Subject Name: Numerical analysis

3rd Class, Second Semester

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Lecture No. 1

Lecture Title: Introduction to Numerical Analysis

Module I: Polynomial Equations and system of linear equation.

Module II: Interpolations.

Module III: Numerical Differentiation and Integration.

Module IV: Numerical Solutions of ODE.

Numerical Analysis

Introduction

Numerical analysis: is the study of algorithms, the problems of continuous mathematics, it's concerned with the mathematical derivation, description and analysis of methods to obtaining numerical solution for mathematical problems.

Numerical analysis is similar in that problems solved, but the only procedures that are used are arithmetic: add, subtract, multiply, divide and compare.

Differences between analytical solutions and numerical solutions:

1) An analytical solution is usually given in terms of mathematical functions. The behavior and properties of the function are often apparent. However, a numerical solution is always an approximation. It can be plotted to show some of the behavior of the solution.

2) An analytical solution is not always meaningful by itself.

Example: √3 as one of the roots of $x^3 - x^2 - 3x + 3 = 0$

3) While the numerical solution is an approximation, it can usually be evaluated as accurate as we need. Actually, evaluating an analytic solution numerically is subject to the same errors.

Error analysis

There are four common ways to express the size of the error in a computed result:

1- Absolute error (true error) = $|True\ value - Approximate\ value|$

2- Relative true error ε_t % = $\left|\frac{True\ error}{True\ valve}\right| * 100\ \%$

3- Approximate error (ε_a)= $|New\ approximate\ value - old\ approximate\ value|$

Relative Approximate error (ε_a) % = $\left|\frac{Approxmaite\ error}{approxmiate\ \ valve}\right| * 100\ \%$

Module I: Polynomial Equations and system of linear Equation

1. Polynomial Equations

ROOT FINDING

a. Methods for Finding Location of Real Roots

1. Graphical Method:

$$f(x) = x^2 + 2x - 1 = 0$$

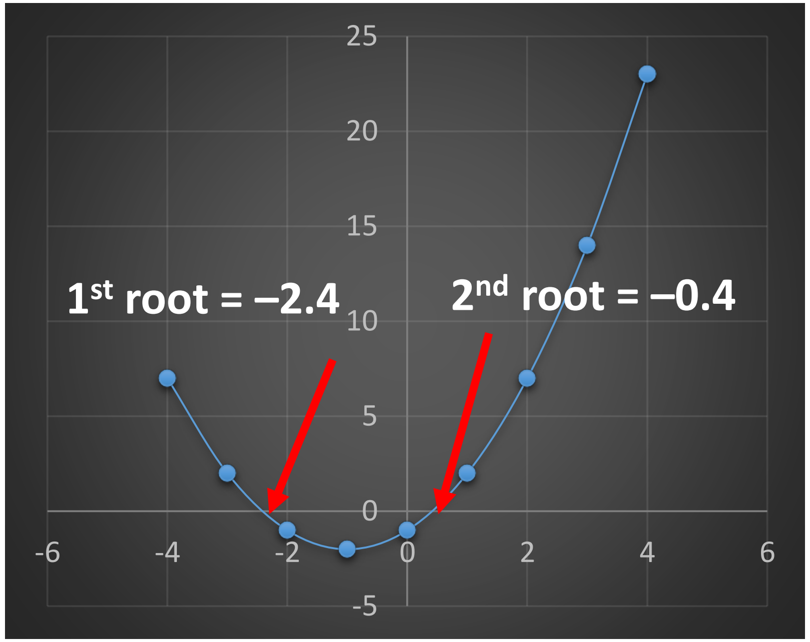
$$x1 = -\sqrt{2} - 1 = -2.4142$$
$$x2 = +\sqrt{2} - 1 = +0.4142$$

