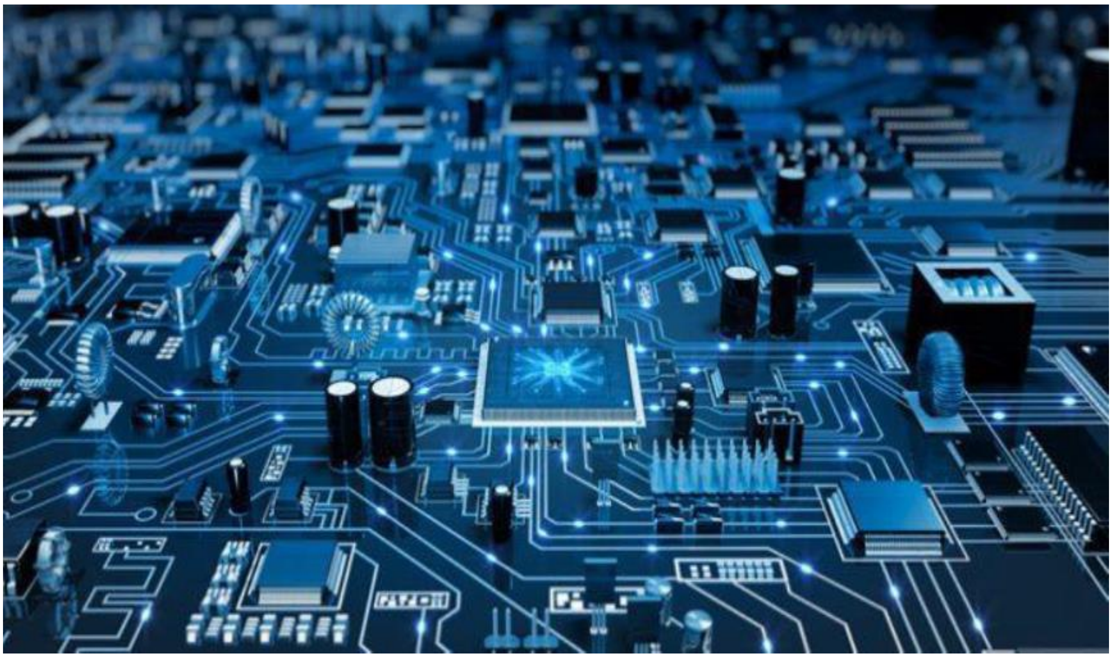
Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Prof. Adnan Ali
Lecture:10

Mode Unit 8

Applications of Operational Amplifiers (Part 3)

For
Students of Third Stage
Department of Medical Instrumentation Techniques Engineering



By

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Dep. Medical
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Engineering

2025

1. Overview

a. Target population:

For students of third class of Department of Medical Instrumentation Techniques Engineering, Electrical Engineering Technical College, Middle Technical University, Baghdad, Iraq.

b. Rationale:

An operational amplifier (often op amp or Op-Amp) is a DC-coupled high-gain electronic voltage amplifier with a differential input and, usually, a single-ended output. In this configuration, an op amp produces an output potential (relative to circuit ground) that is typically 100,000 times larger than the potential difference between its input terminals. Operational amplifiers had their origins in analog computers, where they were used to perform mathematical operations in linear, non-linear, and frequency-dependent circuits.

c. Objectives:

The student will be able after finishing lecture on:

- Draw the waveform of Operational Amplifier (Op-Amp).
- Identify the main types of Operational Amplifier (Op-Amp).

