Failure theories

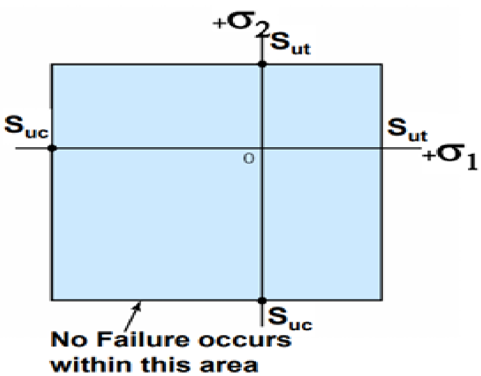
1. Maximum Principal Stress Theory (Rankine's theory):

- The theory states that the failure of the mechanical component subjected to bi-axial or tri-axial stresses occurs when the maximum principal stress reaches the yield or ultimate Strength of the material.

- If σ1 , σ2 and σ3 are the three principal stresses at a point on the component and **σ1 > σ2 > σ3**

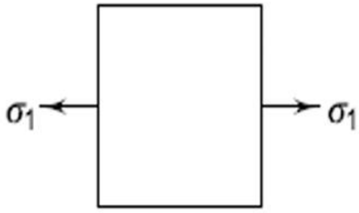
$$\sigma_{yt} < \sigma_1 < \sigma_{yc}$$

Failure occurs,
when *greatest tensile stress*
exceeds *uniaxial tensile strength*.

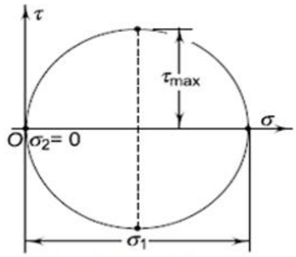


2.Maximum shear stress theory (Tresca criterion):

The theory states that the failure of a mechanical component subjected to bi-axial or tri-axial stresses occurs when the maximum shear stress at any point in the component becomes equal to the maximum shear stress in the standard specimen of the tension test, when yielding starts.



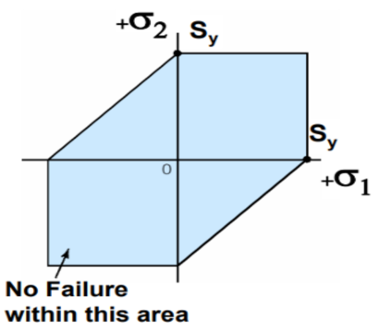
Stresses in Simple Tension Test



Mohr's Circle for Stresses

$$\tau_{max} = \frac{\sigma_{yield\ point}}{2n} = \frac{\sigma_2 - \sigma_1}{2}\ [2D]$$

$$\tau_{max} = \frac{\sigma_{yield\ point}}{2n} = \frac{\sigma_1 - \sigma_3}{2}\ [3D]$$



Failure occurs,
when *maximum shear stress*
exceeds *shear* strength in *uniaxial tension test.*

3. Distortion energy theory (von Mises theory):

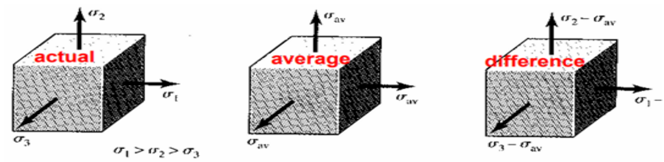
Failure occurs,
when *distortion-energy* in unit volume (arbitrary load condition)
equals *distortion-energy* in same volume for uniaxial yielding.

$$u_\sigma = \frac{\varepsilon_1 \cdot \sigma_1}{2} + \frac{\varepsilon_2 \cdot \sigma_2}{2} + \frac{\varepsilon_3 \cdot \sigma_3}{2}$$

with 3-D stress-strain rel.

$$\varepsilon_1 = \frac{\delta_1}{E} - \frac{v\delta_2}{E} - \frac{v\delta_3}{E}$$
$$\varepsilon_2 = \frac{\delta_2}{E} - \frac{v\delta_1}{E} - \frac{v\delta_3}{E}$$
$$\varepsilon_3 = \frac{\delta_3}{E} - \frac{v\delta_1}{E} - \frac{v\delta_2}{E}$$

$$u_\sigma = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2v(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$



with 3-D averaged stress: $\sigma_{av} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$

$$u_{av} = \frac{3\sigma_{av}^2}{2E}[1 - 2v]$$

Distortion energy $\quad u_d = u_\sigma - u_{av}$

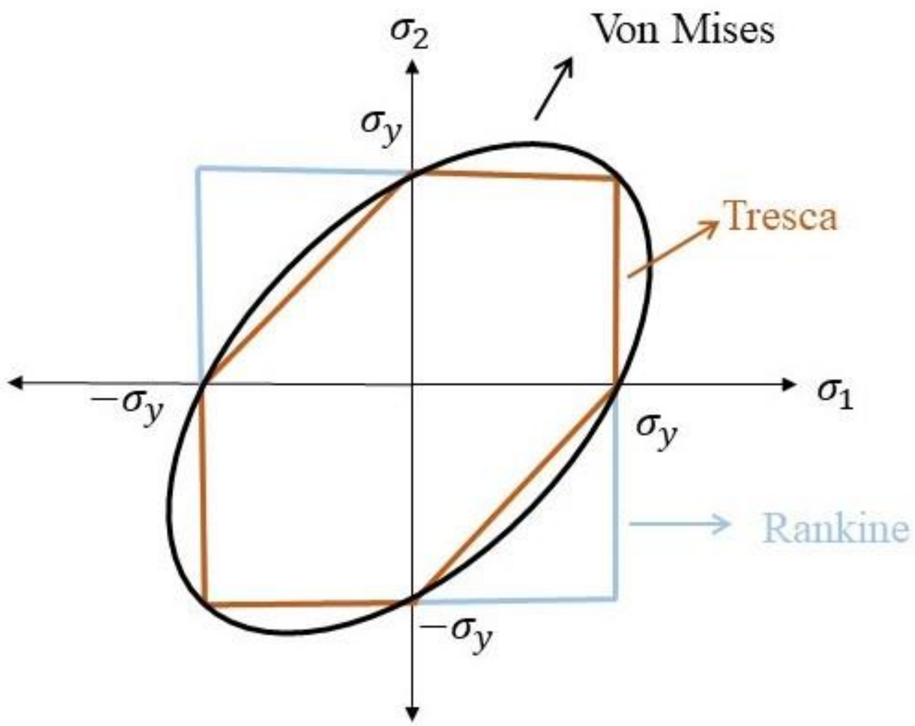
$$u_d = \frac{1+v}{3E}\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right]$$

$$u_d = \frac{1+v}{3E} S_y^2$$

$$S_y > \sigma_e = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

2D(Bi axial stress $\sigma_3 = 0$)

$$\sigma_{yield} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$


σ2
Von Mises
σy
Tresca
−σy
σ1
σy
Rankine
−σy