Module Title: Fundamental of Electrical Engineering (AC)

Module Code:	**UOMU024021**

Week (3)

3. EFFECTIVE (rms) VALUES

The effective value (or the rms value) of an alternating waveform is given by the steady (dc) current which when flowing through a given circuit, for a given time produces the same heat produced by the alternating current when flowing the same circuit for the same time.

Effective value of the sinusoidal is:

$$I_{\text{eff}} = 0.707 I_m$$

$$E_{\text{eff}} = 0.707 E_m$$

The effective value of any quantity plotted as a function of time can be found by using the following equation:

$$I_{\text{eff}} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}}$$

$$I_{\text{eff}} = \sqrt{\frac{\text{area}\,(i^2(t))}{T}}$$

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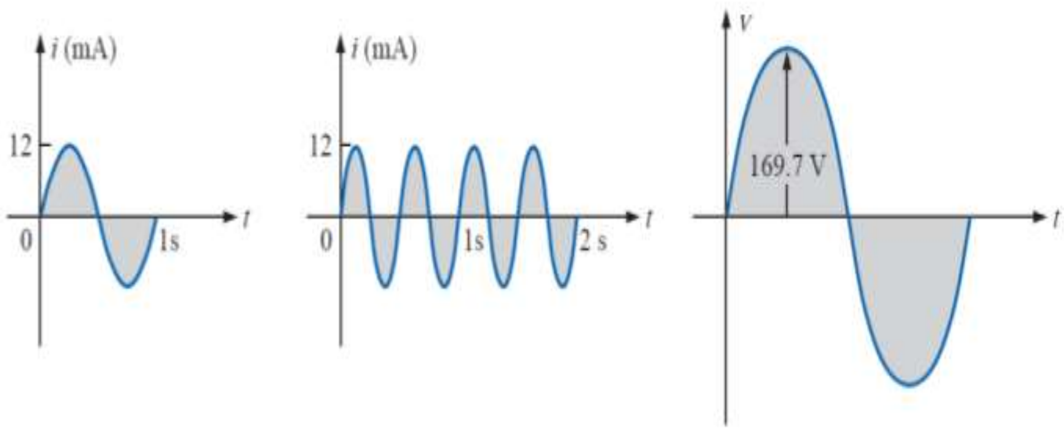
EXAMPLE: Find the **rms** values of the sinusoidal waveform



(a) (b) (c)

Solution:

For part (a), I_{rms} = 0.707(12 * 10^{-3} A) = **8.484 mA.** For part (b), again I_{rms} = **8.484 mA.** Note that frequency did not change the effective value

in (b) above compared to (a). For part (c), V_{rms} = 0.707(169.73 V) ≅ **120 V.**

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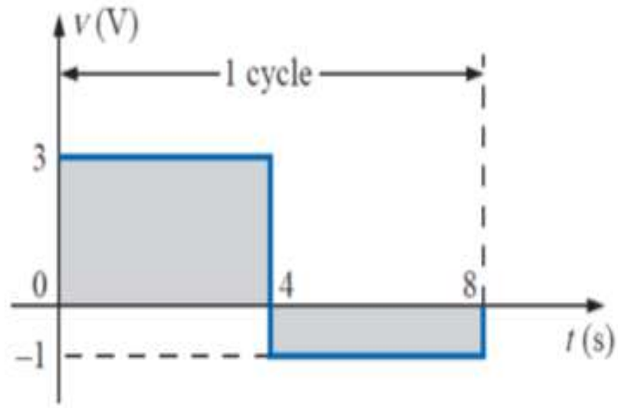
EXAMPLE: Find the effective or rms value of the waveform



Solution:

$$V_{rms} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.236\ V}$$

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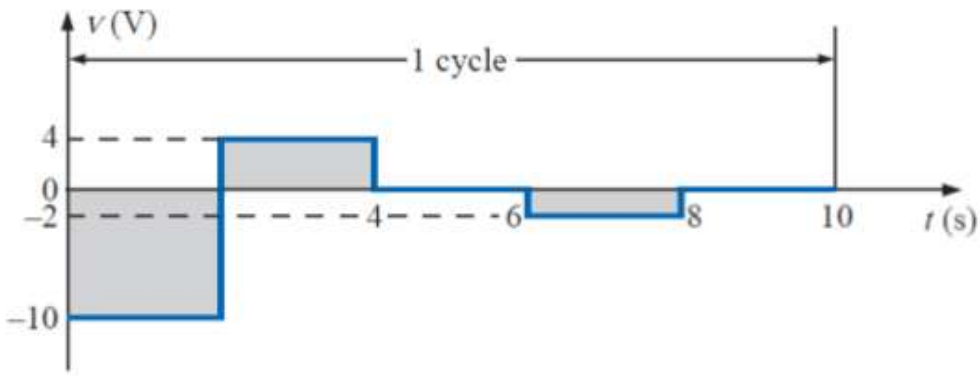
EXAMPLE: Calculate the rms value of the voltage