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Operation amplifier (part 3)

Integrator Op-Amp

Differentiator Op-Amp

2.4 Op-Amp Integrators:

An electronic integrator is a device that produces an output waveform whose value at any instant of time equals the total area under the input waveform up to that point in time. A mathematical integration, the process produces the time varying function $\int_0^t v_{in} dt$. To illustrate this concept, suppose the input to an electronic integrator is the dc level E volts, which is first connected to integrator at an instant of time we will call $t = 0$. Refer to Fig. 2-15. The plot of the dc "waveform" versus time is simply a horizontal line at level E volts, since the dc voltage is constant. The more time that we allow to pass, the greater the area that accumulates under the dc waveform. At any time-point t, the total area under the input waveform between time 0 and time t is (height) × (width) = Et, volts, as illustrated in figure. For example, if $E = 5$ V dc, then the output will be 5 V at $t = 1$ s, 10 V at $t = 2$ s, 15 V at $t = 3$ s, and so forth. We see that the output is the ramp voltage $v(t) = Et$.

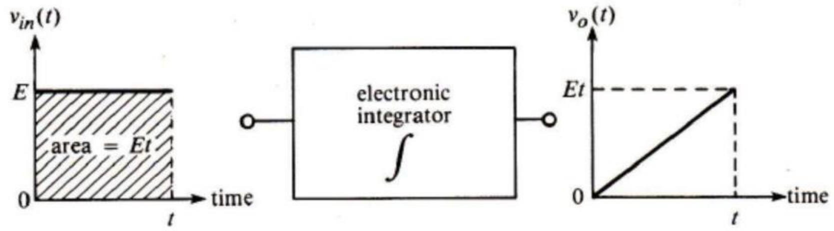


Fig. 2-15

Fig. 2-16 shows how an electronic integrator is constructed using an operational amplifier. The component in the feedback path is capacitor C, and the amplifier is operated in an inverting configuration. To represent integration of the voltage v between time 0 and time t, we are assuming zero input offset, the output of this circuit is

$$v_o(t) = \frac{-1}{C}\int_0^t v_{in} dt \qquad [2\text{-}20]$$

This equation shows that the output is the (inverted) integral of the input, multiplied by the constant $1/R_1C$. If this circuit were used to integrate the dc waveform shown in Fig. 2-15, the output would be a negative-going ramp ($v_o = -Et/R_1C$).

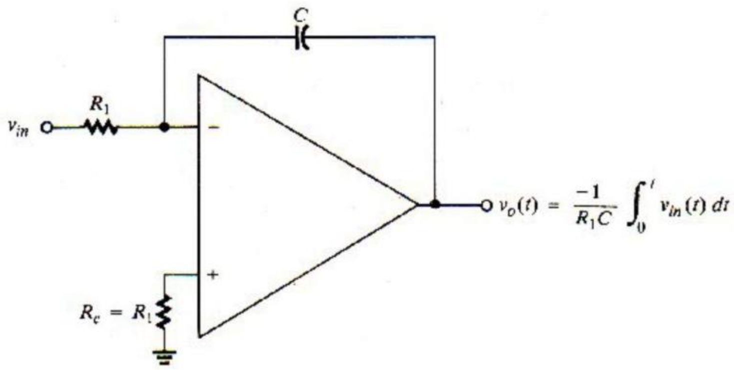


Fig. 2-16

Now we demonstrate why the circuit of Fig. 2-16 performs integration. Since the current into the − input is 0, we have, from Kirchhoff's current law;

$$i_1 + i_C = 0,$$

where i_1 is the input current through R_1 and i_C is the feedback current through the capacitor. Since $v^- = 0$, the current in the capacitor is

$$i_C = C\frac{dvo}{dt} \quad => \quad \frac{v_{in}}{} + C\frac{dvo}{dt} = 0 \quad \text{or} \quad \frac{dv_o}{dt} = \frac{-1}{C}v_{in}.$$

Integrating both sides of the last equation with respect to t, we obtain

$$v_o(t) = \frac{-1}{C}\int_0^t v_{in} dt.$$

It can be shown, using calculus, that the mathematical integral of the sine wave $A \sin \omega t$ is

$$\int(A \sin wt)\, dt = \frac{-A}{}\sin(wt + 90) = \frac{-A}{}\cos(wt).$$

Therefore, if the input to the inverting integrator in Fig. 2-16 is $v_{in} = A \sin \omega t$, the output is

$$v_o = \frac{-1}{C}\int(A \sin wt)\, dt = \frac{-A}{C}(-coswt) = \frac{A}{C}coswt \qquad [2\text{-}21]$$

The most important fact revealed by Eqn. [2-21] is that the output of an integrator with sinusoidal input is a sinusoidal waveform whose amplitude is inversely proportional to its frequency. This observation follows from the presence of ω (= $2\pi f$) in the denominator of Eqn. [2-21].

