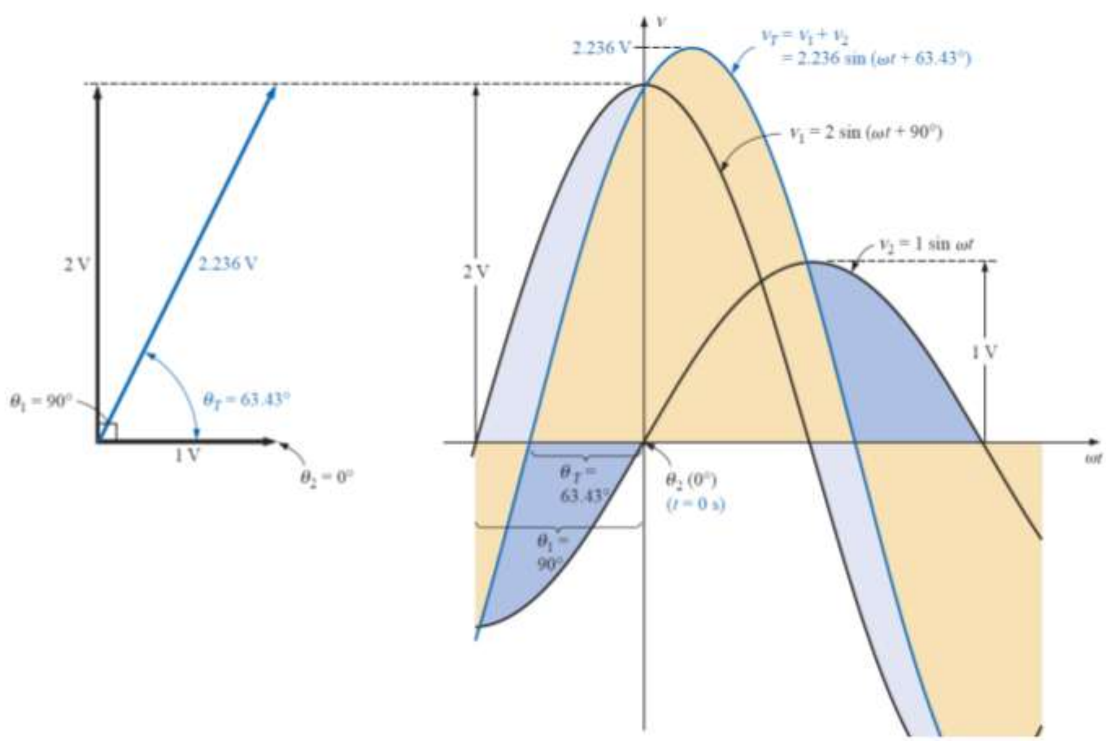
Module Title: Fundamental of Electrical Engineering (AC)

Module Code:	**UOMU024021**

Week 4

4. PHASORS

The radius vector, having a constant magnitude (length) with one end fixed at the origin, is called a phasor when applied to electric circuits.



$$e = V_m \sin(\omega t + \theta) \quad \Rightarrow \quad e = \frac{V_m}{\sqrt{2}} \angle \theta = 0.707\, V_m \angle \theta$$

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EXAMPLE: Convert the following from the time to the phasor domain:

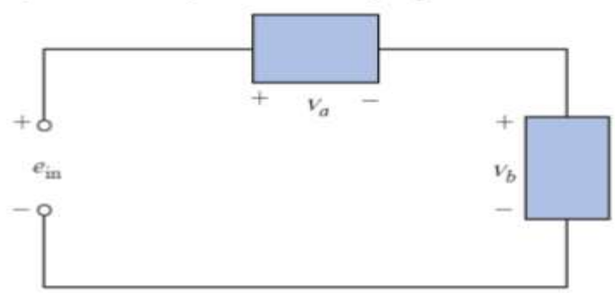
Time Domain	Phasor Domain
a. $\sqrt{2}(50)\sin\omega t$	$\mathbf{50\angle 0^\circ}$
b. $69.6\sin(\omega t + 72^\circ)$	$(0.707)(69.6)\angle 72^\circ = \mathbf{49.21\angle 72^\circ}$
c. $45\cos\omega t$	$(0.707)(45)\angle 90^\circ = \mathbf{31.82\angle 90^\circ}$

EXAMPLE: Write the sinusoidal expression for the following phasors if the frequency is 60 Hz:

