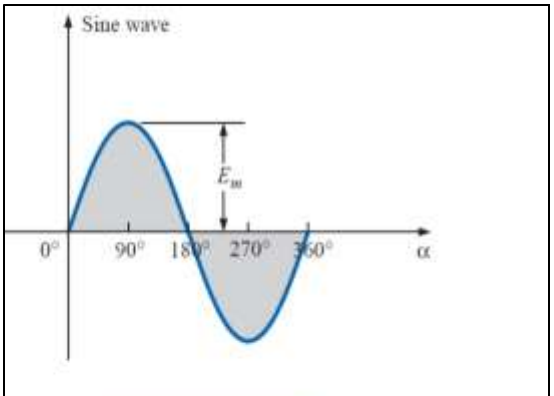
Module Title: Fundamental of Electrical Engineering (AC)

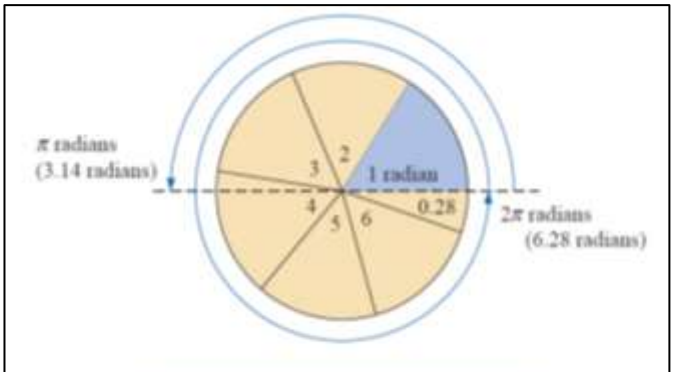
Module Code:	**UOMU024021**

Week (2)

1. THE SINE WAVE

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements. In other words, if the voltage across (or current through) a resistor, coil, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics.





$$\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees})$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians})$$

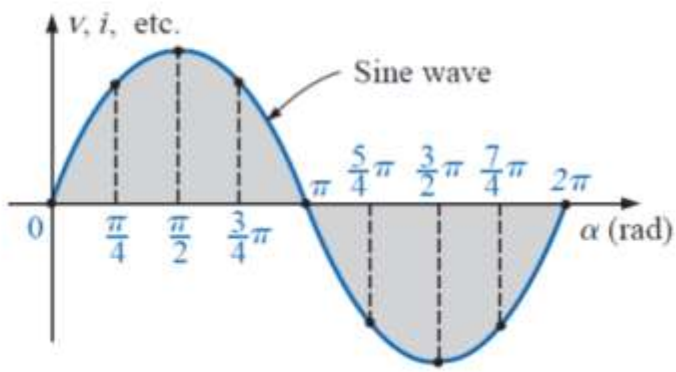
1.1 Angles and Radians:

$$\mathbf{90°:}\quad \text{Radians} = \frac{\pi}{180°}(90°) = \frac{\pi}{2}\ \textbf{rad}$$

$$\mathbf{30°:}\quad \text{Radians} = \frac{\pi}{180°}(30°) = \frac{\pi}{6}\ \textbf{rad}$$

$$\frac{\pi}{3}\ \textbf{rad:}\quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60°}$$

$$\frac{3\pi}{2}\ \textbf{rad:}\quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270°}$$