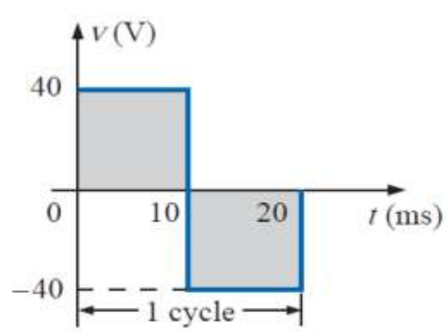


Solution:

$$V_{rms} = \sqrt{\frac{(100)(2) + (16)(2) + (4)(2)}{10}} = \sqrt{\frac{240}{10}}$$

$$= \mathbf{4.899\ V}$$

EXAMPLE: Determine the average and rms values of the square wave.



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Solution:

$$V_{\text{rms}} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}}$$

$$= \sqrt{\frac{32{,}000 \times 10^{-3}}{20 \times 10^{-3}}} = \sqrt{1600}$$

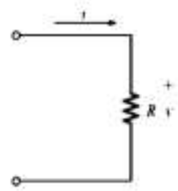
$$V_{\text{rms}} = \mathbf{40\ V}$$

4. RESPONSE OF BASIC R, L, AND C ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURREN

R, L, and C circuit elements each have quite different electrical properties. Resistance, for example, opposes current, while inductance opposes changes in current, and capacitance opposes changes in voltage.

1) Resistor

For a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.



For $v = V_m \sin \omega t$,

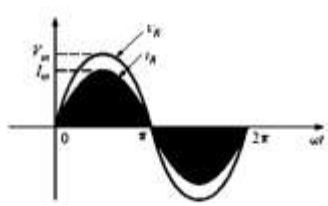
$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

$$\boxed{I_m = \frac{V_m}{R}}$$

Or

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

$$\boxed{V_m = I_m R}$$



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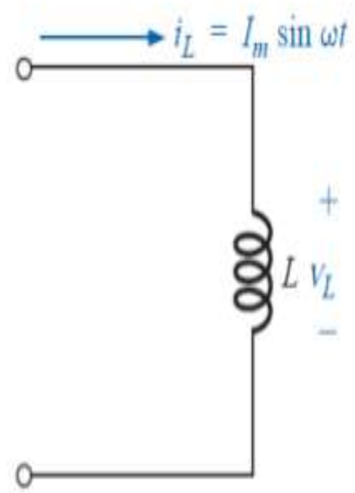
$$X_L = \frac{V_m}{I_m} \qquad \text{(ohms, } \Omega\text{)}$$

$$X_L = \omega L = 2\pi f L = 2\pi L f$$

2) Inductor

For an inductor, vL leads iL by 90°, or iL lags vL by 90°.

$$v_L = L\frac{di_L}{dt}$$



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$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

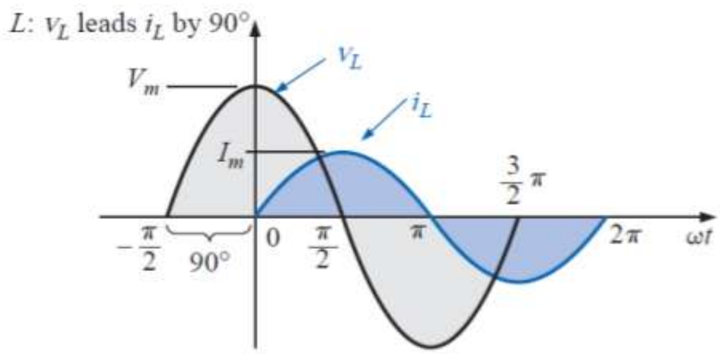
$$v_L = L\frac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$$

$$v_L = V_m \sin(\omega t + 90°)$$

$$V_m = \omega L I_m$$

$$i_L = I_m \sin(\omega t \pm \theta)$$

$$v_L = \omega L I_m \sin(\omega t \pm \theta + 90°)$$



The quantity ωL, called the **reactance** (from the word *reaction*) of an inductor, is symbolically represented by *XL* and is measured in ohms; that is,