Phasor Domain	Time Domain
a. $\mathbf{I} = 10\angle 30^\circ$	$i = \sqrt{2}(10)\sin(2\pi 60t + 30^\circ)$ and $i = \mathbf{14.14\sin(377t + 30^\circ)}$
b. $\mathbf{V} = 115\angle -70^\circ$	$v = \sqrt{2}(115)\sin(377t - 70^\circ)$ and $v = \mathbf{162.6\sin(377t - 70^\circ)}$

EXAMPLE: Find the input voltage of the circuit

$$\left.\begin{aligned} v_a &= 50\sin(377t + 30^\circ) \\ v_b &= 30\sin(377t + 60^\circ) \end{aligned}\right\} f = 60\text{ Hz}$$



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Solutions:

$e_{in} = v_a + v_b$

Converting from the time to the phasor domain yields

$$v_a = 50 \sin(377t + 30°) \Rightarrow \mathbf{V}_a = 35.35 \text{ V} \angle 30°$$
$$v_b = 30 \sin(377t + 60°) \Rightarrow \mathbf{V}_b = 21.21 \text{ V} \angle 60°$$

Converting from polar to rectangular form for addition yields

$$\mathbf{V}_a = 35.35 \text{ V} \angle 30° = 30.61 \text{ V} + j\,17.68 \text{ V}$$
$$\mathbf{V}_b = 21.21 \text{ V} \angle 60° = 10.61 \text{ V} + j\,18.37 \text{ V}$$

$$\mathbf{E}_{in} = \mathbf{V}_a + \mathbf{V}_b = (30.61 \text{ V} + j\,17.68 \text{ V}) + (10.61 \text{ V} + j\,18.37 \text{ V})$$
$$= 41.22 \text{ V} + j\,36.05 \text{ V}$$

$$\mathbf{E}_{in} = 41.22 \text{ V} + j\,36.05 \text{ V} = 54.76 \text{ V} \angle 41.17°$$
$$\mathbf{E}_{in} = 54.76 \text{ V} \angle 41.17° \Rightarrow e_{in} = \sqrt{2}(54.76) \sin(377t + 41.17°)$$
$$e_{in} = \mathbf{77.43 \sin(377t + 41.17°)}$$

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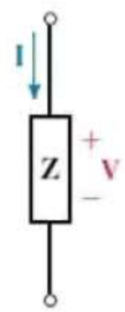
IMPEDANCE AND THE PHASOR DIAGRAM

Impedance

The opposition that circuit elements present to current in the phasor domain is defined as its Impedance.

For example, the impedance of the element of the Figure below is the ratio of its voltage phasors to its current phasor.



$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \text{ ohms}$$

The impedance is symbolically represented by Z. Thus,

$$Z = \frac{V}{I} \quad \text{(ohms)}$$

Since phasor voltages and currents are complex, Z is also complex. That is,

$$Z = \frac{V}{I}\angle\theta$$

Where V and I are rms magnitudes of **V** and **I respectively, and** θ is the angle between them

$$Z = Z\angle\theta$$

Once the impedance of a circuit is known , the current and voltage can be determined using:

$$I = \frac{V}{Z}$$

And

$$V = IZ$$

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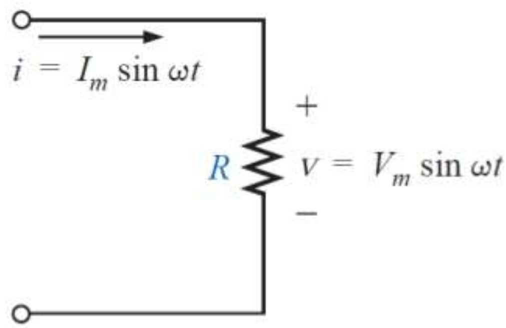
Let us now determine impedance for the basic circuit elements $R, L,$ and C

1) Resistive Elements



$$\mathbf{Z}_R = R \angle 0^\circ$$

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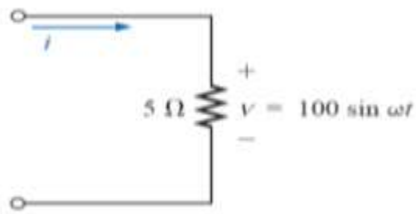
EXAMPLE: find the current i for the circuit. Sketch the waveforms of v and i.

