1 Motor Principle

An electrical motor operates based on the fundamental principle of converting electrical energy into mechanical motion through the interaction of a magnetic field.

In the motor's construction, there is a stationary magnetic field established in the stator, which can be created using permanent magnets or electromagnets. This magnetic field serves as a foundation for the motor's functionality.

Within the motor, there is a component known as a rotor that is capable of rotating and consists of wires. When an electric current is flowing through this coil, it transforms into an electromagnet. The key intersection occurs as the magnetic fields of the stator and the rotor coil interact, which leads to generating the Lorentz force, and this force is crucial in initiating the rotational motion of the rotor.

The rotational motion is sustained by the continuous generation of electromotive force (EMF) in the coil as it rotates. This induced EMF ensures that the current flow persists and maintains the rotational momentum of the motor.

As further the working of the electrical motor has the ability of the motor convert electrical energy into mechanical motion makes it a versatile and widely used components in various applications.

When the terminals of the motor are connected to an external source of DC supply:

(i) The field magnets are excited, developing alternate N and S poles.

(ii) The armature conductors carry currents. All conductors under the N-pole carry currents in one direction, while all the conductors under the S-pole carry currents in the opposite direction.

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Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it. It is clear that the force on each conductor is tending to rotate the armature in an anticlockwise direction. All these forces add together to produce a driving torque that sets the armature rotating.

When the conductor moves from one side of a brush to the other, the current in that conductor is reversed, and at the same time, it comes under the influence of the next pole, which is of opposite polarity. Consequently, the direction of force on the conductor remains the same.

2 Back EMF and Voltage Equation of DC Motor

When the armature of a DC motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field, and hence EMF is induced in them as in a generator. The induced EMF acts in the opposite direction to the applied voltage V (Lenz's law) and is known as back or counter EMF (Eb). The back EMF is always less than V, although this difference is small when the motor is running under normal conditions.

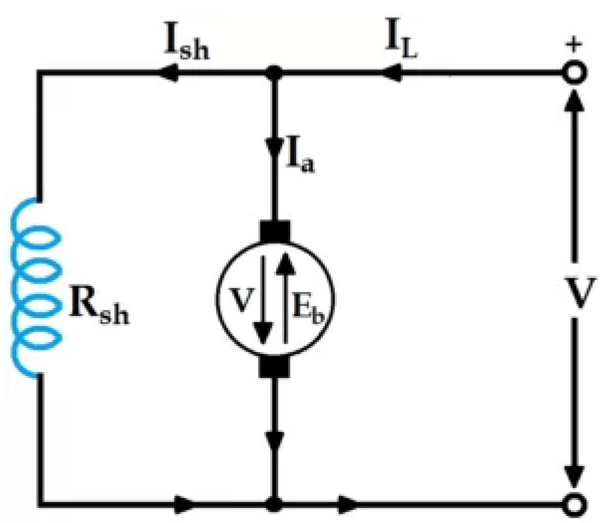

Fig. 1 Shunt DC motor.

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$$E_b = \frac{Z\,P\,\emptyset\,N}{A\;60}\,, N\ in\ rpm \tag{1}$$

Consider the shunt-wound motor shown in the figure above. When DC voltage V is applied across the motor terminals, the field magnets are excited, and current is supplied to the armature conductors. Hence, the driving torque acts on the armature, which starts rotating.

As the armature rotates, a back EMF E_b is induced, which opposes the applied voltage V. The applied voltage V is to force the current through the armature against the back EMF E_b.

The electric work done in overcoming and causing the current to flow against E_b is converted into mechanical energy developed in the armature. Therefore, it follows that energy conversion in a DC motor is possible only due to the production of back EMF E_b.

