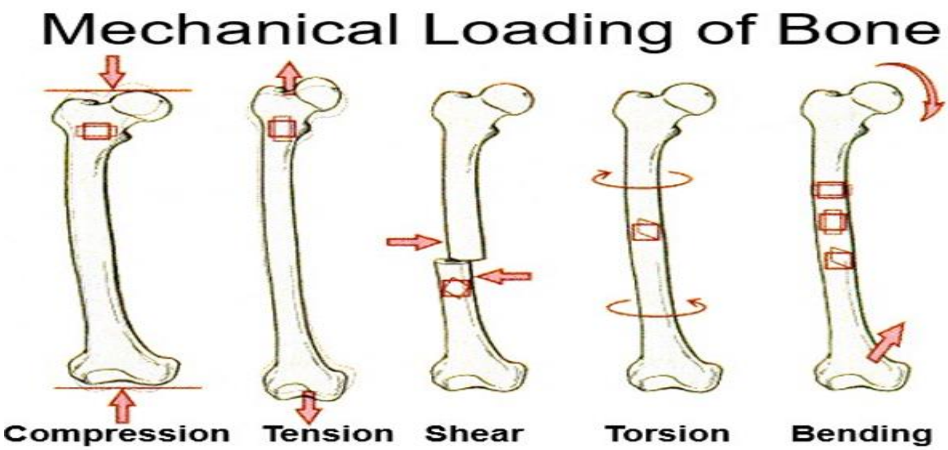
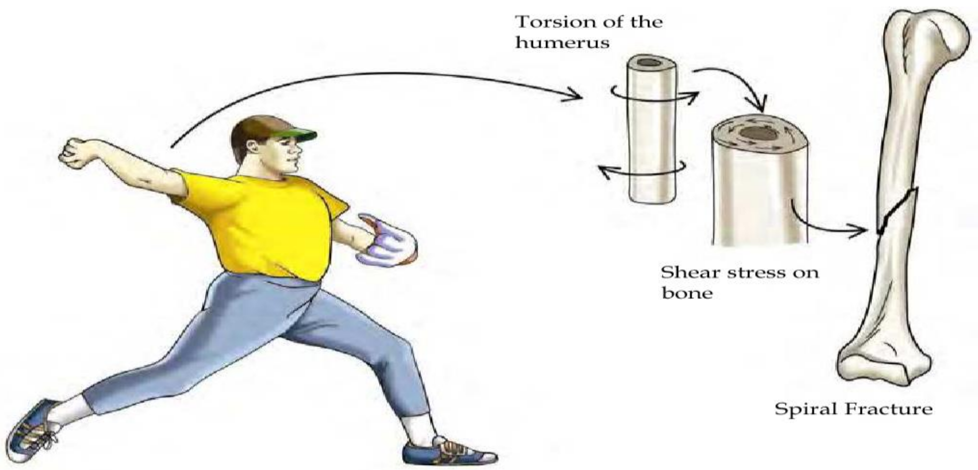
# Combined Loading

Examples of Combined Load in Biomechanics:



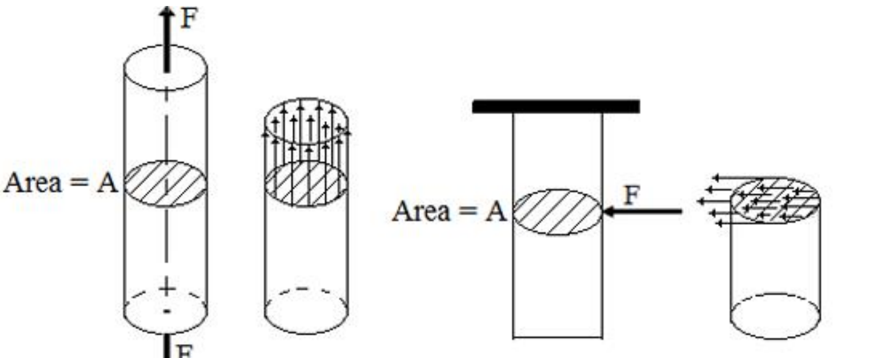




## 1. Combined Loading

Members or parts of any structure or machine are often subjected to more than one type of loading. The basic types of loadings and the corresponding stress formula and its distribution may be summarized as follows:

$$\textit{Axial loading} \rightarrow \sigma_{axial} = \frac{F}{A}$$

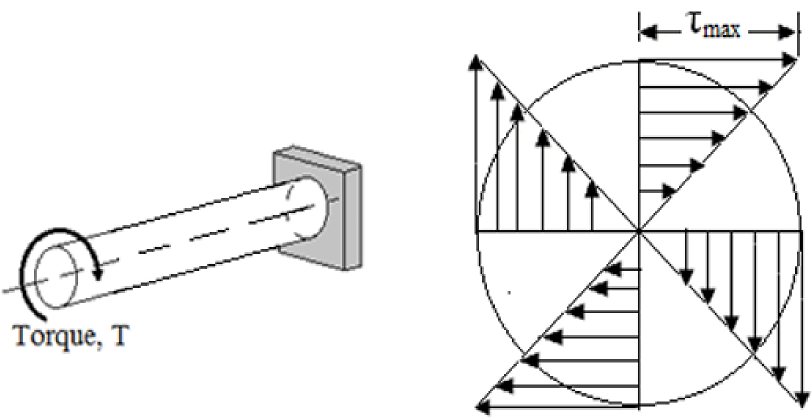
$$\textit{Direct shearing load} \rightarrow \tau = \frac{F}{A}$$