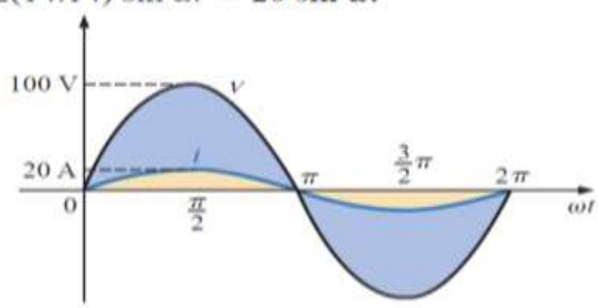


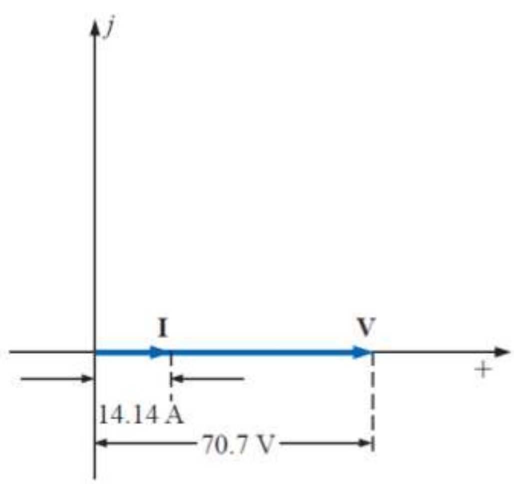
Solutions:

$v = 100 \sin \omega t \Rightarrow$ phasor form $\mathbf{V} = 70.71\ \text{V} \angle 0°$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_R} = \frac{V \angle \theta}{R \angle 0°} = \frac{70.71\ \text{V} \angle 0°}{5\ \Omega \angle 0°} = 14.14\ \text{A} \angle 0°$$

$$i = \sqrt{2}(14.14) \sin \omega t = \mathbf{20 \sin \omega t}$$





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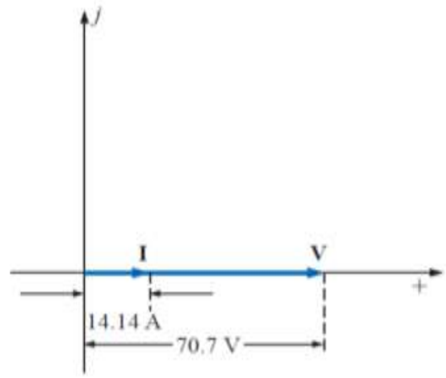
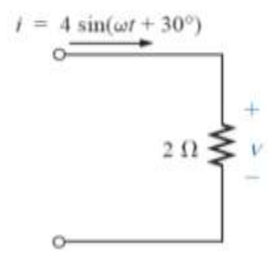


EXAMPLE: find the voltage v for the circuit. Sketch the waveforms of v and i.

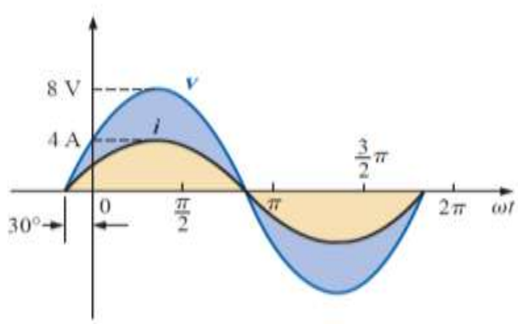


Solutions:

$i = 4 \sin(\omega t + 30°) \Rightarrow$ phasor form $\mathbf{I} = 2.828\text{ A} \angle 30°$

$\mathbf{V} = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0°) = (2.828\text{ A} \angle 30°)(2\ \Omega \angle 0°)$
$= 5.656\text{ V} \angle 30°$

$v = \sqrt{2}(5.656) \sin(\omega t + 30°) = \mathbf{8.0 \sin(\omega t + 30°)}$



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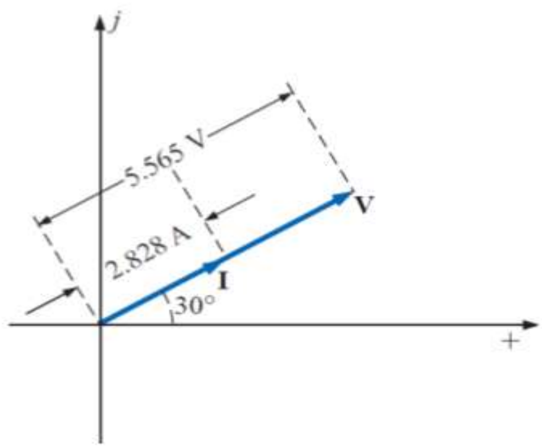
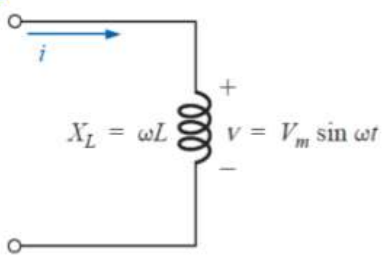