Net voltage in armature circuit = $V - E_b$

If R_a is the armature circuit resistance, then,

$$I_a = \frac{V - E_b}{R_a} \tag{2}$$

$$V = I_a R_a + E_b \tag{3}$$

This is known as voltage equation of the DC motor. Now, multiplying both sides by I_a, we get:

$$V I_a = I_a^2 R_a + E_b I_a \tag{4}$$

VI_a= Eectrical input to the armature

$E_b I_a$ = Electrical equivalent of mechanical power developed in the armature

$I_a^2 R_a$= Cu loss in the armature

3 Condition for Maximum Power

The gross mechanical power developed by a motor is

$$P_m = VI_a - I_a^2 R_a. \tag{5}$$

Differentiating both sides with respect to I_a and equating the result to zero, we get

$$\frac{dP_m}{dI_a} = V - 2I_a R_a = 0$$

$$V = \frac{I_a R_a}{2} \tag{6}$$

Thus, gross mechanical power developed by a motor is maximum when back EMF is equal to half the applied voltage. This condition is, however, not realized in practice, because in that case current would be much beyond the normal current of the motor. Moreover, half the input would be wasted in the form of heat, and taking other losses (mechanical and magnetic) into consideration, the motor efficiency will be well below 50 percent.

Example 1: A 440-V, shunt motor has armature resistance of 0.8 Ω and field resistance of 200 Ω. Determine the back EMF when giving an output of 7.46 kW at 85 % efficiency.

Solution

Motor input power = 7.46 × 10^3/0.85 = **8776 W**

Motor input current = 8776 / 440 = **19.95 A**

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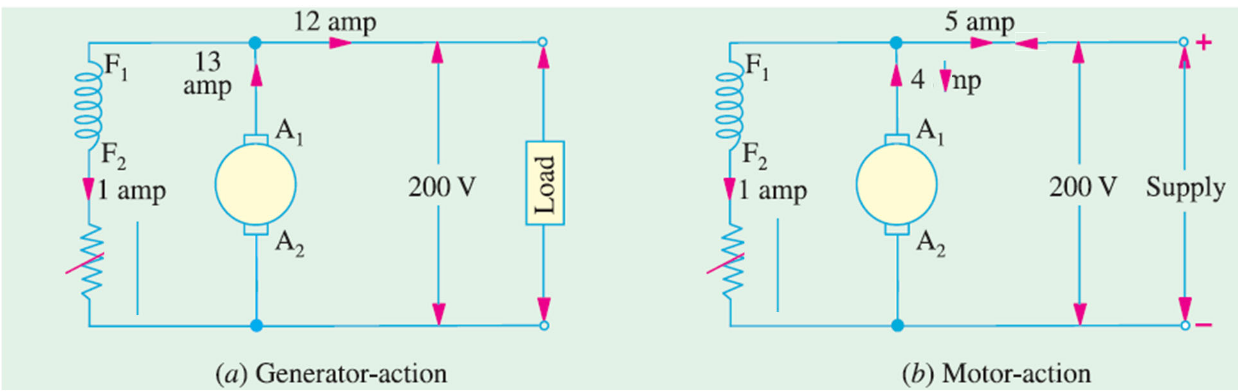
I_{sh} = 440/200 = **2.2 A**

I_a = 19.95 − 2.2 = **17.75 A**

E_b= V − $I_a R_a$ = 440 − (17.75 × 0.8) = **425.8 V**

Example 2: A 4 pole, 32 conductor, lap-wound DC shunt generator with terminal voltage of 200 volts delivering 12 amps to the load has R_a = 2 Ω and field circuit resistance of 200 Ω. It is driven at 1000 RPM. Calculate the flux per pole in the machine. If the machine has to be run as a motor with the same terminal voltage and drawing 5 A from the mains, maintaining the same magnetic field, find the speed of the machine.