Normal stress (Tension)
Uniformly distributed stress

Direct shearing stress
Uniformly distributed stress

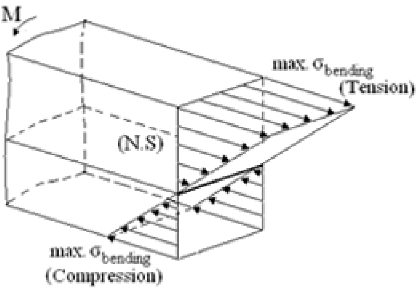
$$\textit{Torsional loading} \rightarrow \tau_{torsional} = \frac{T.r}{J} (\textit{for a circular section})$$



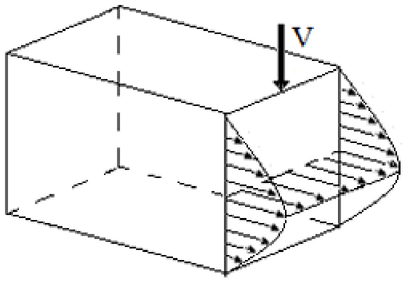
## *Torsional shearing stress*

$$-Flexural\ \ loading\ \rightarrow \sigma_{Flexural} = \frac{M.Y}{I}$$

$$-shearing\ force\ in\ beam\ \rightarrow \tau = \frac{VQ}{I.b}$$



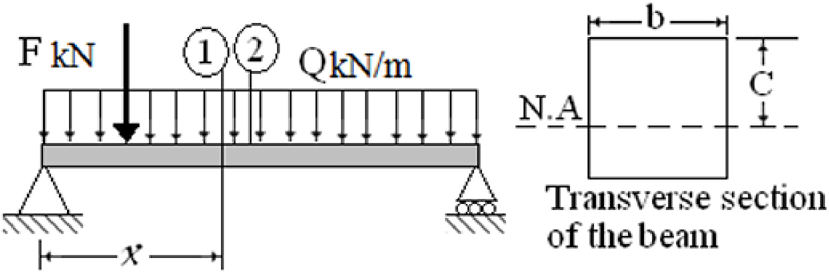
Bending stress



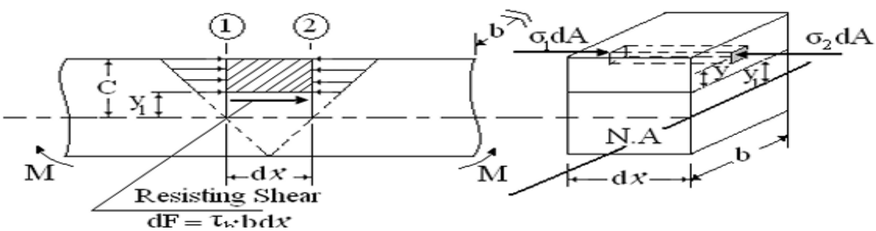
Shearing stress in beams

All these types of stresses may act simultaneously. The stress analyses of any member or parts subjected to such combined loadings begin by determining each stress separately. Then these stresses are combined at the required point to obtain the resultant stresses at that point. The values and the locations of the maximum and minimum stresses should be determined. There are many possible combinations of the loadings. Firstly, combinations of similar stresses: normal stresses or shearing stresses. The other combinations including any shearing stress with any normal stress require a preliminary discussion before they can be considered.

Formula for horizontal shearing stress: Consider a prismatic simply supported beam with a width of "b". The beam supports various concentrated and distributed loads as shown.



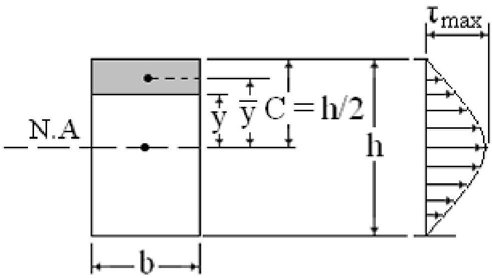

At a distance "x" from the left end, an element with a length of "dx" is detached. The element is locating between section "1" and section "2" and extending across the width of the beam from its upper fibers to a horizontal plane at a distance "y1" from the neutral axis as shown.