2) Inductive Reactance



$$\mathbf{Z}_L = X_L \angle 90°$$

EXAMPLE: find the current i for the circuit. Sketch the v and i curves.

Solutions:

$$v = 24 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 16.968 \text{ V} \angle 0°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_L} = \frac{V \angle \theta}{X_L \angle 90°} = \frac{16.968 \text{ V} \angle 0°}{3\ \Omega \angle 90°} = 5.656 \text{ A} \angle -90°$$

$$i = \sqrt{2}(5.656) \sin(\omega t - 90°) = \mathbf{8.0 \sin(\omega t - 90°)}$$

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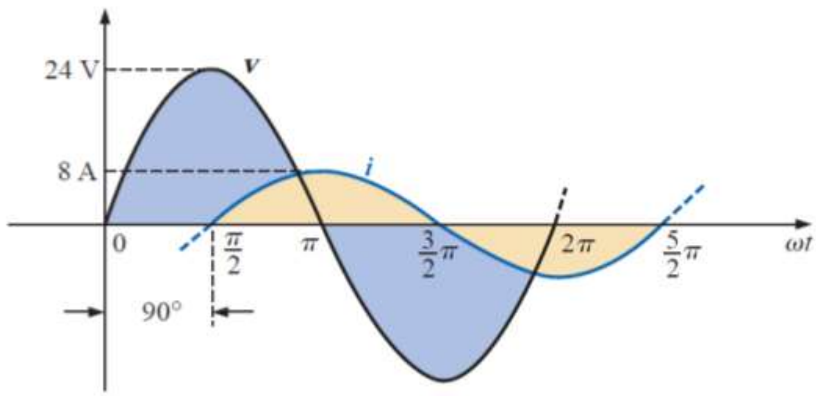
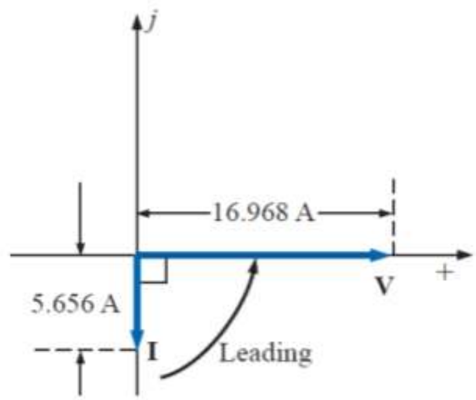
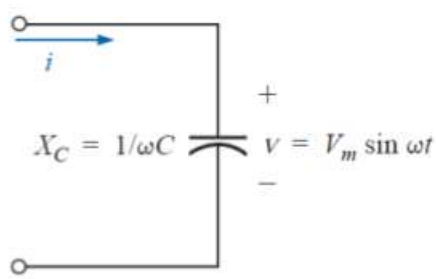




3) Capacitive Reactance



$$\mathbf{Z}_C = X_C \angle -90^\circ$$

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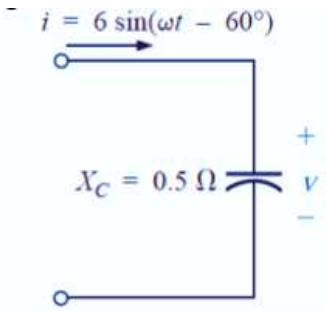
EXAMPLE: find the voltage v for the circuit. Sketch the v and i curves.