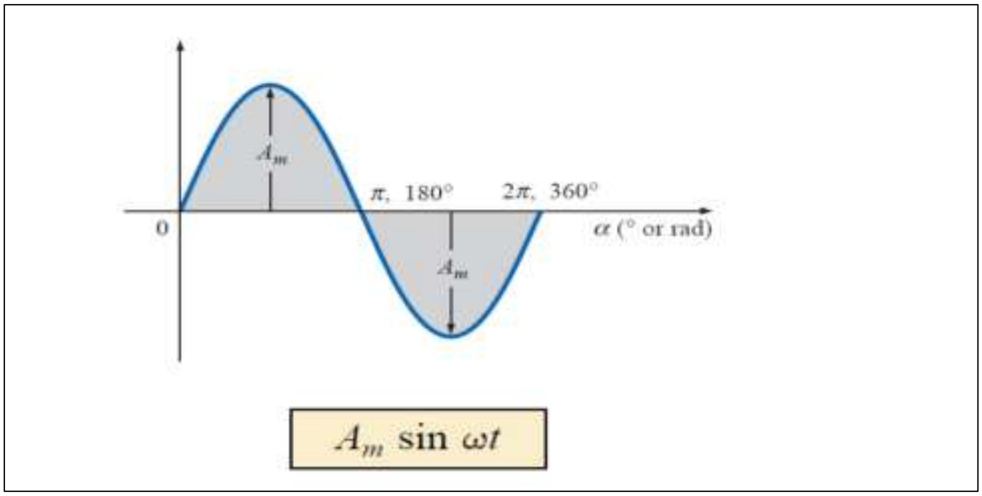
Angular velocity: The velocity, with which the radius vector rotates about the center, can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}$$

$$\omega = \frac{2\pi}{T} \qquad \text{(rad/s)}$$

2.GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is:



where ***Am*** is the peak value of the waveform and α is the unit of measure for the horizontal axis.

$$\alpha = \omega t$$

For electrical quantities such as current and voltage, the general format is:

$$i = I_m \sin \omega t = I_m \sin \alpha$$
$$e = E_m \sin \omega t = E_m \sin \alpha$$

2. EXAMPLES AND SOLVED PROBLEMS

EXAMPLE: Given $e = 5 \sin \alpha$, determine e at $\alpha = 40°$ and $\alpha = 0.8\pi$.

Solution:

For $\alpha = 40°$,

$$e = 5 \sin 40° = 5(0.6428) = \mathbf{3.214\ V}$$

For $\alpha = 0.8\pi$,

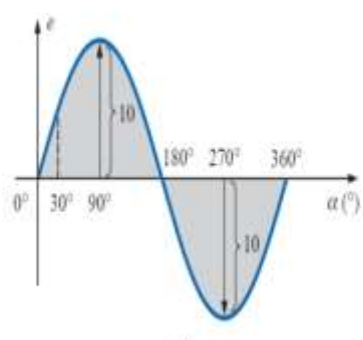
$$\alpha\ (°) = \frac{180°}{\pi}(0.8\pi) = 144°$$

and $$e = 5 \sin 144° = 5(0.5878) = \mathbf{2.939\ V}$$

EXAMPLE: Sketch $e = 10 \sin 314\, t$ with the abscissa

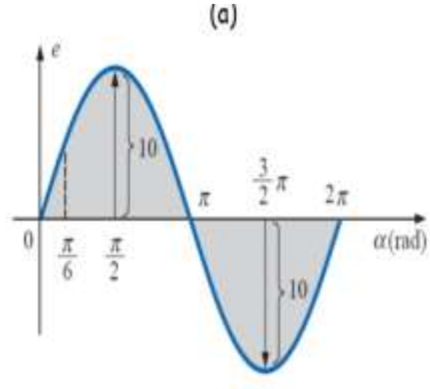
a. angle (α) in degrees.
b. angle (α) in radians.
c. time (t) in seconds.

Solution:



(a)

(a)



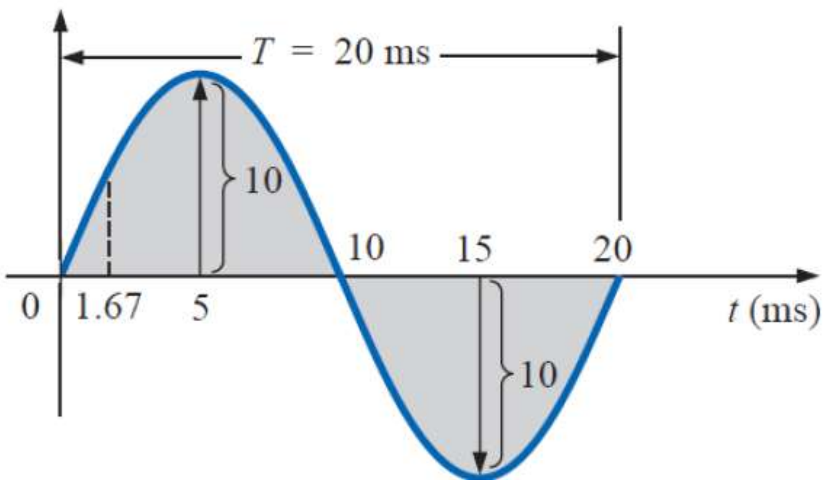
(b)

c. 360°: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{314} = 20 \text{ ms}$