Solution



(a) Generator-action (b) Motor-action

As a generator,

I_a = 13 A

E_g = 200 + (13 × 2) = **226 V**

$$\frac{Z\ P\ \emptyset\ N}{A\ 60} = 226\text{ V} \Longrightarrow \emptyset = \frac{226 \times 60}{1000 \times 32} = \mathbf{0.42375}\ \boldsymbol{wb}$$

As a motor,

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$I_a = 13\text{A}$

$E_b = 200 - (4 \times 2) = 192$

$$E_b = \frac{Z\,P\,\emptyset\,N}{A\;60} \Longrightarrow N = \frac{60 \times 192}{0.42375 \times 32} = \mathbf{850\ RPM}$$

4 Torque

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts. Consider a pulley of radius **r** (meter) acted upon by a circumferential force of **F** (Newton) which causes it to rotate at **N** (RPM)

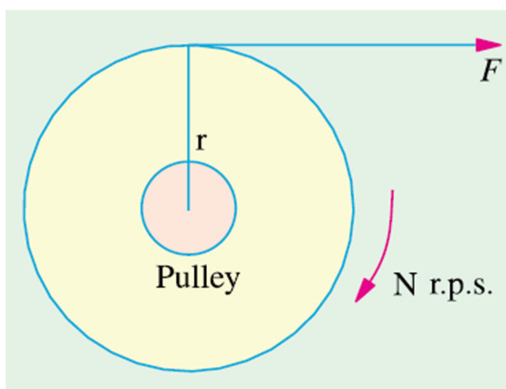

Fig. 2 Torque in DC motors.

Then torque

$$T = F \times r \quad \text{Newton-meter} \qquad (7)$$

Work done by this force in one revolution = Force × distance = F × 2πr Joule

$$\text{Power developed} = F \times 2\pi r \times N/60 \quad \text{Joule/second or Watt} \qquad (8)$$

or

$$\text{Power developed} = \frac{T \times 2\pi \times N}{60} = \frac{NT}{9.55} \quad \text{Joule/second or Watt} \qquad (9)$$

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4.1 Armature Torque of a Motor

Let T_a be the torque developed by the armature of a motor running at N RPM The power developed = $T_a \times 2\pi \times N/60$ watt

We also know that electrical power converted into mechanical power in the armature = $E_b I_a$ then

$$E_b I_a = T_a \times 2\pi \times N/60$$

But $E_b = \frac{Z\,P\,\emptyset\,N}{A\,60}$ then we have,

$$\frac{Z\,P\,\emptyset\,N}{A\,60} I_a = \frac{T_a \times 2\pi \times N}{60}$$

$$T_a = \frac{0.159 \times I_a Z\,P\emptyset}{A} \quad \text{N-m} \qquad (10)$$

Or

$$T_a = \frac{9.55 \times E_b I_a}{N} \quad \text{N-m} \qquad (11)$$

4.2 Shaft Torque (T_{sh})

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of it is required for supplying iron and friction losses in the motor. The torque which is available for doing useful work is known as shaft torque T_{sh}. It is so called because it is available at the shaft. The motor output is given by:

$$\text{Output} = (T_{sh} \times 2\pi N)/60 \quad \text{Watt} \qquad (12)$$

Provided T_{sh} is in N-m and N in RPM

$$T_{sh} = \frac{9.55 \times \text{Output}}{N} \quad \text{N-m} \qquad (13)$$

The difference ($T_a - T_{sh}$) is known as lost torque and is due to iron and friction losses of the motor.

Example 3: Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors' wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.

Solution

Developed torque or gross torque is the same thing as armature torque.

$$T_a = \frac{0.159 \times I_a Z\, P\emptyset}{A} = \frac{0.159 \times 45 \times 800 \times 4 \times 25 \times 10^{-3}}{2} = \mathbf{286.2\ N.m}$$

$$E_b = V - I_a R = 220 - 45 \times 0.6 = \mathbf{193\ V}$$

$$E_b = \frac{Z\ P\ \emptyset\ N}{A\ 60} \Longrightarrow 193 = \frac{800 \times 4 \times 25 \times 10^{-3} \times N}{2 \times 60}$$

$$N = \mathbf{289.5\ r.p.m.}$$

$$\text{Output} = (T_{sh} \times 2\pi N)/60 \quad \Longrightarrow \quad T_{sh} = \frac{8200 \times 60}{2\pi \times 289.5} = \mathbf{270.5\ N.m}$$

Example 4: A 4-pole, 240 V, wave connected shunt motor gives 11.19 kW when running at 1000 RPM and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1 Ω. Assuming a drop of 1 volt per brush, find (a) total torque (b) useful torque (c) useful flux / pole (d) Armature input, losses and rotational losses (e) efficiency.