Detached element from the loaded beam

## Application of the derived formula to Rectangular sections:

$$\tau = \frac{V.Q}{I_{N.A} \cdot b}$$




$$I_{N.A} = \frac{bh^3}{12}$$

$$Q = \bar{A}\bar{y}$$

$$\bar{y} = \frac{1}{2}\left(\frac{h}{2} - y\right) + y = \frac{h}{4} - \frac{y}{2} + y$$

$$= \frac{1}{2}\left(\frac{h}{2} + y\right)$$

or $$Q = b\left(\frac{h}{2} - y\right) * \frac{1}{2}\left(\frac{h}{2} + y\right) = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

$$\rightarrow \tau = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

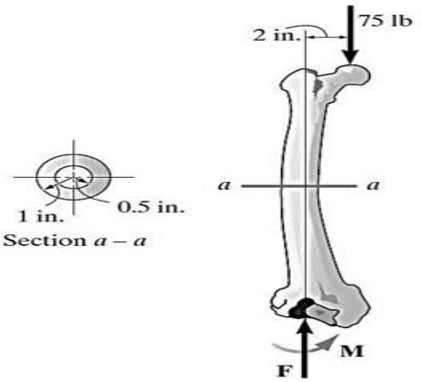
This shows that the shearing stress is distributed parabolically across the depth of the section and its maximum value is located at the neutral axis, when (y = 0).

So: $$\tau_{max} = \frac{6V}{bh^3}\left(\frac{h^2}{4} - 0\right) = \frac{3V}{2bh}$$

$$\boldsymbol{\tau_{max} = \frac{3V}{2A}}$$

## Combined direct normal stress and bending stress:

Example #1: If the cross section of the femur at section a-a can be approximated as a circular tube as shown, determine the maximum normal stress developed on the cross section at section a-a due to the load of 75 lb




Solution:

| | | |
|---|---|---|
| $+\uparrow \Sigma F_y = 0;$ | $N - 75 = 0$ | $N = 75\,\text{lb}$ |
| $\circlearrowleft + \Sigma M_O = 0;$ | $M - 75(2) = 0$ | $M = 150\,\text{lb}\cdot\text{in}$ |

**Section Properties:** The cross-sectional area and the moment of inertia about the centroidal axis of the femur's cross section are

$$A = \pi\left(1^2 - 0.5^2\right) = 0.75\pi\ \text{in}^2$$

$$I = \frac{\pi}{4}\left(1^4 - 0.5^4\right) = 0.234375\pi\ \text{in}^4$$

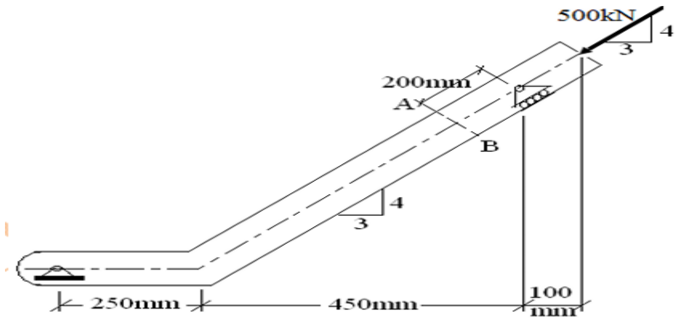
**Normal Stress:** The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress is in compression.

$$\sigma_{\max} = \frac{-75}{0.75\pi} - \frac{150(1)}{0.234375\pi} = -236\ \text{psi} = 236\ \text{psi (C)}$$

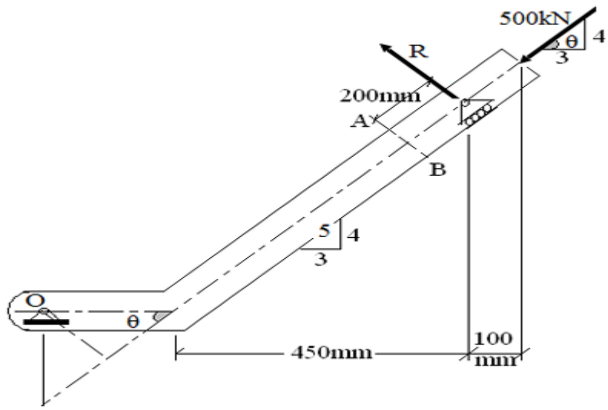
Example #2:The bent steel bar shown in the figure is 200mm square. Determine the normal stresses at A&B.




**Solution:**

$$\sum M_0 = 0; R(\frac{0.45}{\cos\theta} + 0.25\ \cos\theta) = 500 * 0.25\ \sin\theta$$

R=111.111 KN




$$M_{A-B} = 0.2R = 0.2\ * 111.111 = 22.22kN$$

Total stress at "A" and at "B" will be:

$$\sigma_{total} = \pm\sigma_{axial} \pm \sigma_{bending}$$

$$\sigma_{total} = \pm\frac{force}{area} \pm \frac{M.y}{I}$$

$$\sigma_A = -\frac{500 * 10^3}{0.2 * 0.2} - \frac{22.22 * 10^3 * \frac{0.2}{2}}{\frac{0.2 * 0.2^3}{12}} = -29.2\ MPa$$

$$\sigma_B = -\frac{500 * 10^3}{0.2 * 0.2} + \frac{500 * 10^3 * \frac{0.2}{2}}{\frac{0.2 * 0.2^3}{12}} = 4.2\ MPa$$