x	y
4	23
3	14
2	7
1	2
0	-1
-1	-2
-2	-1
-3	2
-4	7

$$Error1\% = \frac{-2.4142 + 2.4}{2.4142} = 0.6\%$$

$$Error2\% = \frac{0.4142 + 0.4}{0.4142}$$
$$= 3.42\%$$

$$Average\ Error\% = \frac{0.6 + 3.42}{2} = \mathbf{2.0\%}$$



$$x^3 - 15.2x - 13.2 = 0$$

$$Error1\% = \frac{-3.357 + 3.2}{-3.357} = 4.7\%$$

$$Error2\% = \frac{-0.919 + 0.8}{0.919} = 12.9\%$$

$$Error3\% = \frac{4.276 - 4.1}{4.276} = 4.1\%$$

$$Average\ Error\% = \frac{4.7 + 12.9 + 4.1}{3} = 7.2\%$$

$$x1 = -3.357$$
$$x2 = -0.919$$
$$x3 = +4.276$$

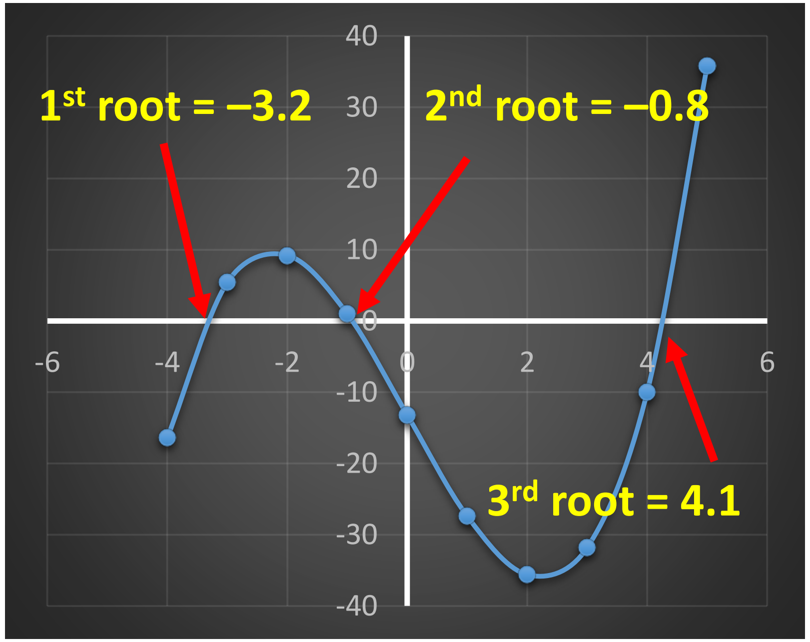
$$x^3 = 15.2x + 13.2$$

$$Error1\% = \frac{-3.357 + 3.2}{-3.357} = 4.7\%$$

$$Error2\% = \frac{-0.919 + 0.8}{0.919} = 12.9\%$$

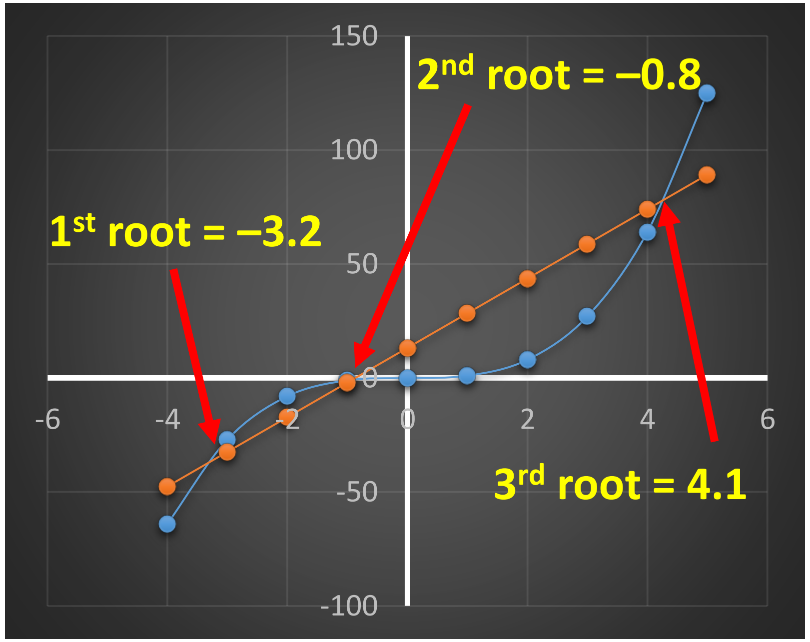
$$Error3\% = \frac{4.276 - 4.1}{4.276} = 4.1\%$$

