Derivative Rules:- قوانين المشتقة

What is a derivative ? ما هي المشتقة ؟

The derivative is finding a slope at any point.
المشتقة هي ايجاد الميل في اي نقطة.

Before we go over the derivative Rules, lets introduce the definitions of the derivative formula.

Definition of the Derivative formula :-

By using the limit process as

$$\frac{dF}{dx} = F'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} \quad \text{---} (*)$$

Ex①/ Find $\frac{dF}{dx}$ of the following equation by using the definition of the derivative

$F(x) = 5x - 2$

Sol:-

- $F(x+\Delta x) = 5(x+\Delta x) - 2$
- $F(x) = 5x - 2$

plug the above two eqs into eq (*)

$$\frac{dF}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{5(x+\Delta x) - 2 - (5x-2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5x + 5\Delta x - 2 - 5x + 2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 5 = \boxed{5}$$

Ex ② By using the definition of the derivative find $\frac{df}{dx}$ for the following eqs, $F(x) = x^2$

Sol.

$$\frac{df}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$* f(x+\Delta x) = (x+\Delta x)^2$

$f(x) = x^2$

$$\therefore \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)(x+\Delta x) - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + \Delta x \cdot x + \Delta x \cdot x + \Delta x^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x)$$

$$\frac{df}{dx} = 2x + 0 = \boxed{2x}$$

By your own exp. try to find $\frac{df}{dx}$ for the following eqs using the def.

1- $f(x) = \frac{1}{x}$

2- $F(x) = \frac{1}{\sqrt{x}}$

3- $F(x) = \frac{5}{\sqrt{x}}$

4- $f(x) = x^2 - 2x + 4$

the Derivative Rules :- قوانين المشتقة

① Constant derivative مشتقة الثابت

$$F(x) = a \Longrightarrow \frac{dF}{dx} = f'(x) = \text{zero} \quad , \; a = \text{constant}$$

② Variable derivative مشتقة المتغير

$$f(x) = x^n \Longrightarrow \frac{df}{dx} = f'(x) = n\,x^{n-1} \quad , \; n = \text{any no.}$$

③ Multi-variable Funs مشتقة الدوال ذات المتغيرات المتعددة

$$f(x) = h(x) \mp g(x) \Longrightarrow \frac{df}{dx} = f'(x) = h'(x) \mp g'(x)$$

④ Quotient Funs مشتقة قسمة دالتين

$$f(x) = \frac{h(x)}{g(x)} \Longrightarrow \frac{df}{dx} = f'(x) = \frac{g(x)\cdot h'(x) - h(x)\cdot g'(x)}{(g(x))^2}$$

⑤ Product Funs مشتقة ضرب دالتين

$$F(x) = h(x)\cdot g(x) \Longrightarrow \frac{dF}{dx} = f'(x) = h(x)\cdot g'(x) + g(x)\cdot h'(x)$$

⑥ Power raised Funs مشتقة دالة مرفوعة لأس

$$f(x) = [h(x)]^n \Longrightarrow \frac{df}{dx} = f'(x) = n\,[h(x)]^{n-1}\cdot h'(x)$$

Examples (3):-

1- $F(x) = 4 \Rightarrow F'(x) = \text{zero}$

2- $F(x) = x \Rightarrow F'(x) = 1$

3- $F(x) = x^4 \Rightarrow F'(x) = 4x^3$

4- $F(x) = 5x^3 \Rightarrow F'(x) = 5*3x^2 = 15x^2$

5- $F(x) = x^{-3} \Rightarrow F'(x) = -3x^{-3-1} = -3x^{-4} = \frac{-3}{x^4}$

6- $F(x) = \sqrt{x} \Rightarrow F(x) = x^{1/2} \Rightarrow F'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}}$

7- $F(x) = \sqrt[5]{x^2} \Rightarrow F(x) = x^{\frac{2}{5}} \Rightarrow F'(x) = \frac{2}{5}x^{\frac{2}{5}-1} = \frac{2}{5}x^{-\frac{3}{5}} = \frac{2}{5\sqrt[5]{x^2}}$

8- $F(x) = 3x^5 + 7x \Rightarrow F'(x) = 3*5x^{5-1} + 7 = 15x^4 + 7$

9- $F(x) = (x^4 - x^2 + 1)(5x^6 - 3x) \Rightarrow F'(x) = (x^4 - x^2 + 1)(30x^5 - 3) + (5x^6 - 3x)(4x^3 - 2x)$

10- $F(x) = \frac{x^3+1}{x^4+1} \Rightarrow F'(x) = \frac{(x^4+1)(3x^2) - (x^3+1)(4x^3)}{(x^4+1)^2}$

11- $F(x) = (x^3 + x^2 + x + 1)^5 \Rightarrow F'(x) = 5(x^3 + x^2 + x + 1)^4 * (3x^2 + 2x + 1)$

12- $F(x) = \sqrt{x^2 - 2x + 1} \Rightarrow F'(x) = \frac{2x - 2}{2\sqrt{x^2 - 2x + 1}}$

Ex (4)/ Find the derivative of the quotient fun. at $x = 1$, $F(x) = \frac{x^3+1}{x^4+1}$

Sol/

From Ex3 # 10 $\Rightarrow$ $F'(x)\Big|_{x=1} = \frac{(1^4+1)(3*1^2) - (1^3+1)(4*1^3)}{(1^4+1)^2}$

$= \frac{2*3 - 2*4}{2^2} = \frac{6-8}{4}$

$= \boxed{\frac{-1}{2}}$