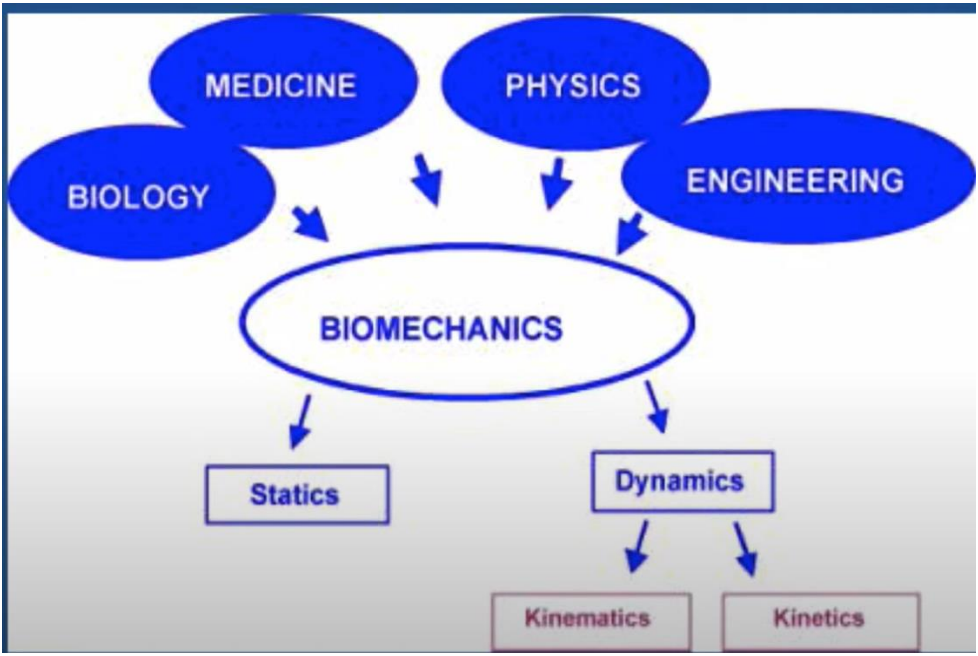
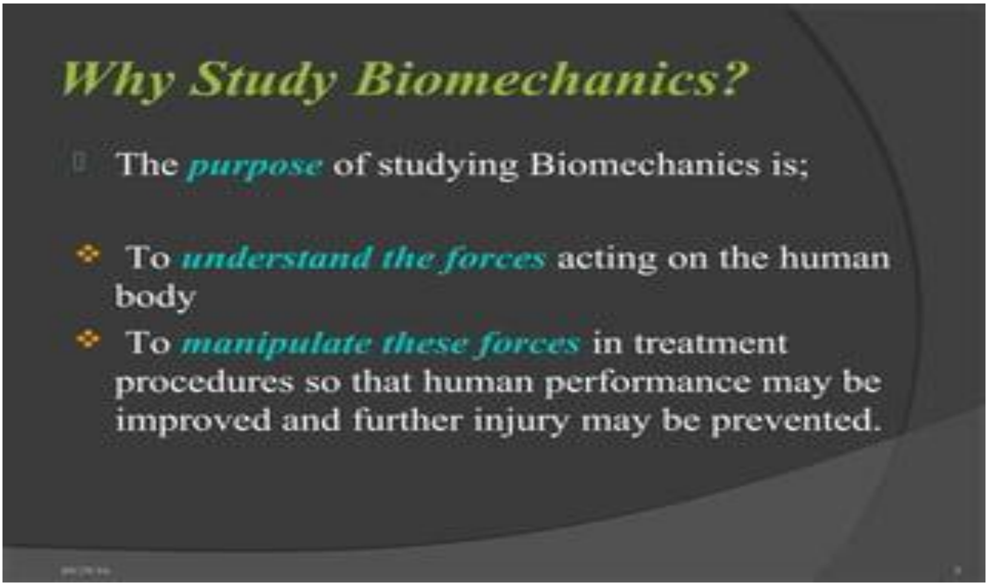
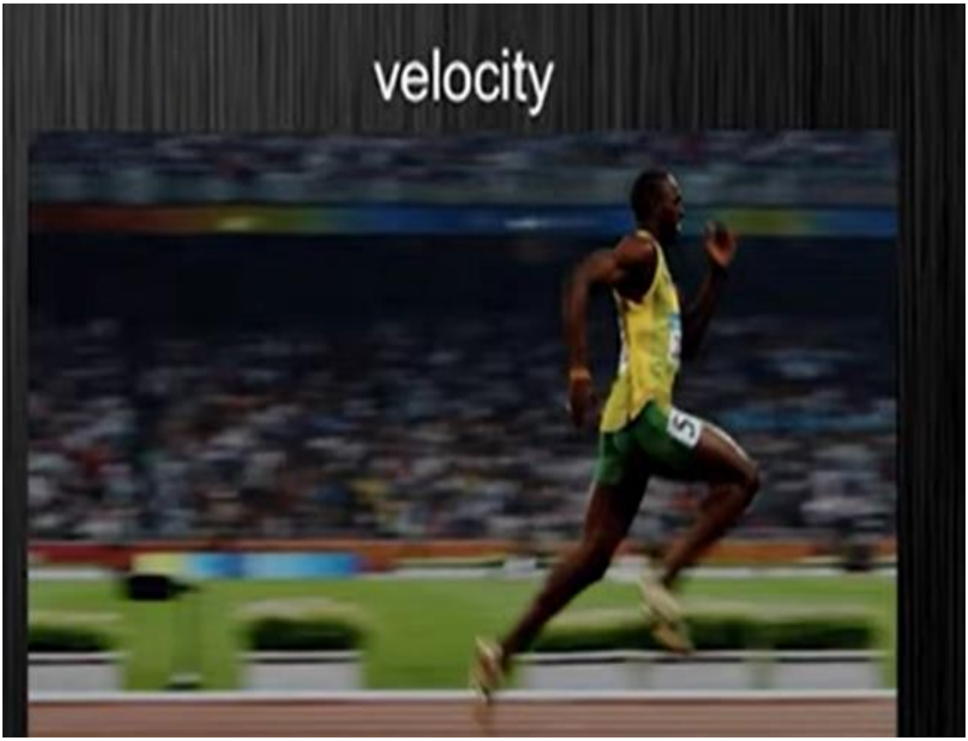
Biomechanics.