$$Average\ Error\% = \frac{4.7 + 12.9 + 4.1}{3} = 7.2\%$$



x	y
5	35.8
4	-10
3	-31.8
2	-35.6
1	-27.4
0	-13.2
-1	1
-2	9.2
-3	5.4
-4	-16.4

x	y1	y2
5	125	89.2
4	64	74
3	27	58.8
2	8	43.6
1	1	28.4
0	0	13.2
-1	-1	-2
-2	-8	-17.2
-3	-27	-32.4
-4	-64	-47.6



Example: Find the location of the root of the equation $x.lnx = 1$

$$Error1\% = \frac{1.763 - 1.8}{1.763} = \mathbf{2.1\%}$$

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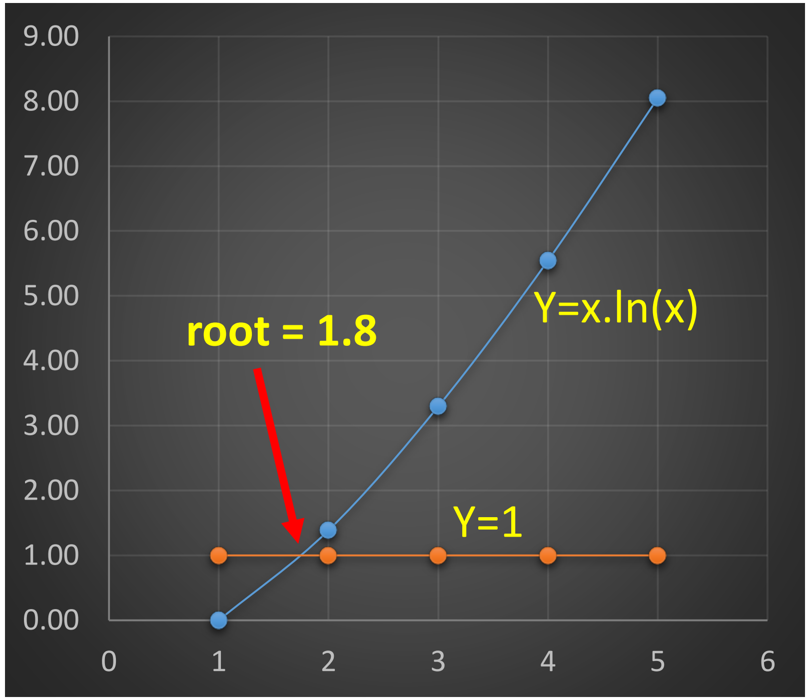
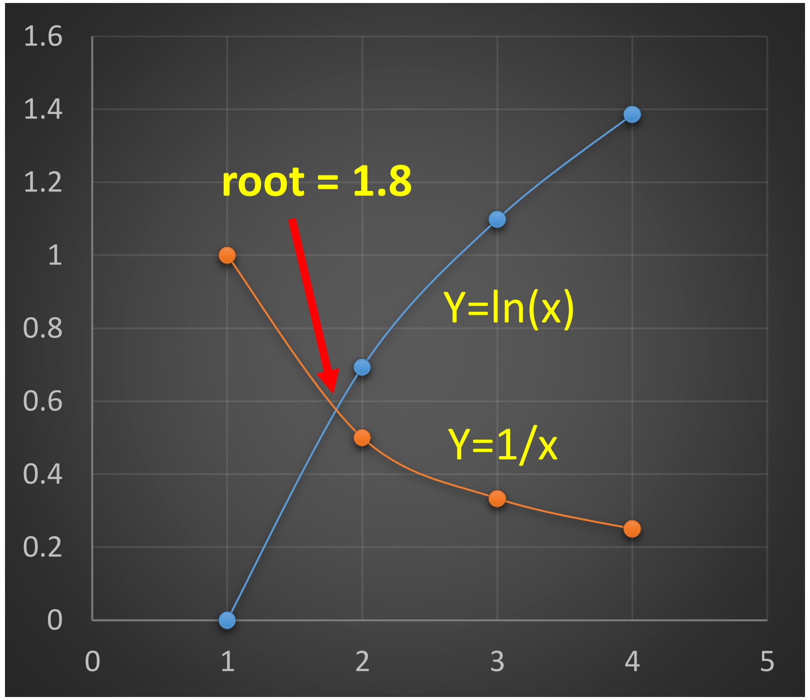
$$x = 1.763$$



