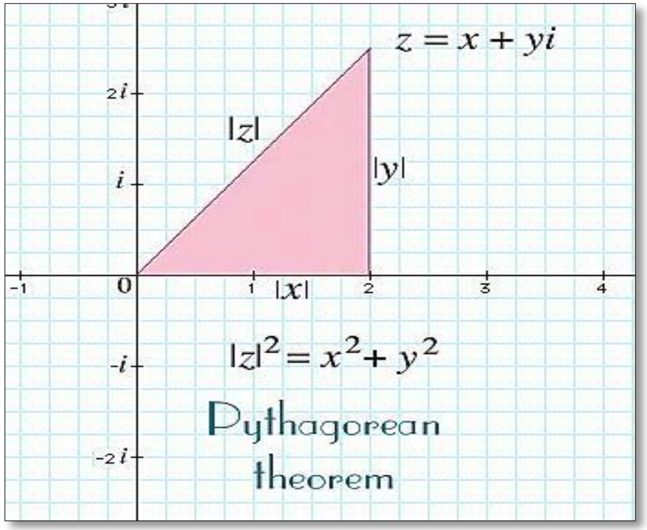
Absolute value of z

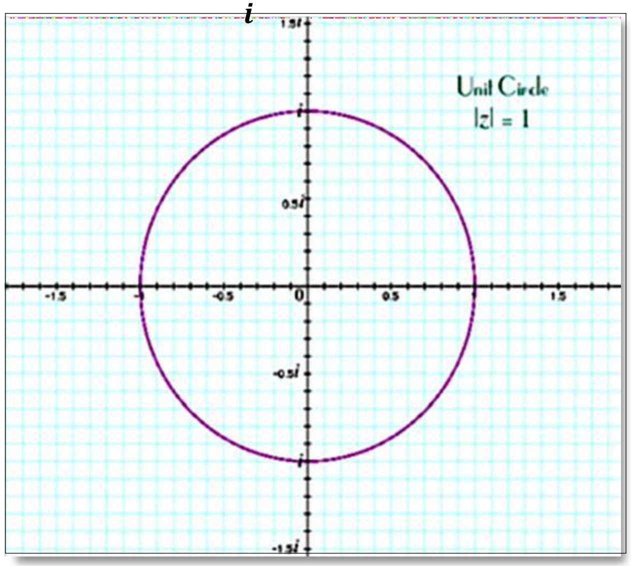
Also called ,
- **Length** of z
- **Amplitude** z
- **Modulus** z

r is the " ***radius of circle*** "

$$|z| = \sqrt{x^2 + y^2} = r$$

centered at $(x\ ,y)$.





- **Distance between** z_1 *and* z_2

Represented by $|z_1 - z_2|$ and given by :

$$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- **Power of** z

Or ,

$$z^n = r^n e^{(in\theta)}$$

$$(cos\theta + isin\theta)^n = \cos(n\theta) + isin(n\theta)$$

"*De Moivers theorem*"

PASCAL TRIANGLE

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1a + 1b$$

$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

$$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$$

- **Roots of** z

$$z^{1/n} = r^{1/n}\left[\cos\left(\frac{+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right], \qquad k=0,1,2,3,\ldots,(n-1)$$

$z_0, z_1, z_2 \ldots$ *the roots of* z

COMPLEX NUMBERS

Examples

1. Verify

a) $(\sqrt{2}-i)-i(1-\sqrt{2}i)=-2i$

b) $\left(\frac{1+2i}{3-4i}\right)+\left(\frac{2-i}{i}\right)=\frac{-2}{5}$

$$\Longrightarrow \frac{1+2i}{3-4i}*\frac{3+4i}{3+4i}+\frac{2-i}{5i}*\frac{-5i}{-5i}$$

$$=\boxed{\frac{3+4i+6i-8}{25}=\frac{-1}{5}-\frac{2i}{5}}$$

c) $\frac{5}{(1-i)(2-i)(3-i)}=\frac{1}{2}\,i$

$$\frac{5}{(1-i)(2-i)(3-i)}=\frac{5}{(1-3i)(3-i)}=\frac{5}{3-i-9i-3}=\frac{5}{-10i}=\frac{-i}{2i^2}$$

$$=+\frac{i}{2}\quad \text{since } i^2=-1$$

2. Simplify $\left(\frac{1}{2-3i}\right)\left(\frac{1}{1+i}\right)$

Solution:

$$\left(\frac{1}{2-3i}\right)\left(\frac{1}{1+i}\right) = \frac{1}{2+3-3i+2i} = \frac{1}{5-i} * \frac{5+i}{5+i} = \frac{5+i}{25-1} = \frac{5+i}{24}$$

$$= \frac{5}{24} + \frac{1}{24}i$$

Question : Express $X, Y, and\ |z|^2$ in terms of $Re(z)$ and $Im(z)$

Solution:

$$z = x + iy \quad \dots(1)$$

$$\underline{\bar{z} = x - iy} \quad \dots(2)$$

Add (1) and (2) ,

$$z + \bar{z} = 2x \Longrightarrow \boxed{X \equiv Re(z) = \frac{z+\bar{z}}{2}}$$

Now, subtract (1) and (2),

$$z = x + iy$$

$$\overline{z} = x - iy$$

$$z - \bar{z} = 2yi \Longrightarrow \boxed{Y \equiv Im(z) = \frac{z - \bar{z}}{2i}}$$

$$|z|^2 = zz^* = \boxed{x^2 + y^2} = \left(Re(z)\right)^2 + \left(Im(z)\right)^2$$

Basic of algebraic properties of *z*, verify a few algebraic properties of *z*.

1. The commutative laws.

$$z_1 \pm z_2 = z_2 \pm z_1, \qquad z_1 z_2 = z_2 z_1$$

2. The associative laws

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3), \quad (z_1 z_2) z_3 = z_1 (z_2 z_3)$$

Now,

$$\frac{1}{z_1 z_2} = \left(\frac{1}{z_1}\right)\left(\frac{1}{z_2}\right), \quad (z_1 \neq 0, z_2 \neq 0, z_1 z_2 \neq 0)$$

$$\frac{z_1 + z_2}{z_3} = \frac{z_1}{z_3} + \frac{z_2}{z_3}, \quad \frac{z_1}{z\ z_4} = \left(\frac{z_1}{z_3}\right)\left(\frac{z_2}{z_4}\right), \quad (z_3 \neq 0, z_4 \neq 0, z_3, z_4 \neq 0)$$

Absolute value of Z

With that interpretation in mind, we introduce the length, amplitude, absolute value or modulus of the complex number its the length when thinking of it as a vector:

If $z = x + iy$ then $|z| = \sqrt{X^2 + y^2}$

$|z|$ is the distance between the point (x, y) and the origin.

Example:

Compute the absolute value for each of the complex numbers:

$z_1 = 1 + i, z_2 = 2 - 3i$, find $|z_1 - z_2|$

Solution:

$$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(1-2)^2 + (1+3)^2} = \sqrt{1^2 + 4^2} = \sqrt{17}$$