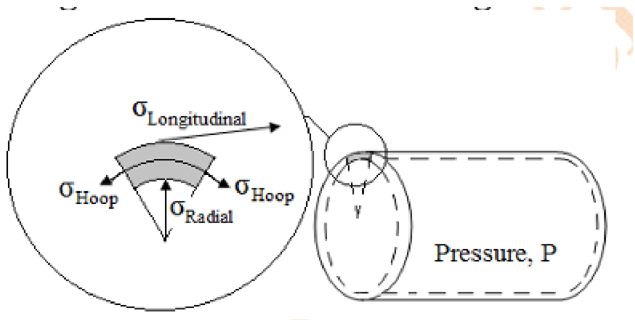
# 3.1Stresses in thin-walled cylinders and shells:

Thin-walled cylinders subjected to internal (or external) important application of in-plane pressure provide an diameter "d" to the normal stress. The ratio of cylinders cylinders thickness "t" is the criterion to classify the cylinders to thin-walled and thick- walled cylinder), so cylinders: for thin-walled

$$Diameter \geq 20 * Thickness$$

Consider a thin-walled closed cylinder shown in the pressure "P", inner diameter "d", wall figure with internal stresses will be set up thickness "t", and length "l". Three described in the wall of this cylinder. These stresses are by the enlarged element shown in the figure.



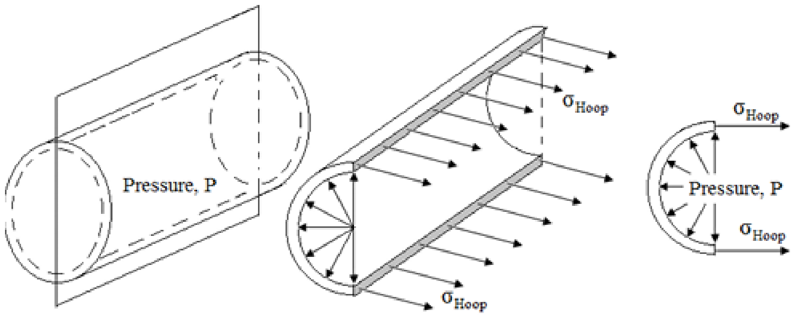

The stresses are:

1- Radial stress "$\sigma_R$" is very small, so neglected.

2. Hoop stress, "$\sigma_H$" (also called circumferential or tangential). It is a tensile stress acting along the circumference of the cylinder at all its length. Figure a, b, and c describe its

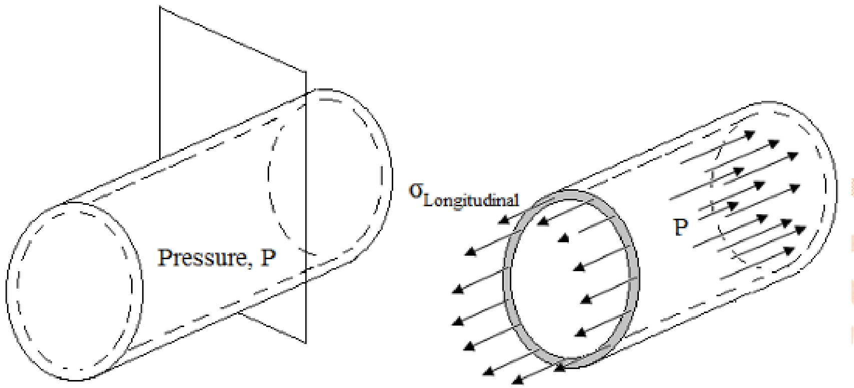
location. For equilibrium, the internal forces due to the hoop stress must equal the resultant force due to pressure, so:

$$\sum f_x = 0 \quad OR \quad \sigma_x(2 * l * t) = p(d * l), \rightarrow \sigma_H = \frac{pd}{2t}$$




(a) (b) (c)

2- Longitudinal stress " $\sigma_l$ ". It is a tensile stress acting along the length of the cylinder. Figure a, and b, describe the location of the longitudinal stress.




(a) (b)

For equilibrium the internal forces due to this stress must equal the resultant .

$$\sigma_l * (\pi d t) = P * \frac{\pi}{4} d^2$$

$$\rightarrow \boldsymbol{\sigma_l} = \frac{pd}{4t}$$

You can see that: $\boldsymbol{\sigma_H} = 2\boldsymbol{\sigma_l}$

**Example #1** Unpressurized vertical tank is filled with water (density = 1000kg/m3). The tank is cylindrical with an outer diameter of 3m, a wall thickness of 10mm, and a height of 8m. Determine the maximum normal stress induced in the wall of the tank.

Solution:- The hoop stress is the maximum normal stress in the wall of the tank, so:

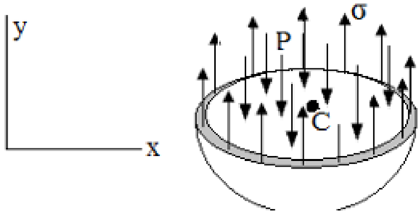
$$\boldsymbol{\sigma_{max}} = \boldsymbol{\sigma_H} = \frac{pd}{2t}$$

when: $P = \rho g h = 1000 * 9.81 * 8 = 78.48 * 10^3 N/m^2$

$$\rightarrow \sigma_H = \frac{78.48*10^3*(3-2*0.01)}{2*0.01} = 11.693 MPa$$

**Example #2** A spherical pressure vessel is 3m in diameter with a wall thichness of 12mm. Knowing that for the steel used the maximum allowable stress is 80MPa, determine the allowable pressure that can be used to the vessel.

**Solution:** As the vessel is spherical, so by symmetry, the stresses induced on any element in any direction at the wall of the vessel must be equal. This stress can determined by passing a section through the center of the vessel and considering the free body diagram as shown in the figure. For equilibrium you can state that the internal forces due to the stress must equal the resultant force due to pressure, so:




$$\sigma_l = \frac{pd}{4t} \qquad p = \frac{4*0.012*80*10^6}{(3-2*0.012)} = 1.29\ MPa$$