Solution

$$E_b = V - I_a R_a - brush\ drop = 240 - (50 \times 0.1) - 2 = \mathbf{233\ V}$$

(a) $T_a = 9.55 \times E_b I_a / N = \mathbf{111\ \ N.m}$

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(b) $T_{sh} = 9.55 \times output/N = \mathbf{106.9\ N.m}$

(c) $E_b = \frac{Z\,P\,\emptyset\,N}{A\;60}$

$$233 = \frac{540 \times\ 4 \times\ \emptyset\ \times 1000}{2 \times 60} \quad \Longrightarrow \quad \emptyset = \mathbf{12.9\ mWb}$$

(d) Armature input $= VI_a = 240 \times 50 = \mathbf{12000\ W}$

Armature Cu loss = $I_a^2 R_a$ = $50^2 \times 0.1$ = **250 W**

Brush contact loss = 50 × 2 = **100 W**

Power developed = $12000 - 250 - 100 = \mathbf{11650\ W}$

Output = 11190 W

Rotational losses = $11650 - 11190 = \mathbf{460\ W}$

(e) motor input = VI = 240 × 51=**12340 W**

$$Efficiency = \frac{motor\ output}{motor\ input} \times 100\% = \frac{11190}{12240} \times 100\% = \mathbf{91.4}\%$$

4.3 Speed of DC Motor

In order to study the relation of the change in Eb and Ø over N and from the voltage equation of a motor, we get:

$$E_b = V - I_a R_a \tag{14}$$

$$\frac{Z\,P\,\emptyset\,N}{A\;60} = V - I_a R_a \tag{15}$$

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$$N = \frac{V - I_a R_a}{Z\ P\ \emptyset} \times A\ 60 \qquad (16)$$

Or

$$N = \frac{E_b\ A\ 60}{Z\ P\ \emptyset} = k\frac{E_b}{\emptyset} \qquad (17)$$

It shows that speed is directly proportional to back EMF

4.3.1 Series Motor

N_1 = Speed in the first case

I_{a1} = armature current in the first case

$\emptyset_1$ = flux/pole in the first case

N_2, I_{a2}, $\emptyset_2$ = corresponding quantities in the second case.

Then, using the above relation, we get

$$N_1 = k\frac{E_{b1}}{\emptyset_1}, \qquad N_2 = k\frac{E_{b2}}{\emptyset_2}$$

Then

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\emptyset_1}{\emptyset_2} \qquad (18)$$

Prior to saturation of magnetic poles, $\emptyset \propto I_a$ then

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \qquad (19)$$

4.3.2 Shunt Motor

In this case the same equation applies as in Eq 18.

If $\emptyset_2 = \emptyset_1$ then

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \tag{20}$$

Example 5: A DC series motor operates at 800 RPM with a line current of 100 A from 230-V mains. Its armature circuit resistance is 0.15 Ω and its field resistance 0.1 Ω. Find the speed at which the motor runs at a line current of 25 A, assuming that the flux at this current is 45 per cent of the flux at 100 A.

Solution

$$\emptyset_2 = 0.45\ \emptyset_1$$

$$E_{b1} = V - I_{a1}(R_a + R_{sh}) = 230 - \ 100\ (0.15 + 0.1) = \mathbf{205\ V}$$

$$E_{b2} = V - I_{a2}(R_a + R_{sh}) = 230 - \ 25\ (0.15 + 0.1) = \mathbf{223.75\ V}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\emptyset_1}{\emptyset_2} \Longrightarrow \frac{N_2}{800} = \frac{223.75}{205} \times \frac{\emptyset_1}{0.45\emptyset_1}$$

$$N_2 = \mathbf{1940\ r.p.m}$$