180°: $\dfrac{T}{2} = \dfrac{20 \text{ ms}}{2} = 10 \text{ ms}$

90°: $\dfrac{T}{4} = \dfrac{20 \text{ ms}}{4} = 5 \text{ ms}$

30°: $\dfrac{T}{12} = \dfrac{20 \text{ ms}}{12} = 1.67 \text{ ms}$



EXAMPLE: Given $i = 6 * 10^{-3} \sin 1000\, t$, determine i at t = 2 ms.

Solution:

$$\alpha = \omega t = 1000t = (1000 \text{ rad/s})(2 \times 10^{-3} \text{ s}) = 2 \text{ rad}$$

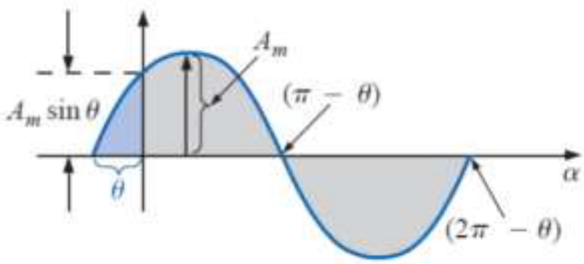
$$\alpha\,(°) = \frac{180°}{\pi \text{ rad}}(2 \text{ rad}) = 114.59°$$

$$i = (6 \times 10^{-3})(\sin 114.59°)$$
$$= (6 \text{ mA})(0.9093) = \mathbf{5.46 \text{ mA}}$$

PHASE RELATIONS

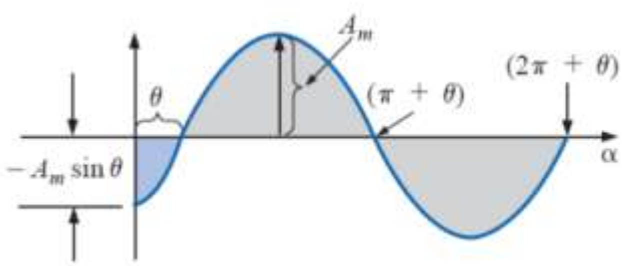
* If the waveform passes through the horizontal axis with a *positive going* (increasing with time) slope *before* 0°.

$$A_m \sin(\omega t + \theta)$$



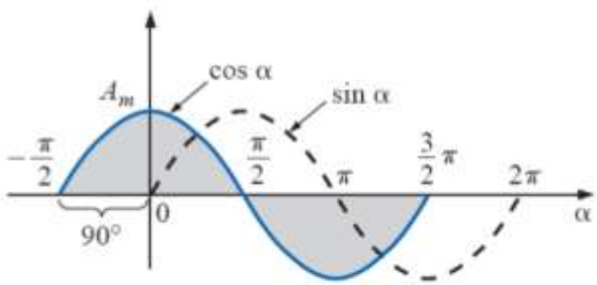
* If the waveform passes through the horizontal axis with a positive-going slope *after* 0°,

$$A_m \sin(\omega t - \theta)$$



* If the waveform crosses the horizontal axis with a positive-going slope 90°(π/2) sooner, it is called a *cosine wave*.