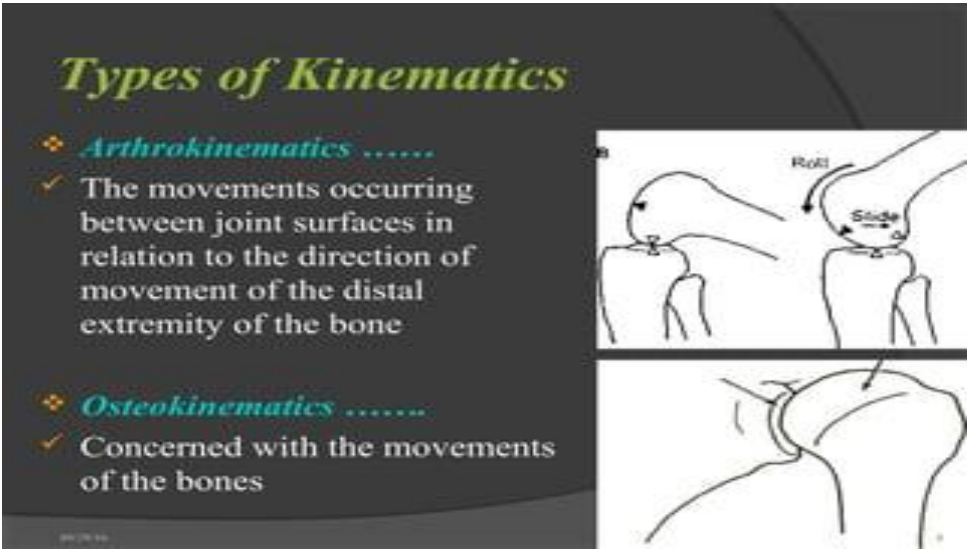




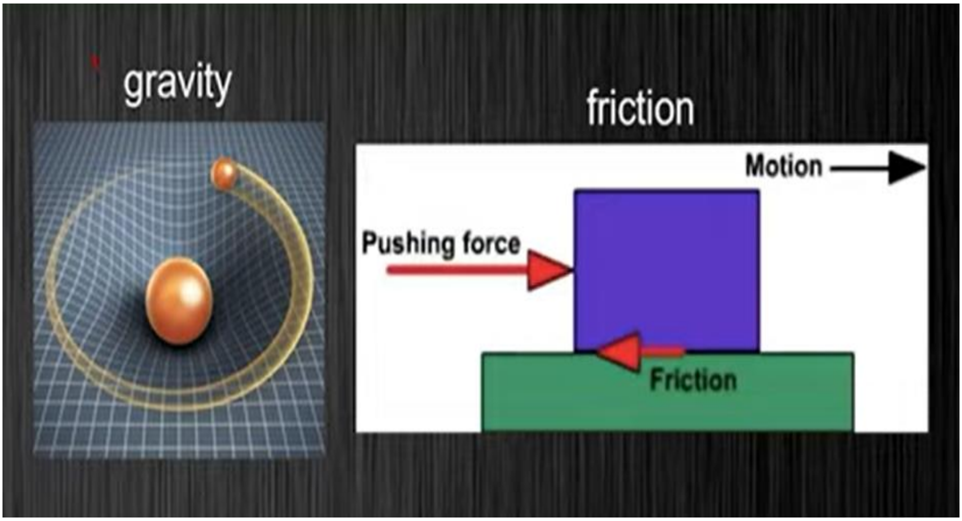
1.Sub-branches of Biomechanics:-

- **statics:** study of systems in constant motion without acceleration (including zero motion)
- **dynamics:** study of systems subject to acceleration.
- **kinematics:** study of the appearance or description of motion.





• kinetics: study of forces causing motion



2.Reference planes:

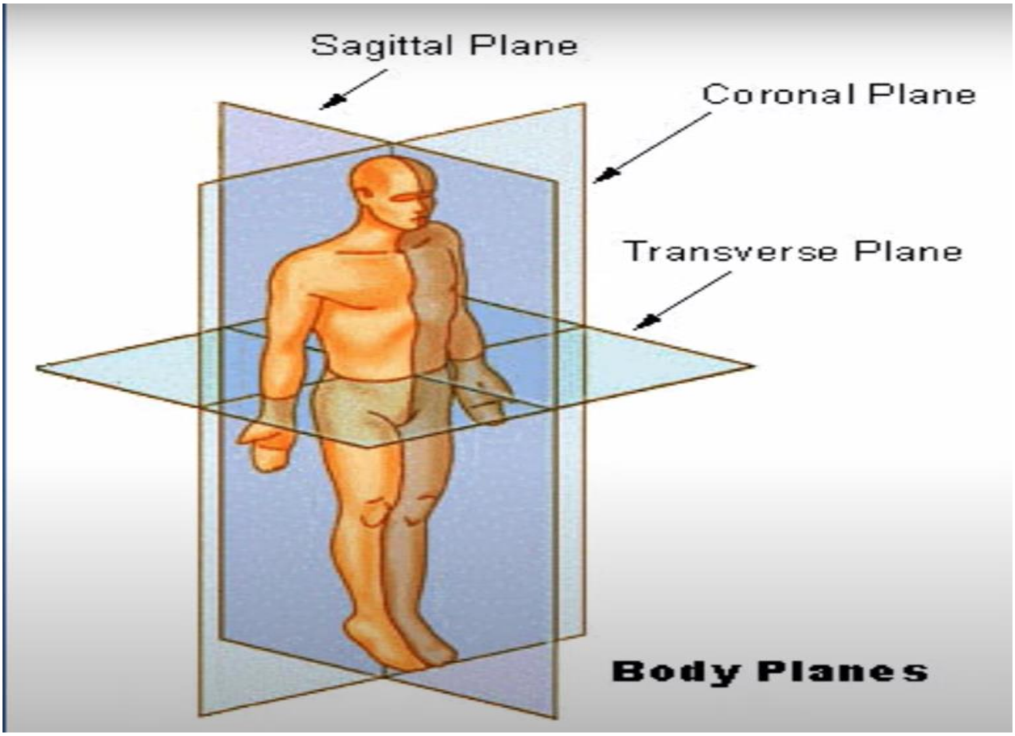
A three-dimensional analysis is necessary for a complete representation of human motion. Such analyses require a coordinate system, which is typically composed of anatomically aligned axes:

a **two-** It is often convenient to consider only **dimensional, or planar, analysis, in which only two of the three axes are considered. In the human body, there are three perpendicular anatomical planes, which are referred to** as the cardinal planes.

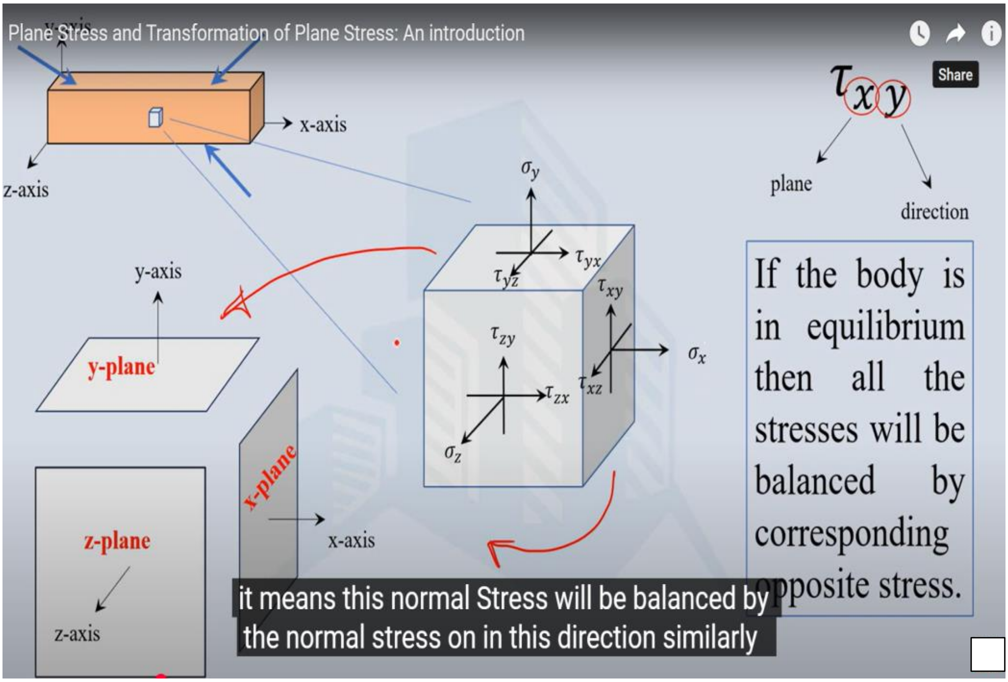
• **sagittal plane** - in which forward and backward movements occur

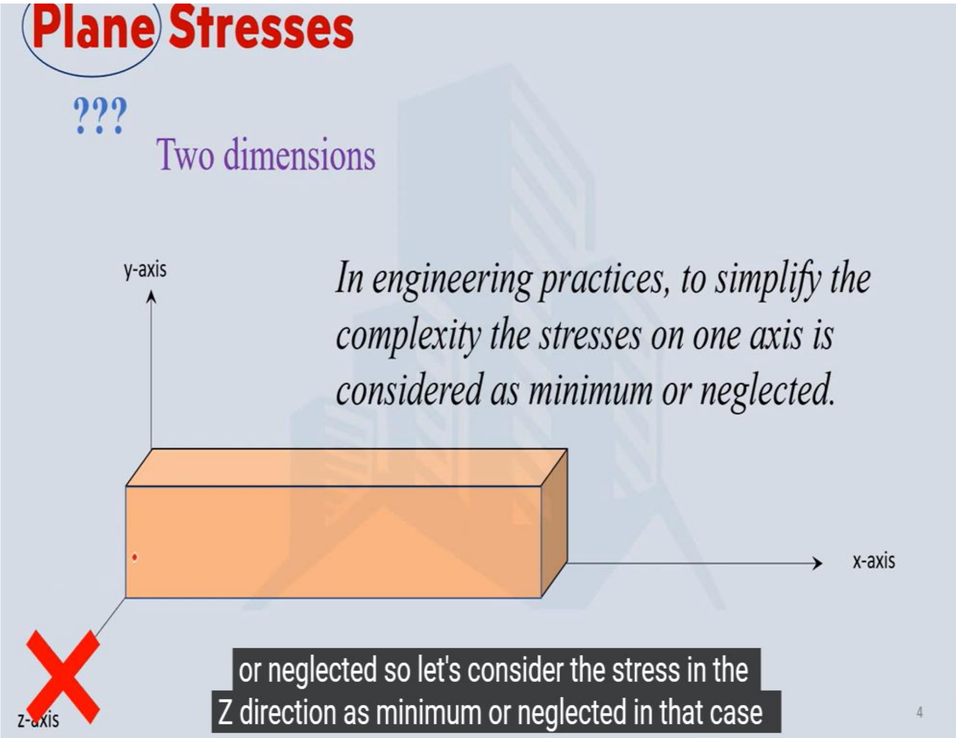
• **frontal plane** - in which lateral movements occur