$$X_L = \omega L \quad \text{(ohms, } \Omega)$$

$$X_L = \frac{V_m}{I_m} \quad \text{(ohms, } \Omega)$$

$$X_L = \omega L = 2\pi f L = 2\pi L f$$

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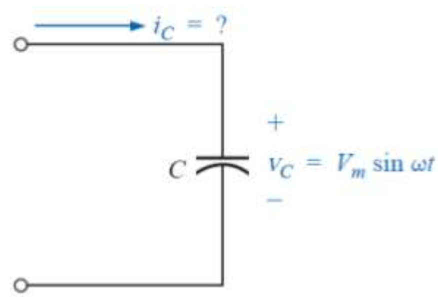
3) Capacitor

For a particular capacitance, the greater the rate of change of voltage across the capacitor, the greater the capacitive current.

$$i_C = C\,\frac{dv_C}{dt}$$



$$\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin \omega t) = \omega V_m \cos \omega t$$

$$i_C = C\,\frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

$$i_C = I_m \sin(\omega t + 90°)$$

$$I_m = \omega C V_m$$

For a capacitor, iC leads vC by 90°, or vC lags iC by 90°.

$$v_C = V_m \sin(\omega t \pm \theta)$$

$$i_C = \omega C V_m \sin(\omega t \pm \theta + 90°)$$

The quantity $1/\omega C$, called the **reactance** of a capacitor, is symbolically represented by XC and is measured in ohms; that is,

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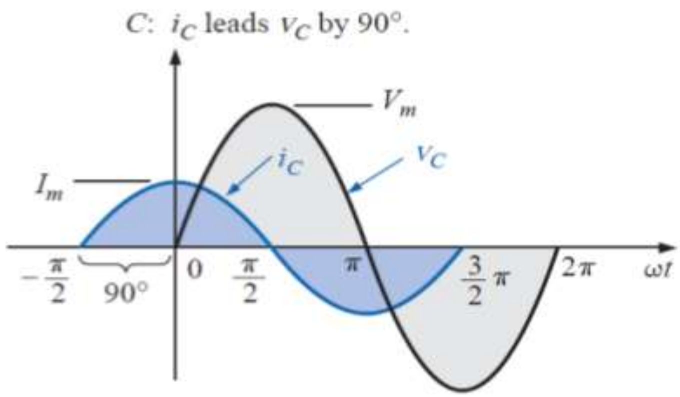
$$X_C = \frac{1}{\omega C} \quad \text{(ohms, } \Omega\text{)}$$

$$X_C = \frac{V_m}{I_m} \quad \text{(ohms, } \Omega\text{)}$$

$$X_C = \frac{1}{2\pi f C}$$



EXAMPLE 7: The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ohms . Sketch the curves for v and i.

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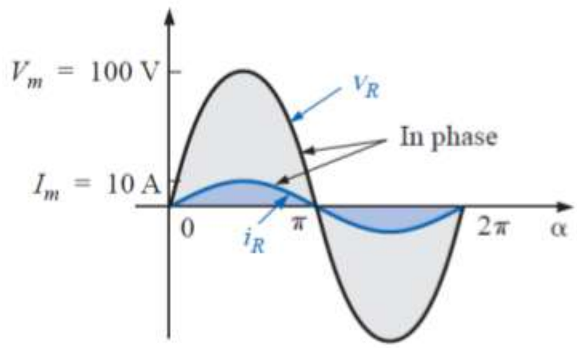
Solutions:

a)

$$I_m = \frac{V_m}{R} = \frac{100\ \text{V}}{10\ \Omega} = 10\ \text{A}$$

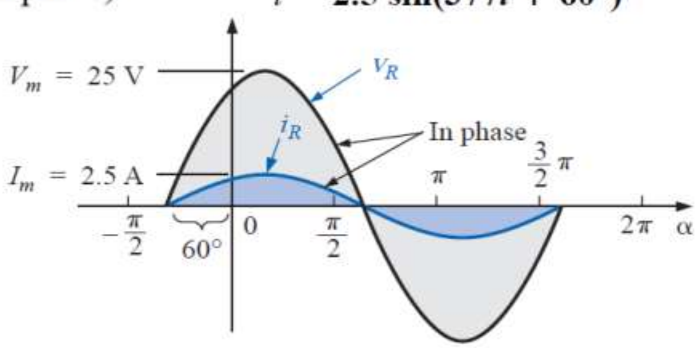
(v and i are in phase) $i = \mathbf{10 \sin 377}t$