$$\sin(\omega t + 90^\circ) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

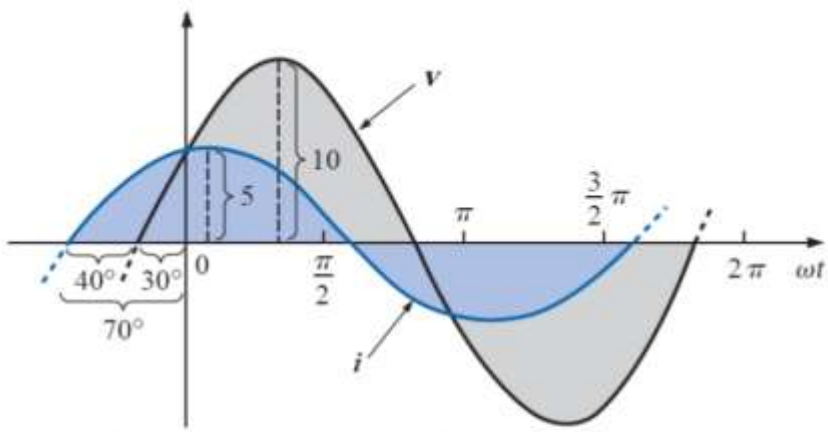


EXAMPLE 13.12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

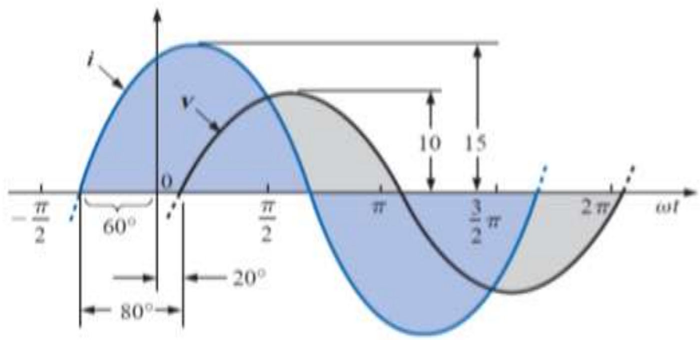
a. $v = 10 \sin(\omega t + 30^\circ)$
   $i = 5 \sin(\omega t + 70^\circ)$
b. $i = 15 \sin(\omega t + 60^\circ)$
   $v = 10 \sin(\omega t - 20^\circ)$
c. $i = 2 \cos(\omega t + 10^\circ)$
   $v = 3 \sin(\omega t - 10^\circ)$
d. $i = -\sin(\omega t + 30^\circ)$
   $v = 2 \sin(\omega t + 10^\circ)$
e. $i = -2 \cos(\omega t - 60^\circ)$
   $v = 3 \sin(\omega t - 150^\circ)$

Solutions:

a) *i* leads *v* by 40°, or *v* lags *i* by 40°.



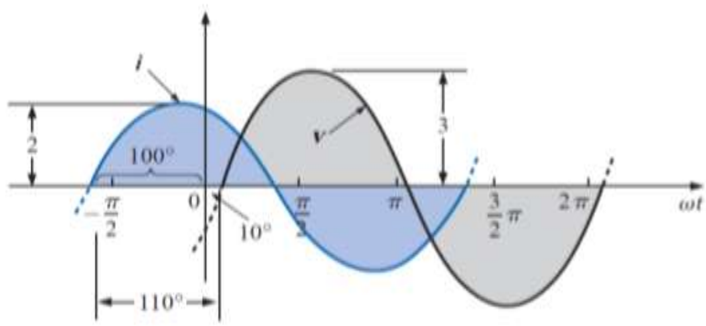
b) *i* leads *v* by 80°, or *v* lags *i* by 80°.



c)

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$$
$$= 2\sin(\omega t + 100°)$$

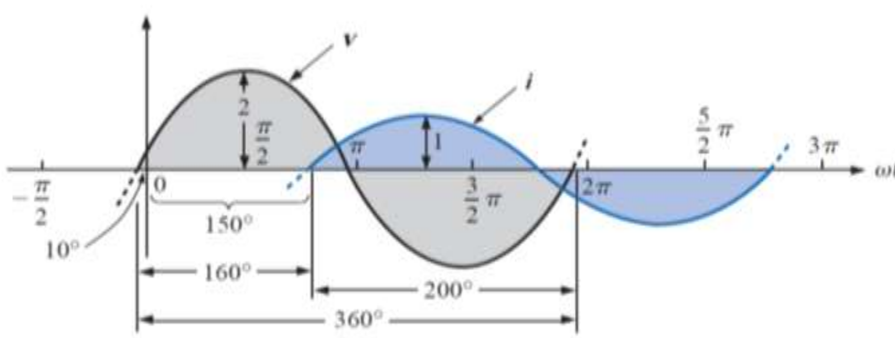
***i* leads *v* by 110°, or *v* lags *i* by 110°.**



d)