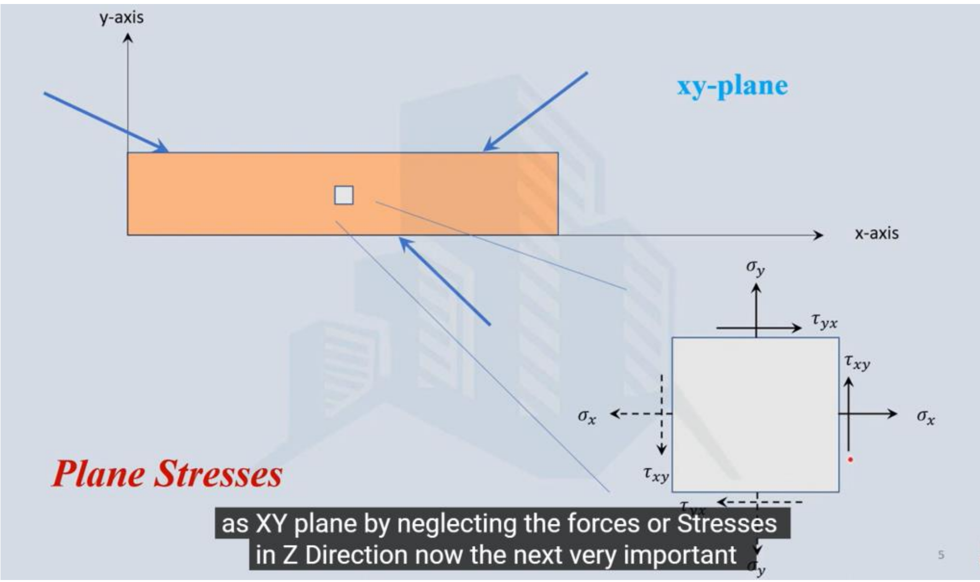
•**transverse plane** - in which rotational