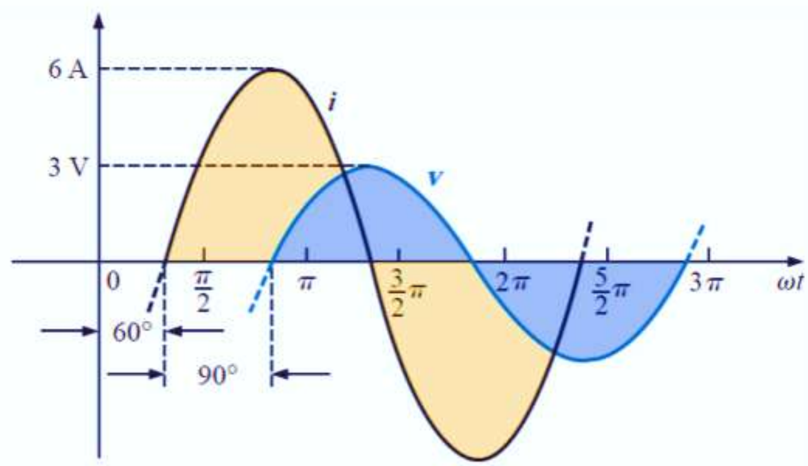


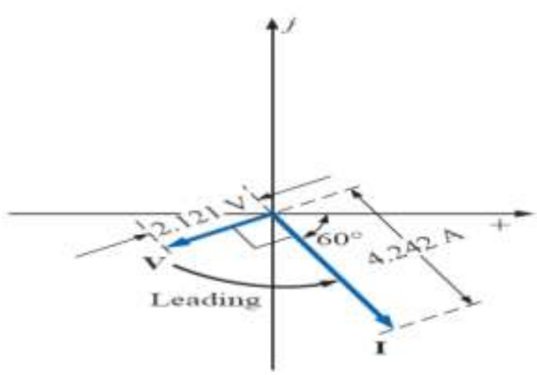
Solutions:

$i = 6\sin(\omega t - 60^\circ) \Rightarrow$ phasor notation $\mathbf{I} = 4.242\text{ A}\angle -60^\circ$

$$\mathbf{V} = \mathbf{IZ}_C = (I\angle\theta)(X_C\angle -90^\circ) = (4.242\text{ A}\angle -60^\circ)(0.5\ \Omega\angle -90^\circ)$$
$$= 2.121\text{ V}\angle -150^\circ$$

and $\quad v = \sqrt{2}(2.121)\sin(\omega t - 150^\circ) = \mathbf{3.0\sin(\omega t - 150^\circ)}$





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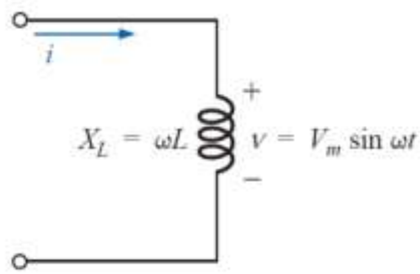
Review week 4 with examples:

Inductive Reactance

The voltage leads the current by 90° and that the reactance of the coil XL is determined by ωL.

$$v = V_m sin\omega t = V\angle 0$$



By Ohm's law,

$$\mathbf{I} = \frac{V\angle 0}{X_L\angle 90} = \frac{V}{X_L}\angle -90$$

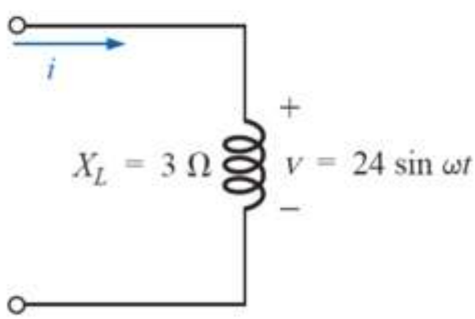
so that in the time domain,

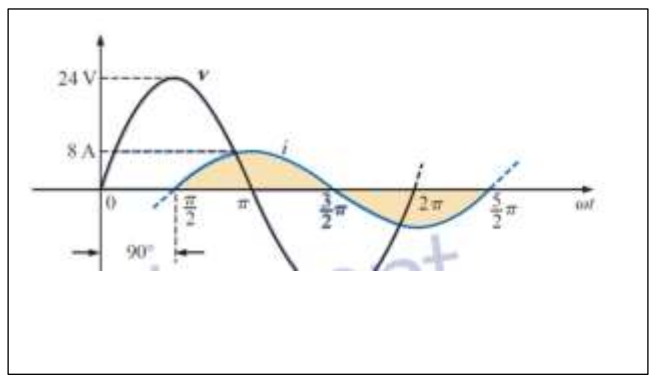
$$i = \sqrt{2}\frac{V}{X_L}sin(\omega t - 90)$$

$$Z_L = X_L\angle 90$$

Example:

Using complex algebra, find the current i for the circuit shown below. Sketch the v and i curves.