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° - 180°)$$
$$= \sin(\omega t - 150°)$$

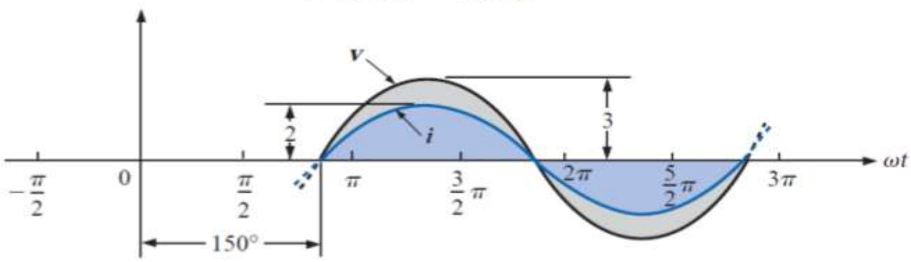
***v* leads *i* by 160°, or *i* lags *v* by 160°.**



e)

$$i = -2\cos(\omega t - 60^\circ) = 2\cos(\omega t - 60^\circ - 180^\circ)$$ (180°: By choice)

$$= 2\cos(\omega t - 240^\circ)$$



However, $\cos\alpha = \sin(\alpha + 90^\circ)$

so that $2\cos(\omega t - 240^\circ) = 2\sin(\omega t - 240^\circ + 90^\circ)$

$$= 2\sin(\omega t - 150^\circ)$$

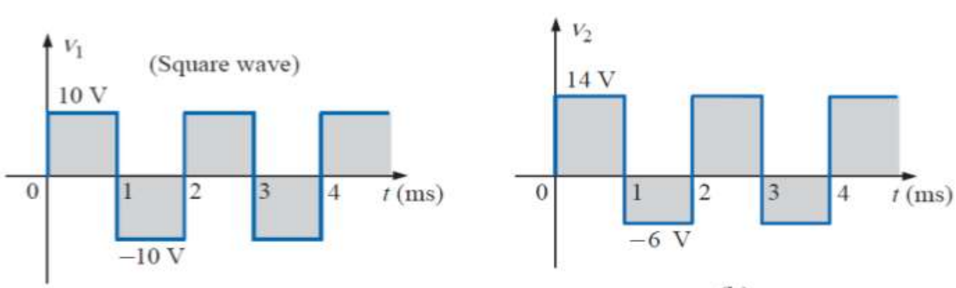
***v* and *i* are in phase.**

AVERAGE VALUE (mean)

The average value of alternating waveform is the equivalent (DC) value over a complete cycle. In general the average value of a waveform is given as:

$$G \text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$

EXAMPLE: Determine the average value of the waveforms



(a) (b)

Solutions:

a) By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts.

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}}$$

$$= \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

b)

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}}$$

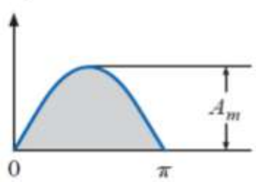
$$= \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

EXAMPLE 4: The area of sine wave (for one half) can be calculated by the following:

$$\text{Area} = \int_0^{\pi} A_m \sin \alpha \, d\alpha$$

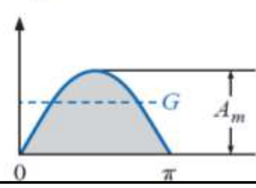
$$\begin{aligned}\text{Area} &= A_m[-\cos \alpha]_0^{\pi} \\ &= -A_m(\cos \pi - \cos 0°) \\ &= -A_m[-1 - (+1)] = -A_m(-2)\end{aligned}$$