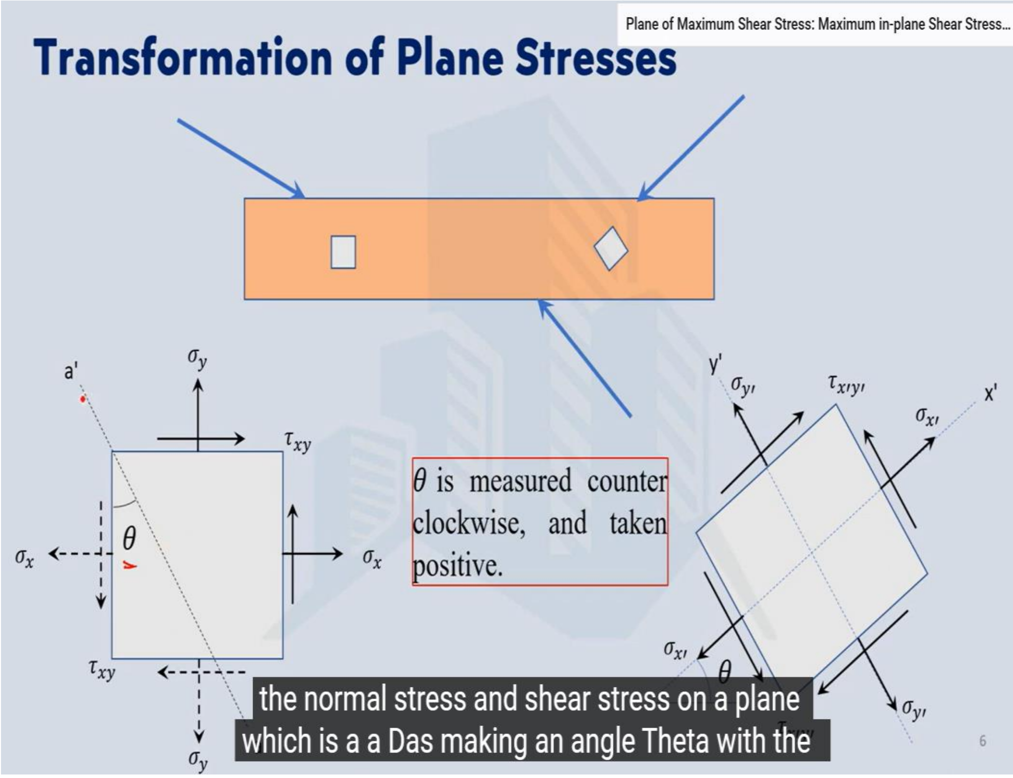

Sagittal Plane
Coronal Plane
Transverse Plane
Body Planes

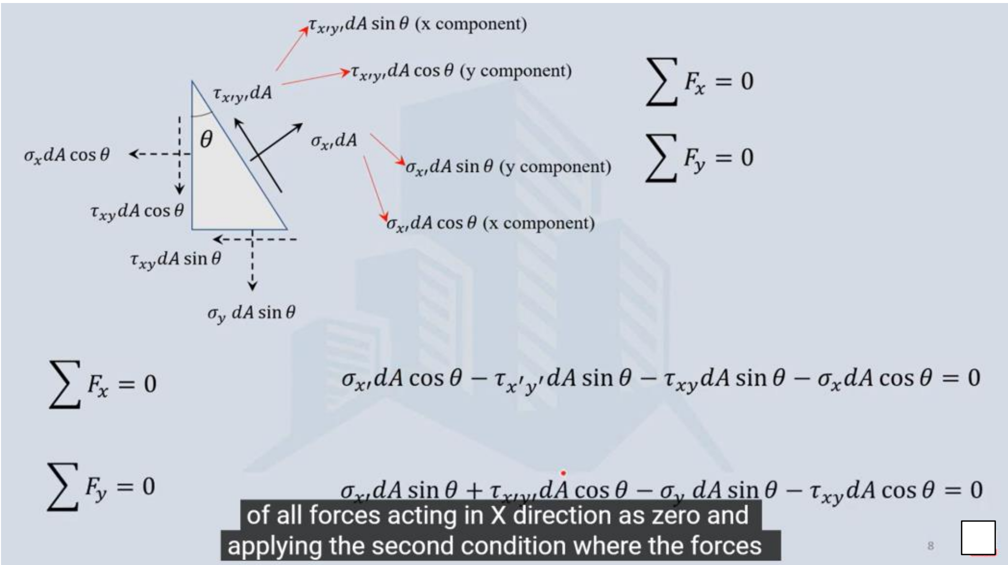
3.Plane Stress, Stress Transformations and Principal Stresses.

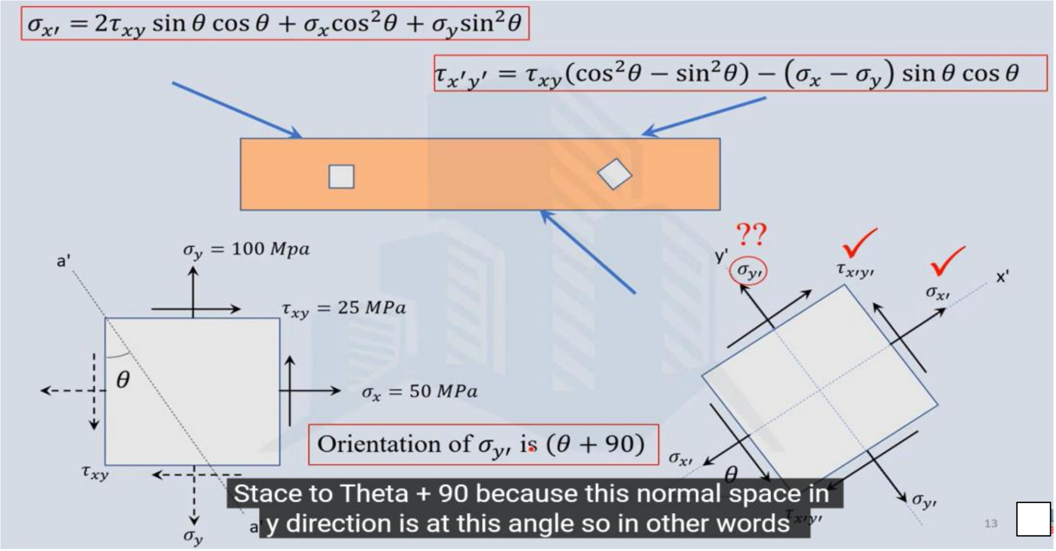



Plane Stresses
???
Two dimensions
y-axis
In engineering practices, to simplify the complexity the stresses on one axis is considered as minimum or neglected.
x-axis
z-axis
or neglected so let's consider the stress in the
Z direction as minimum or neglected in that case
4


y-axis
xy-plane
x-axis
σ_y
τ_{yx}
τ_{xy}
σ_x
σ_x
τ_{xy}
Plane Stresses
as XY plane by neglecting the forces or Stresses
in Z Direction now the next very important
σ_y
5

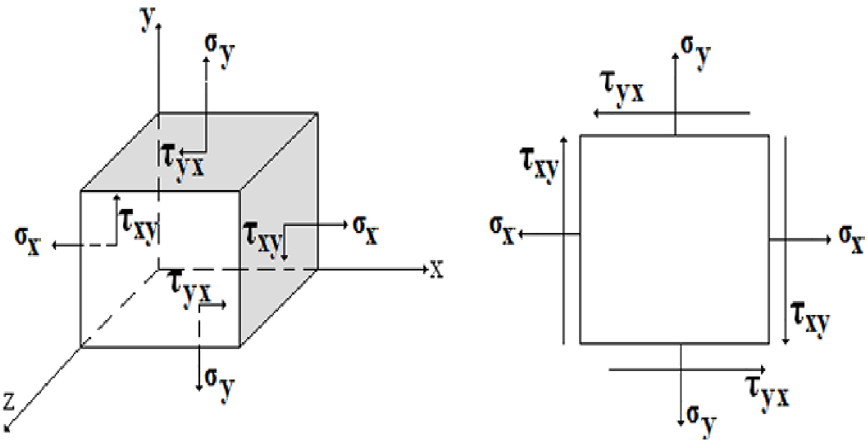

Plane of Maximum Shear Stress: Maximum in-plane Shear Stress...
Transformation of Plane Stresses
θ is measured counter clockwise, and taken positive.
the normal stress and shear stress on a plane
which is a a Das making an angle Theta with the
a'
y'
x'
6