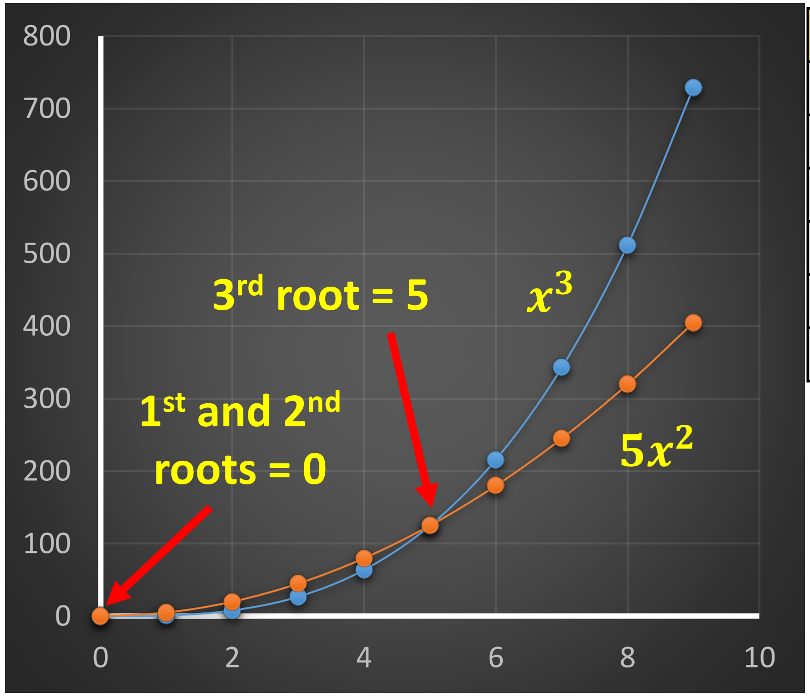
x	y1	y2
5	8.05	1.00
4	5.55	1.00
3	3.30	1.00
2	1.39	1.00
1	0.00	1.00