$\text{Area} = 2A_m$

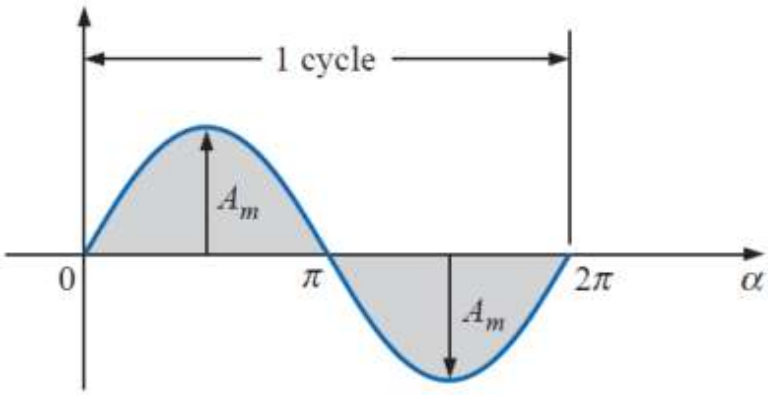


$$G = \frac{2A_m}{\pi}$$

$G = 0.637A_m$



EXAMPLE 5: Determine the average value of the sinusoidal waveform.

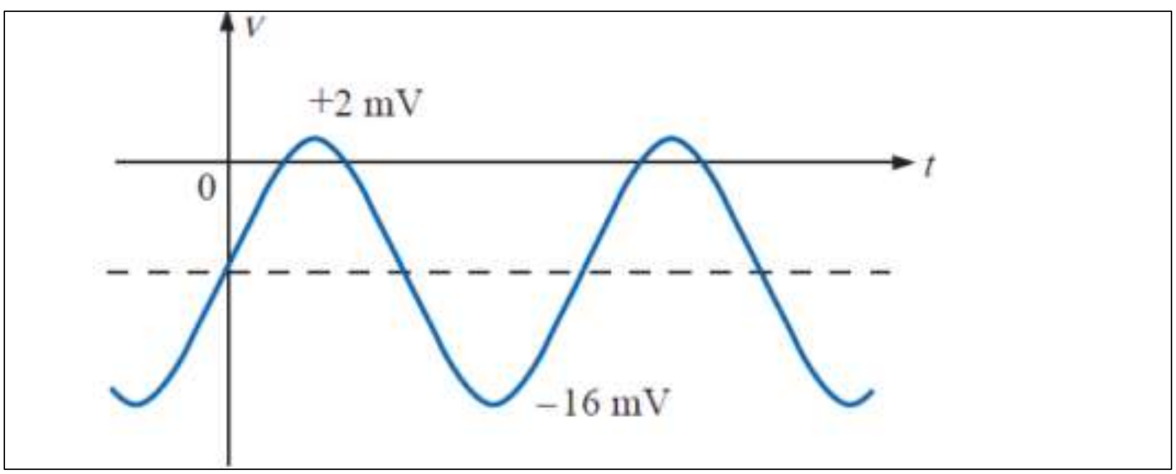


Solutions:

The average value of a pure sinusoidal waveform over one full cycle is zero.

$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$

EXAMPLE 6: Determine the average value of the waveform.

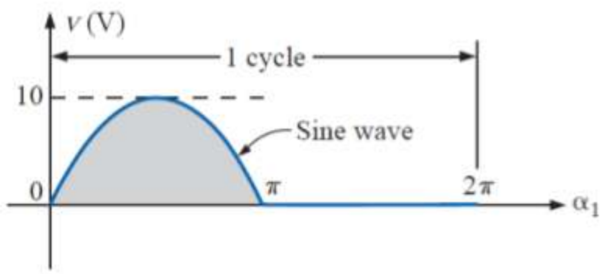


EXAMPLE: Determine the average value of the waveform

Solutions:

The peak-to-peak value of the sinusoidal function is 16 mV + 2 mV = 18 mV. The peak amplitude of the sinusoidal waveform is, therefore, 18 mV/2 = 9 mV. Counting down 9 mV from 2 mV (or 9 mV up from -16 mV) results in an average or dc level of -7 mV.

EXAMPLE 7: Determine the average value of the waveform .



Solutions:

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10\text{ V})}{2\pi} \cong \mathbf{3.18\ V}$$