b)

$$I_m = \frac{V_m}{R} = \frac{25\ \text{V}}{10\ \Omega} = 2.5\ \text{A}$$

(v and i are in phase) $i = \mathbf{2.5 \sin(377}t \mathbf{+ 60°)}$



EXAMPLE: The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a) $i = 10 \sin 377t$

b) $i = 7 \sin(377t \_ 70°)$

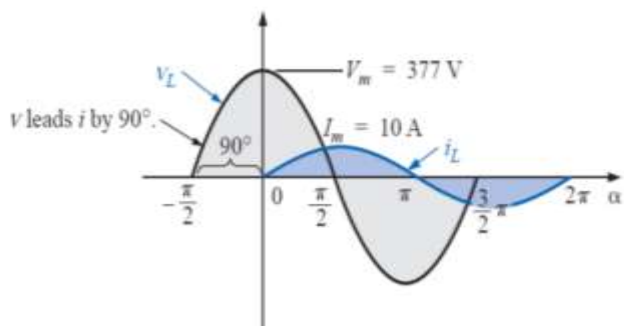
Solutions:

a)

$X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7\ \Omega$

$V_m = I_m X_L = (10 \text{ A})(37.7\ \Omega) = 377 \text{ V}$

v leads i by 90° $\quad\quad$ $v = \mathbf{377 \sin(377t + 90°)}$

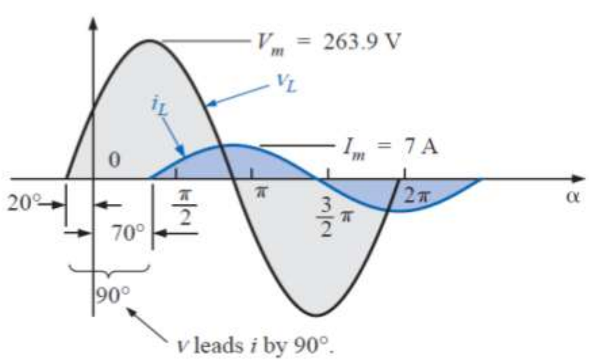


b)

$V_m = I_m X_L = (7 \text{ A})(37.7\ \Omega) = 263.9 \text{ V}$

v leads i by 90° $\quad\quad$ $v = 263.9 \sin(377t - 70° + 90°)$

$$v = \mathbf{263.9 \sin(377t + 20°)}$$



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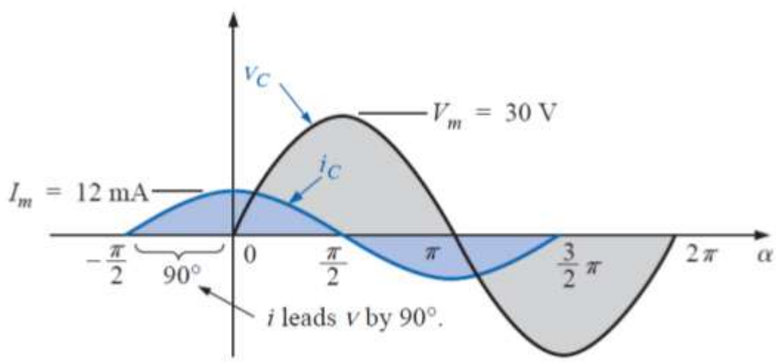
EXAMPLE: The voltage across a 1-μF capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.
v =30 sin 400t

Solutions:

$$X_C = \frac{1}{\omega C} = \frac{1}{(400\ \text{rad/s})(1 \times 10^{-6}\ \text{F})} = \frac{10^6\ \Omega}{400} = 2500\ \Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{30\ \text{V}}{2500\ \Omega} = 0.0120\ \text{A} = 12\ \text{mA}$$

i leads v by 90° $\mathbf{i = 12 \times 10^{-3} \sin(400t + 90°)}$



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EXAMPLE: At what frequency will the reactance of a 200-mH inductor match the resistance level of a 5-kΩ resistor?

Solutions:

$$5000\ \Omega = X_L = 2\pi fL = 2\pi Lf$$
$$= 2\pi(200 \times 10^{-3}\ \text{H})f = 1.257f$$

$$f = \frac{5000\ \text{Hz}}{1.257} \cong \mathbf{3.98\ kHz}$$

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