x	y1	y2
5	1.61	0.20
4	1.39	0.25
3	1.10	0.33
2	0.69	0.50
1	0.00	1.00



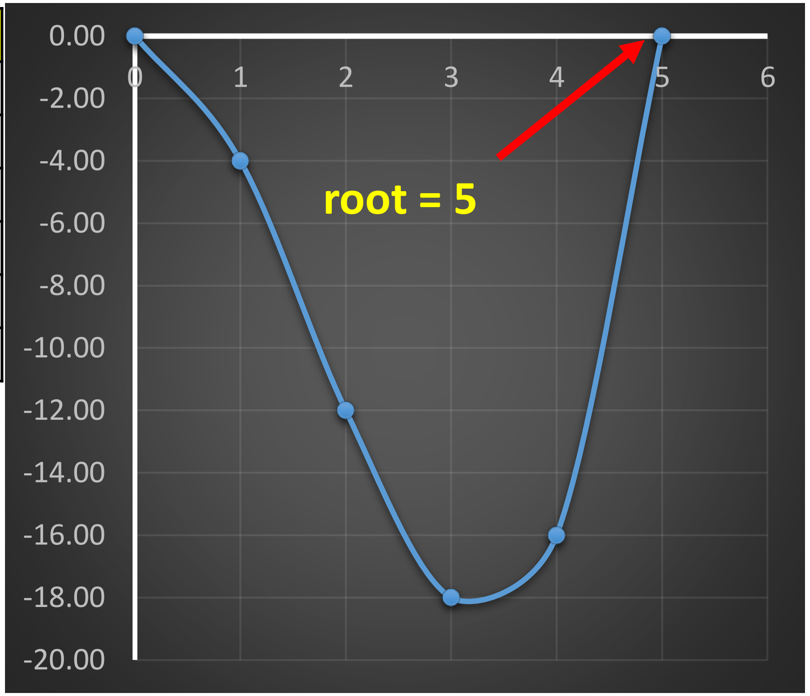
$$x^3 = 5x^2$$

$$x^3 - 5x^2 = 0$$



x	y1	y2
5	125.00	125.00
4	64.00	80.00
3	27.00	45.00
2	8.00	20.00
1	1.00	5.00
0	0.00	0.00

x	y1
5	0.00
4	-16.00
3	-18.00
2	-12.00
1	-4.00
0	0.00

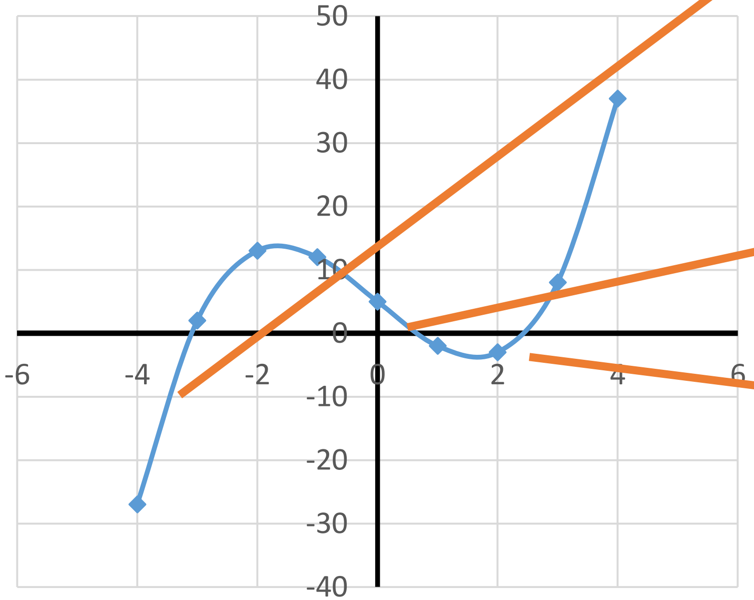


2. Tabulated Method:

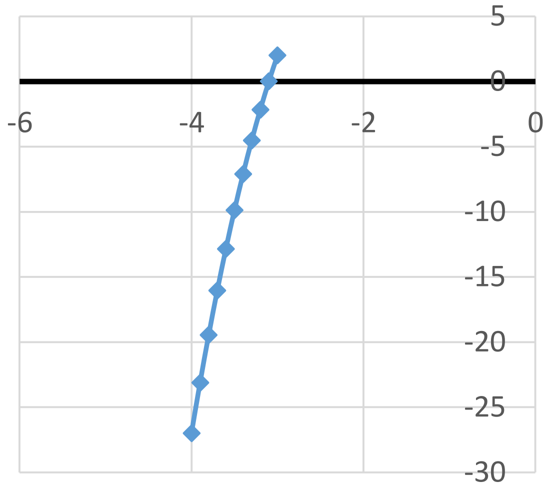
1. In the tabulation method, a table of values of f (x) is made for values of x in a particular range.
2. Then, we look for the **change in sign** n the values of f (x) for two consecutive values of x.
3. We conclude that a **real root lies between** these values of x. This is true if we make use of the following theorem on continuous functions.

