Work and heat (part2)

Polytropic Process

One common example of this second type of functional relationship is a process called a polytropic process, one in which

$$PV^n \text{ constant}$$

throughout the process. The exponent (n) may possibly be any value from -∞ to +∞, depending on the particular process. For this type of process, we can integrate Eq. 4.4 as follows:

$$PV^n = constant = \boldsymbol{P_1V_1^n = P_2V_2^n}$$

$$P = \frac{constant}{V^n} = \frac{P_1V_1^n}{V^n} = \frac{P_2V_2^n}{V^n}$$

$$W = \int_1^2 P\,dV = constant\int_1^2 \frac{dV}{V^n} = constant\left(\frac{V^{-n+1}}{-n+1}\right)\Big|_1^2$$

$$= \frac{\boldsymbol{P_2V_2 - P_1V_1}}{\boldsymbol{1-n}}(\boldsymbol{General})$$

Note that the resulting Eq. is valid for any exponent n, except n = 1. Where **n = 1**, is the **isothermal expansion** $P_1V_1 = P_2V_2$

$$\int_1^2 \boldsymbol{P\,dV} = P_1V_1\int_1^2 \frac{dV}{V^n} = \boldsymbol{P_1V_1 ln\frac{V_2}{V_1}}$$

Notes:

1.the work=0 when volume constant

$$\boldsymbol{2.W = \int_1^2 P\,dV = P(V_2 - V_1) When\ pressure\ constant\ n = 0}$$

$$\boldsymbol{3.W = P_1V_1 ln\frac{V_2}{V_1}\ When\ temp.of\ gas\ is\ constant(\ isothermal) n = 1}$$

$$4.\ W = \frac{P_2V_2 - P_1V_1}{1-n} \quad When\ PV^n = constant (P_1V_1^n = P_2V_2^n)$$

Example:

Consider as a system the gas in the cylinder shown in Fig. 4.7; the cylinder is fitted with a piston on which a number of small weights are placed. The initial pressure is 200 kPa, and the initial volume of the gas is 0.04 m^3.

a. Let a Bunsen burner be placed under the cylinder, and let the volume of the gas increase to 0.1 m^3 while the pressure remains constant. Calculate the work done by the system during this process.

b. Consider the same system and initial conditions, but at the same time as the Bunsen burner is under the cylinder and the piston is rising, let weights be removed from the piston at such a rate that, during the process, the temperature of the gas remains constant.

c. Consider the same system, but during the heat transfer let the weights be removed at such a rate that the expression $PV^{1.3}$ = constant describes the relation between pressure and volume during the process. Again the final volume is 0.1 m^3. Calculate the work.

d. Consider the system and initial state given in the first three examples, but let the piston be held by a pin so that the volume remains constant. In addition, let heat be transferred from the system until the pressure drops to 100 kPa. Calculate the work.

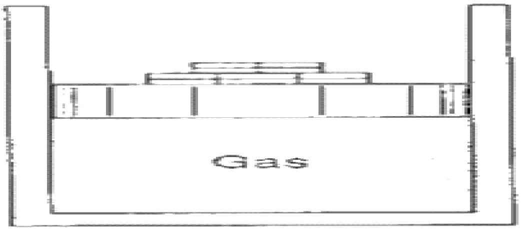

Fig.4.7

Solution :

a)

kJ

b)

$$ {}_1W_2 = \int_1^2 P\,dV = P_1V_1 \ln\frac{V_2}{V_1} $$

$$ = 200\text{ kPa} \times 0.04\text{ m}^3 \times \ln\frac{0.10}{0.04} = 7.33\text{ kJ} $$

c)

$$ P_2 = 200\left(\frac{0.04}{0.10}\right)^{1.3} = 60.77\text{ kPa} $$

$$ {}_1W_2 = \int_1^2 P\,dV = \frac{P_2V_2 - P_1V_1}{1-1.3} = \frac{60.77\times 0.1 - 200\times 0.04}{1-1.3}\text{ kPa m}^3 $$

$$ = 6.41\text{ kJ} $$

d)

Since $\delta W = P\,dV$ for a quasi-equilibrium process, the work is zero, because there is no change in volume.

4.DEFINITION OF HEAT

The thermodynamic definition of heat is somewhat different from the everyday under-standing of the word. It is essential to understand clearly the definition of heat given here, because it plays a part in so many thermodynamic problems.

If a block of hot copper is placed in a beaker of cold water, we know from experi-ence that the block of copper cools down and the water warms up until the copper and water reach the same temperature. What causes this decrease in the temperature of the copper and the increase in the temperature of the water? We say that it is the result of the transfer of energy from the copper block to the water. It is out of such a transfer of energy that we arrive at a definition of heat.

degrees a change from state I to state 2 depends on the path that the system follows during the change of state. Since heat is an inexact differential, the differential is written $\boldsymbol{\delta Q}$. On integrating, we write

$$1Q_2 = \int_1^2 \delta Q$$

In words, $_1Q_2$ is the heat transferred during the given process between states 1 and 2. **The rate at which heat is transferred to a system is designated by symbol $\dot{Q}$.**

$$\dot{Q} = \frac{\dot{Q}}{dt}$$

It is also convenient to speak **of the heat transfer per unit mass of the system**, q, often termed **specific heat transfer**, which is defined as n

$$q = \frac{Q^{\cdot}}{m}$$

Sign convention: work done by a system is positive, and the work done on a system is negative. Heat transfer to the system is positive and from a system will be negative.

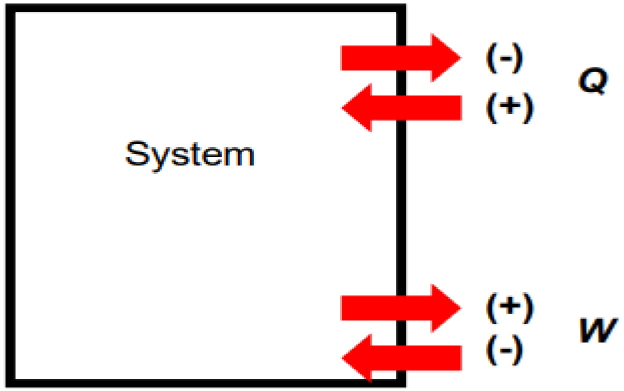

Fig. Sign convention for heat and work.