$\tau_{x'y'} dA \sin\theta$ (x component)
$\tau_{x'y'} dA \cos\theta$ (y component)
$\sigma_{x'} dA \sin\theta$ (y component)
$\sigma_{x'} dA \cos\theta$ (x component)
$\sum F_x = 0$
$\sum F_y = 0$
$\sum F_x = 0 \qquad \sigma_{x'} dA\cos\theta - \tau_{x'y'} dA\sin\theta - \tau_{xy} dA\sin\theta - \sigma_x dA\cos\theta = 0$
$\sum F_y = 0 \qquad \sigma_{x'} dA\sin\theta + \tau_{x'y'} dA\cos\theta - \sigma_y dA\sin\theta - \tau_{xy} dA\cos\theta = 0$
of all forces acting in X direction as zero and
applying the second condition where the forces
8



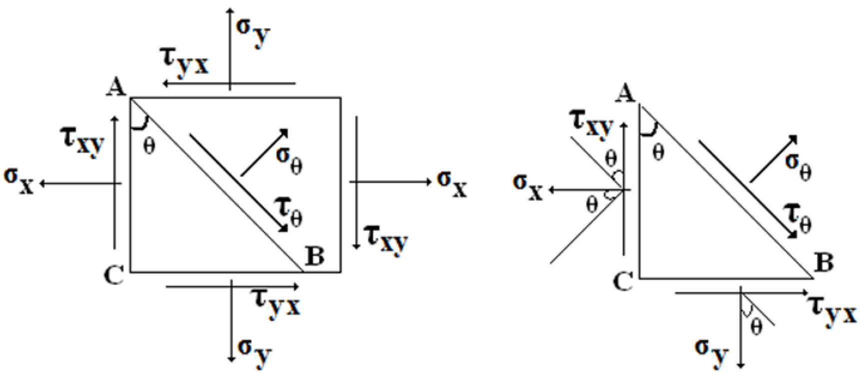
4.Variation of stresses with inclination of elements:

The stresses on an element in a stressed body vary with the orientation of the elements, so to determine the stresses acting on an element in a stressed body, the stress of the body must be analyzed with respect to the inclination of the element. First of all a representation of stresses at a point in any body must be established according to a two dimensional state (see the figure below).



5. Stresses on oblique planes:

A- Stresses on oblique planes caused by combined direct and shear stresses:



Assuming the area of the inclined plane (with the angle θ).is "A", then the forces can be resolved as follows: - Forces perpendicular to AB:

$$\sigma_\theta \cdot A = (\sigma_x \cdot A cos\theta) cos\theta + (\sigma_y \cdot A sin\theta) sin\theta - (\tau_{xy} \cdot A cos\theta) sin\theta - (\tau_{yx} \cdot A sin\theta) cos\theta \ldots\ldots(1)$$

- Forces parallel to **AB:**

$$\tau_\theta \cdot A = (\sigma_x \cdot A cos\theta) sin\theta - (\sigma_y \cdot A sin\theta) co + (\tau_{xy} \cdot A cos\theta) cos\theta - (\tau_{yx} A sin\theta) sin\theta \ldots.(2)$$

But:

A is constant, $\tau_{xy}=\tau_{yx}$

$$(\sin^2 x = \frac{1}{2}(1 - cos\ 2x) \quad or\ (cos^2 x = \frac{1}{2}(1 + cos\ 2x)$$

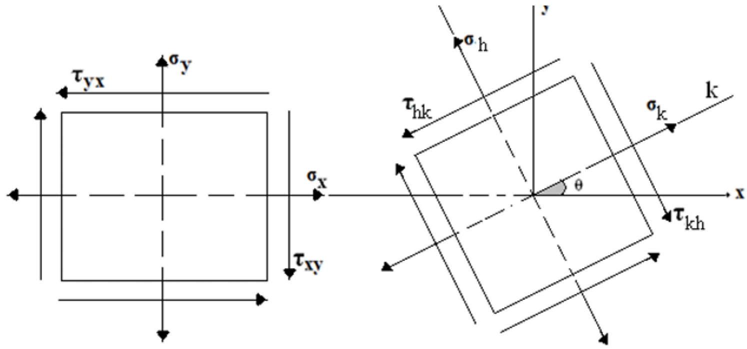
$$sin\ 2\theta = 2\ sin\theta\ cos\theta$$

Sub. and get that:

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) cos2\theta - \tau_{xy} sin2\theta \quad \ldots(3)$$

$$\tau_{\theta_s} = \frac{1}{2}(\sigma_x - \sigma_y) sin2\theta_s + \tau_{xy} cos2\theta_s \quad \ldots\ldots\ldots.(4)$$

Equations 3&4 are called the transformation equations for plane stress. These equations can be used totransform the stress components from one set of axes to another, for example from x-y coordinate to k-h coordinate as shown in the figure.



The sum of the normal stresses has the same value in each coordinate system. For this reason, the sum $\sigma_x + \sigma_y$ is called "stress invariant". So we can write that:

stress invariant $= \sigma_x + \sigma_y = \sigma_k + \sigma_h$

It can also be verified by simple algebrai

manipulation that: c

$$\sigma_x\sigma_y - \tau^2_{xy} = \sigma_k\sigma_h - \tau^2_{kh}$$

I- To obtain **<u>the location</u> of the maximum and minimum normal stresses**:

$$\frac{d\sigma_\theta}{d\theta} = 0$$

1-For maximum and minimum normal stresses:

in equation (3), so:

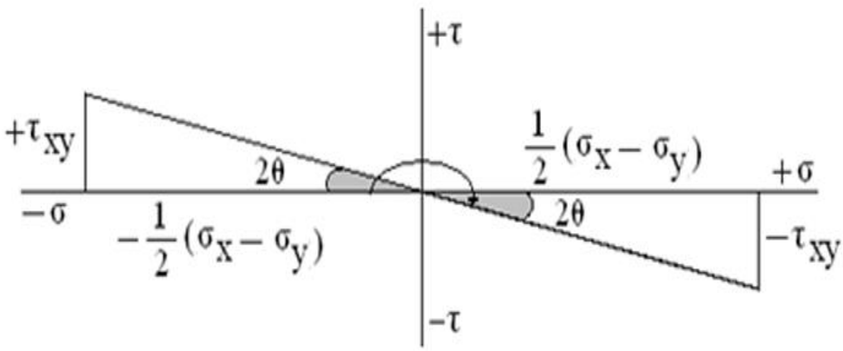
$$\tan 2\theta_p = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \text{.....................(5)}$$

$$\sigma\theta = max. = \sigma y \qquad at\ \boldsymbol{\theta} = \mathbf{90}^{0}$$

This means that the planes of maximum and minimum normal stresses are 90^0 apart

II-To obtain the magnitudes of the principal stresses:

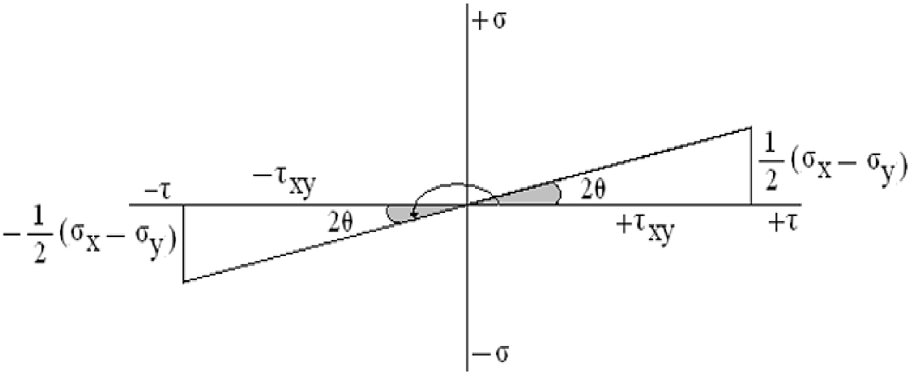
Draw ($\tan 2\theta_p = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$) as shown in Fig.



$$\sigma_{\substack{max \\ min}} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

III- To obtain the magnitudes of the maximum and minimum shearing stresses:

Draw $\left(tan2\theta_s = \frac{\sigma_x-\sigma_y}{2\tau_{xy}}\right)$ as shown in Fig. , so:



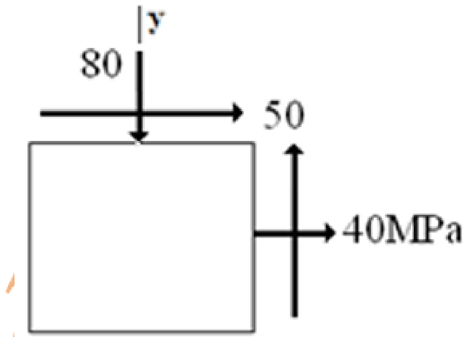
$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

Example 1: For the element stressed as shown find:

a) magnitude and locations of the principal stresses;

b) magnitude and locations of the maximum shear stresses.

c)Draw all results on complete sketches of appropriate element for each case.



Solution:

a) Find magnitude and locations of **the principal stresses**

$$\sigma_{\substack{max \\ min}} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{\substack{max \\ min}} = \frac{40+(-80)}{2} \pm \sqrt{\left(\frac{40-(-80)}{2}\right)^2 + (-50)^2}$$

σ_{max} = 58.1 MPa and σ_{min}=- 98.1 MPa

$$tan2\theta_p = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -\frac{-2*50}{40-(-80)}$$

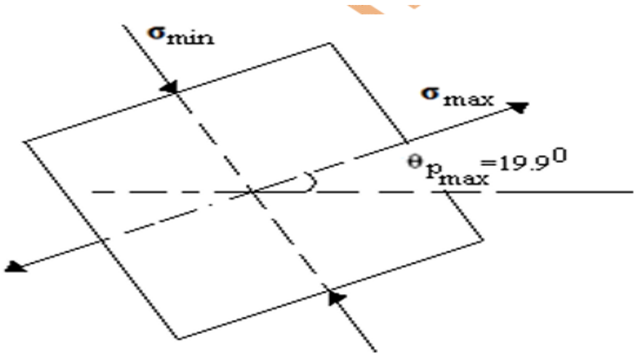
$\rightarrow \theta p = +19.9^0$ and $\theta p = +19.9^0 + 90^0 = 109.9^0$

To determine which one of the two angles represents the location of the maximum principal stress:

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)cos2\theta - \tau_{xy}sin2\theta$$

$$\sigma\theta = \frac{1}{2}(40+(-80)) + \frac{1}{2}(40-(-80))\cos(2*19.9) - (-50)\sin(2*19.9)$$

$$\sigma_{\theta = 19.9^0} = 58.1MPa$$



b) magnitude and locations of the **maximum shear stresses**

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

$$\tau \text{max} = \frac{1}{2}(58.1 - (-98.1))$$

$$\tau_{nim}^{xam} = \pm 78.1 \text{ MPa}$$

$$tan2\theta_s = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$tan2\theta_s = \frac{40-(-80)}{2*(-50)}$$

$\rightarrow \theta s = -25.1^0$and $\theta s = -25.1^0 + 90^0 = 64.9^0$

To determine which one of the two angles represents the location of the maximum shear stress:

$$\tau_{\theta_s} = \frac{1}{2}(\sigma_x - \sigma_y)sin2\theta_s + \tau_{xy}cos2\theta_s$$

$$\tau\theta(-25.1) = \frac{1}{2}(40 + (-80))\sin(2 * (-25.10) + (-50)\cos(2 * (-25.10)$$

$$\tau_{\theta}(-25.10) = 78.10\text{MPa}$$

$$\text{So: } \theta s_{min} = -25.1^{0} \text{and } \theta s_{max} = = 64.9^{0}$$