Consider for example, the equation $f(x) = x^3 - 8x + 5 = 0$. Constructing the following table of x and f (x),

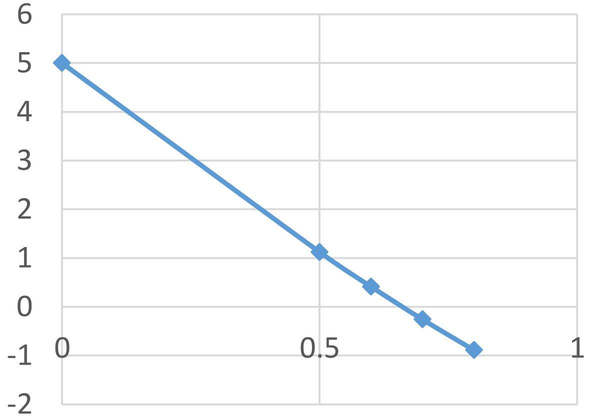
-4	-27
-3	**2**
-2	13
-1	12
0	5
1	-2
2	-3
3	8
4	37



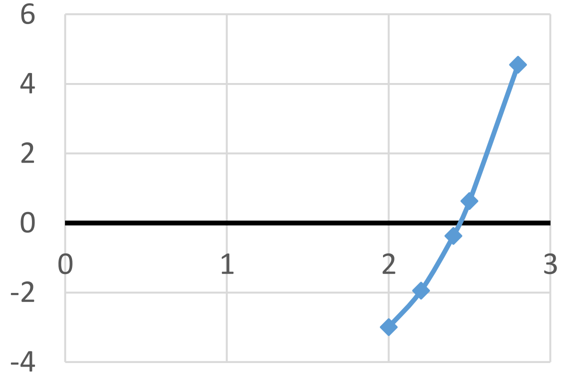
-4	-27
-3.3	-4.537
-3.2	-2.168
-3.1	0.009
-3	2



0	5
0.5	1.125
0.6	0.416
0.7	-0.257
0.8	-0.888



2	-3
2.2	-1.952
2.4	-0.376
2.5	0.625
2.8	4.552



Error Assessment and Reduction

$f(x) = x^3 - 8x + 5 = 0.$

-4	**-27**
-3	**2**
-2	13
-1	12
0	5
1	-2
2	-3
3	8
4	37

$x1 = -3.1004$
$x2 = \quad 0.6611$
$x3 = \quad 2.4393$

Average Error = 21.4%

Average Error = 2.5%

Residual ERROR

-4	**-27**
-3.3	**-4.537**
-3.2	**-2.168**
-3.1	**0.009**
-3	**2**

Error = 0.9%

$$Error1\% = \frac{-3.1004 + 3.1}{3.1004} = 0.13\%$$

Error = 0.9%

0	**5**
0.5	**1.125**
0.6	**0.416**
0.7	**-0.257**
0.8	**-0.888**

Error = 25.7%

0.6	0.416
0.61	0.347
0.63	0.210
0.65	0.075
0.67	-0.059

$$Error2\% = \frac{0.6616 - 0.67}{0.6616} = 0.12\%$$

Error = 5.9%

2	**-3**
2.2	**-1.952**
2.4	**-0.376**
2.5	**0.625**
2.8	**4.552**

Error = %37.6

2.4	-0.376
2.41	-0.282
2.42	-0.188
2.44	0.007
2.45	0.106

$$Error3\% = \frac{2.4393+2.4}{2.4393} = \mathbf{0.3\%}$$

Error = 0.7%