Subject Name: Numerical analysis

3^{rd} Class, Second Semester

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Lecture No. 3

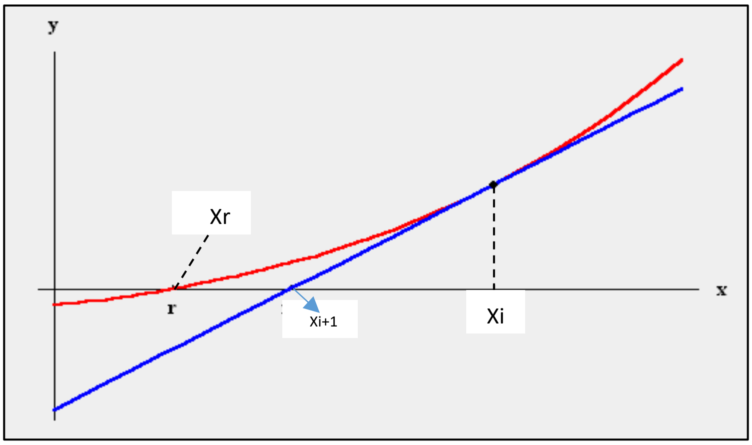
Lecture Title: Methods for Finding the Roots - Approximate (One point methods)

4- Newton-Raphson Technique

The Newton-Raphson method is one of the most widely used methods for root finding. It can be easily generalized to the problem of finding solutions of a system of non-linear equations, which is referred to as Newton's technique. Moreover, it can be shown that the technique is quadratically convergent as we approach the root.

Unlike the bisection and false position methods, the Newton-Raphson (N-R) technique requires only one initial value x_0, which we will refer to as the *initial guess* for the root.

The secant method can also be derived from geometry, as shown in Figure



$$\acute{f}(xi) = \frac{f_{(x+1)} - f_{(xi)}}{x_{i+1} - x_i}$$

$$f_{(x+1)} = 0$$

$$\acute{f}(xi) = \frac{-f_{(xi)}}{x_{i+1} - x_i}$$

$$-f_{(xi)} * \frac{1}{f'(xi)} = \frac{x_{i+1} - x_i}{-f_{(xi)}} * -f_{(xi)}$$

$$x_{i+1} = x_i - \frac{f_{(xi)}}{\acute{f}(xi)}$$

Example 1/ Find the root of the function by newton Raphson method

$$F(x) = x^2 - 5x + 6$$

$$\acute{f}(xi) = 2x - 5$$

Using $X_0=1$, ε_s %= 1%

Sol/

X_i	$F(x_i)$	$\acute{f}(xi)$	X_{i+1}	$\varepsilon_a\%$
1	2	-3	1.66667	40%
1.66667	0.4444	-1.6666	1.9334	13.79
1.9334	0.0711	-1.13332	1.99608	3.14
1.99608	0.00394	-1.00784	1.99998	0.195

Then the Root of the function when $\varepsilon_a\% < \varepsilon_s\%$ =1.99998

Example 1/ Find the root of the function by newton Raphson method

$$F(x) = \frac{1}{x} + 1$$

Using X_0= -0.5 , ε_s %= 0.5%

Sol/

X_i	$F(x_i)$	$\acute{f}(xi)$	X_{i+1}	$\varepsilon_a\%$
-0.5	-1	-4	0.75	33
0.75	-0.333	-1.77	-0.937	19
-0.937	-0.067	-1.137	-0.997	6
-0.997	-0.003	-1.006	-1	0.3

Then the Root of the function when $\varepsilon_a\% < \varepsilon_s\%$ =-1

H.w.

1- Find the root of the function by Newton Raphson method

$$F(x) = x^3 - x^2 - 10x - 8$$

Using X_0=3.75 , ε_s %= 0.01%

5- **FIXED POINT ITERATION METHOD**

Fixed point : A point, say, **s** is called a fixed point if it satisfies the equation **x = g(x)**.

Fixed point Iteration : The transcendental equation **f(x) = 0** can be converted algebraically into the form **x = g(x)** and then using the iterative scheme with the recursive relation

$$x_{i+1} = g(x_i), \qquad i = 0, 1, 2, \ldots,$$

with some initial guess x_0 is called the fixed point iterative scheme.

Algorithm - Fixed Point Iteration Scheme

Given an equation f(x) = 0
Convert f(x) = 0 into the form x = g(x)
Let the initial guess be x_0
Do
    $x_{i+1} = g(x_i)$
while (none of the convergence criterion C1 or C2 is met)

- C1. Fixing apriori the total number of iterations **N** .
- C2. By testing the condition $| x_{i+1} - g(x_i) |$ (where **i** is the iteration number) less than some tolerance limit, say epsilon, fixed apriority.

Example 1/ Find the root of the function by fixed point method

$$F(x) = e^{-x} - x$$

Using X_0=0.5 , ε_s %= 5%

$$F(x) = e^{-x} - x$$

X= e^{-x} …………..(1)

e^{-x}= x …………(2)

-x= ln x

X=-ln x …………(2) ….

Sol/

X_i	$F(x_i)$	ε_a%
0.5	0.60653	17.56
0.60653	0.54523	11.24
0.54523	0.5797	5.94
0.5797	0.56007	3.5

Then the Root of the function when $\varepsilon_a\% < \varepsilon_s\%$ =0.56007

Example 2/ Find the root of the function by fixed point method

$$F(x) = e^x - 2x - 1$$

Using X_0=1 , ε_s %= 3%

$-e^x = -2x - 1) * -1$

$$e^x = 2x + 1$$

Sol/

$x = \frac{e^x-1}{2}$ *The relative approx. error will be divergence……1*

$x = \ln(2x+1)$ *The relative approx. error will be convergence ……2*

So will be use $g_2(x)$

X_i	$F(x_i)$	ε_a%
1	1.0986	8.97
1.0986	1.1622	5.47
1.1622	1.2012	3.25
1.2012	1.2244	2.32

Then the Root of the function when $\varepsilon_a\% < \varepsilon_s\%$ =1.2244

H.w / Find the root of the function by fixed point method

$$F(x) = -x^2 + 1.8x + 2.5$$

Using $X_0=5$, $\varepsilon_s\% = 0.05\%$