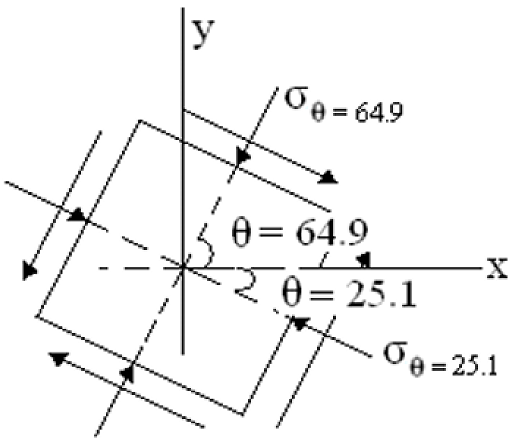
To find the normal stresses at the planes of maximum and minimum shear stresses:

$$\sigma_{\theta((-25.1)} = \frac{1}{2}(40+(-80))+\frac{1}{2}(40-(-80))\cos(2*(-25.1^{0})-(-50)\sin(2*(-25.1^{0})$$

$$\sigma_{\theta((-25.1)} = -20\text{MPa}$$

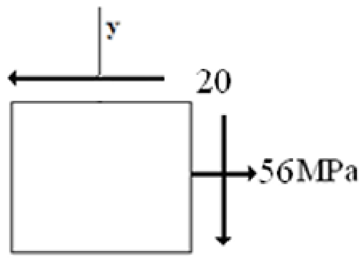
$$\sigma_{\theta((64.9)} = \frac{1}{2}(40+(-80))+\frac{1}{2}(40-(-80))\cos(2*(64.9^{0})-(-50)\sin(2*(64.9^{0})$$

$$\sigma_{\theta((64.9)} = -20\text{MPa}$$



Example#2:

An element in plane stress is subjected to stresses as shown in the figure. Determine (a) the principal stresses, (b) the maximum shearing stresses, and (c) Draw all results on complete sketches of appropriate element for each case.



Solution:

a) Find magnitude and locations of **the principal stresses**

$$\sigma_{\substack{max \\ min}} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{min}^{max} = \frac{56+0}{2} \pm \sqrt{(\frac{56-0}{2})^2 + 20^2}$$

σmax =64.4MPa and σmin=-6.4MPa

$$tan2\theta_p = -\frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

$$= -\frac{2*20}{56-0}$$

$\rightarrow \theta p = -17.8^0$ and $\theta p = -17.8^0 + 90^0 = 72.2^0$

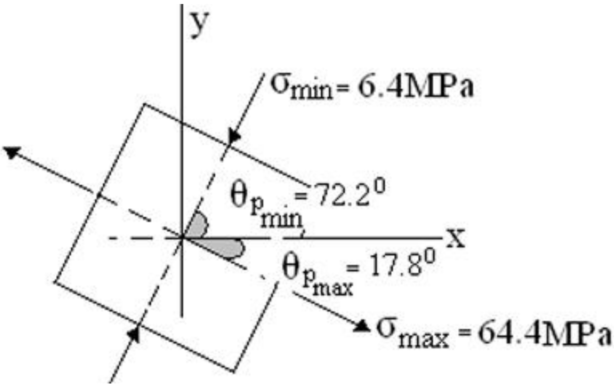
To determine which one of the two angles represents the location of the maximum principal stress:

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)cos2\theta - \tau_{xy}sin2\theta$$

$$\sigma_{\theta(-17.8)} = \frac{1}{2}(56+(0))+ \frac{1}{2}(56-(0))cos\,(2*17.8^0)-(20)sin\,(2*17.8^0)$$

$$\sigma_{\theta = 17.8^0} = 64.4MPa$$

so: $\theta_{p_{max}} = -17.8^0$ and $\theta_{p_{min}} = 72.2^0$



b) magnitude and locations of the **maximum shear stresses**

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

$$\tau\text{max} = \frac{1}{2}(64.4 - (-6.4))$$

${}^{xam}_{nim}\tau = \pm$ 35.4 MPa

$$tan2\theta_s = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tan 2\theta_s = \frac{56-0)}{2*20}$$

$\rightarrow \theta s = 27.2^0$and $\theta s = 27.2^0 - 90^0 = -62.8^0$

To determine which one of the two angles represents the location of the maximum shear stress:

$$\tau_{\theta_s} = \frac{1}{2}(\sigma_x - \sigma_y)sin2\theta_s + \tau_{xy}cos2\theta_s$$

$$\tau_\theta(27.2) = \frac{1}{2}(56 - 0))\sin(2 * (27.20) + (-50)\cos(2 * (27.20)$$

$$\tau_\theta(27.2) = 35.4\text{MPa}$$

So: $\theta s_{max} = 27.2^0$ and $\theta s_{min} = = -62.8^0$

To find the normal stresses at the planes of maximum and minimum shear stresses:

$$\sigma_{\theta((27.2)} = \frac{1}{2}(56-0)) + \frac{1}{2}(56\text{-}0))\cos(2 * (27.2^0) - 20\sin(2 * (27.2^0)$$

$$\sigma_{\theta((27.2)} = 28\text{MPa}$$

$$\sigma_{\theta((-62.8)} = \frac{1}{2}(56-0)) + \frac{1}{2}(56-0))\cos(2*(-62.8^0) - 20\sin(2*(-62.8^0)$$

$$\sigma_{\theta((-62.8)} = 28\text{MPa}$$