Gauss sidel iteration method

We know that the general form of the equations as follows:

$a_{11}x + a_{12}y + a_{13}z = b_1$

$a_{21}x + a_{22}y + a_{23}z = b_2$

$a_{31}x + a_{32}y + a_{33}z = b_3$

Here the diagonal must satisfy the followings rules:

$|a_{11}| > |a_{12}| + |a_{13}|$ …………………1

$|a_{22}| > |a_{21}| + |a_{23}|$ …………………2

$|a_{33}| > |a_{31}| + |a_{32}|$ …………………3

If the above OK we can proceed for solving the problems as follows:

$$x=\frac{1}{a_{11}}[b_1 - a_{12}y - a_{13}z]$$

$$y=\frac{1}{a_{22}}[b_2 - a_{21}x - a_{23}z]$$

$$z=\frac{1}{a_{33}}[b_3 - a_{31}x - a_{32}z]$$

Example:

Solve the following to find the values of x,y .z

$27x + 6y - z = 85$

$x + y + 54z = 110$

$6x + 15y + 2z = 72$

Re arranges the above equations:

27x + 6y −z =85

6x + 15y + 2z =72

x + y + 54z =110

Iteration 1

Now take y=0 and z=0

S0, find x_1 by using the following equation:

$$x=\frac{1}{a_{11}}[b_1 - a_{12}y - a_{13}z]$$

$$x_1 = \frac{1}{27}[85 - 6(0) - 0] = 3.148$$

then y_1 according to the following equation:

$$y=\frac{1}{a_{22}}[b_2 - a_{21}x - a_{23}z]$$

$$y_1=\frac{1}{15}[72 - 6(3.148) - 2(0)] = 3.5407$$

then to find z_1 by using the following formula:

$$z=\frac{1}{a_{33}}[b_3 - a_{31}x - a_{32}z]$$

$$z_1 = \frac{1}{54}[110 - 3.148 - 3{,}5407] = 1.9132$$

Iteration 2

$x_2 = 2.4322$

$y_2 = 3.5720$

$z_2 = 1.9258$

Iteration 3

$x_3 = 2.4257$

$y_3 = 3.5729$

$z_3 = 1.9259$

Iteration 4

$x_4 = 2.4255$

$y_4 = 2.5730$

$z_4 = 1.9259$

Iteration 5

$x_5 = 2.4255$

$y_5 = 3.5730$

$z_5 = 1.9259$

Example 2:

$45x_1 + 2x_2 + 3x_3=58$

$-3x_1 + 22x_2 + 2x_3 = 47$

$5x_1 + x_2 + 20x_3 =67$

$$\begin{matrix} 45 & 2 & 3 \\ -3 & 22 & 2 \\ 3 & 1 & 20 \end{matrix}$$

$45 \geq 2+3$

$22 \geq 3+2$

$20 \geq 5+ 1$

S0 satisfied

First iteration

$X_1 = 1.928889$

$X_2 = 2.31212$

$X_3 = 2.91217$

Second iteration:

$X_1 = 0.991983$

$X_2 = 2.00689$

$X_3 = 3.00166$

And so on……