Solution:

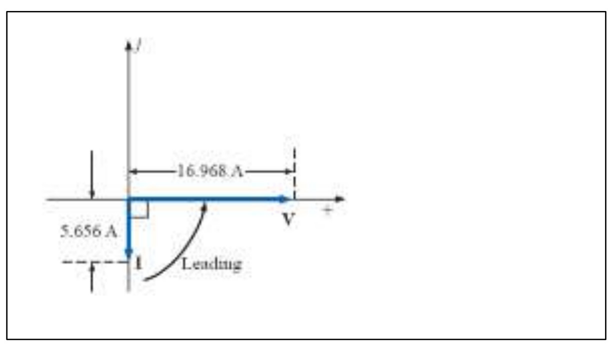
$v = 24sin\omega t$

In polar form

$\mathbf{V} = 16.968\angle 0$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_L}} = \frac{V\angle 0}{X_L\angle 90} = \frac{16.968\angle 0}{3\angle 90} = 5.656\ \text{A}\angle -90$$

$$i = \sqrt{2}(5.656)\ sin(\omega t - 90) = 8\ sin(\omega t - 90)$$



Capacitive Reactance

The current leads the voltage by 90° and that the reactance of the capacitor X_C is determined by $\frac{1}{\omega C}$.

$v = V_m sin\omega t$

In polar form

$\mathbf{V} = V\angle 0$

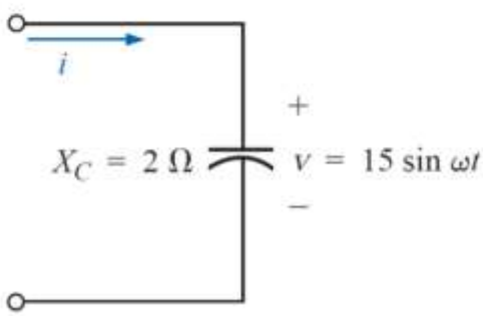
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_C}} = \frac{V\angle 0}{X_C\angle -90} = \frac{V}{X_C}\angle 90$$

$$i = \sqrt{2}\frac{V}{X_C}sin(\omega t + 90)$$

$\mathbf{Z_C} = X_C\angle -90$

Example:

Using complex algebra, find the current i for the circuit shown below. Sketch the v and i curves.



solution:

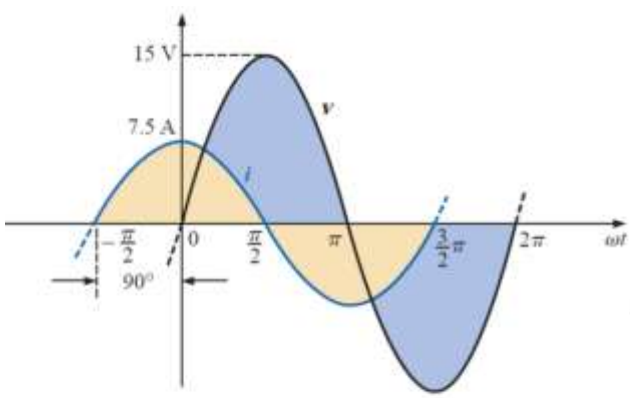
$v = 15\ sin\omega t$

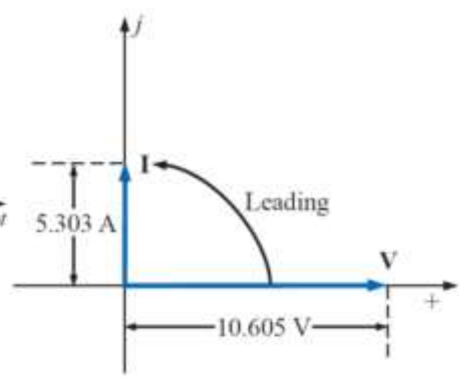
In polar form

$\mathbf{V} = 10.605\ \angle 0$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_C}} = \frac{V\angle 0}{X_C\angle -90} = \frac{10.605\ \angle 0}{2\angle -90} = 5.303\ A\angle 90$$

$$i = \sqrt{2}\frac{V}{X_C}sin(\omega t + 90) = 7.5\ sin(\omega t + 90)$$





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