A gain magnitude is the ratio of the peak value of the output to the peak value of the input:

$$\left|\frac{vo}{v_{in}}\right| = \frac{A}{A \quad C} = \frac{1}{C} \qquad [2\text{-}22]$$

This equation clearly shows that gain is inversely proportional to frequency.

Although high-quality, precision integrators are constructed as shown in Fig. 2-16 for use in low-frequency applications such as analog computers, these applications require high-quality amplifiers with extremely small offset voltages. As mentioned earlier, any input offset is integrated as if it were a dc signal input and will eventually cause the amplifier to saturate. To eliminate this problem in *practical integrators* using general purpose amplifiers, a resistor is connected in parallel with the feedback capacitor, as shown in Fig. 2-17. Since the capacitor is an open circuit as dc is concerned, the dc closed-loop gain of the integrator is $-R_f/R_1$. At high frequencies, X_C is much smaller than R_f, so the parallel combination of C and R_f is essentially the same as C alone, and signals are integrated as usual.

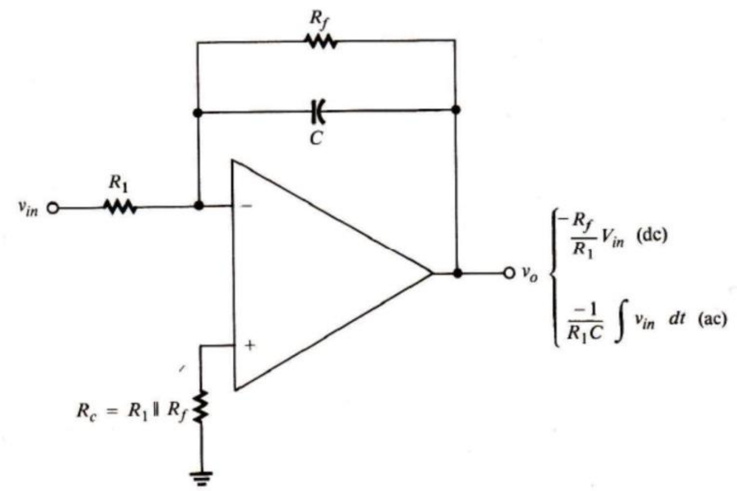


Fig. 2-17

While the feedback resistor in Fig. 2-17 prevents integration of dc inputs, it also degrades the integration of low-frequency signals. At frequencies where the capacitive reactance of C is comparable in value to R_f, the net feedback impedance is not predominantly capacitive and true integration do es not occur. As a rule, we can say that satisfactory integration will occur at frequencies much greater than the frequency at which $X_C = R_f$. That is, for integrator action we want

$$X_C << R_f \;=>\; \frac{1}{2\pi fC} << R_f \;=>$$

$$f >> \frac{1}{2\pi\, _fC} \qquad [2\text{-}23]$$

The frequency f_c where $X_C = R_f$,

$$f_c = \frac{1}{2\pi\, _fC} \qquad [2\text{-}24]$$

Eqn. [2-24] defines a break frequency, f_c, in the Bode plot of the practical integrator. As shown in Fig. 2-18, at frequencies well above f_c, the gain falls off at the rate of −20 dB/decade, like that of an ideal integrator, and at frequencies below f_c, the gain approaches its dc value of R_f/R_1. Because the integrator's output amplitude or gain decreases with frequency, it is a kind of low-pass filter.

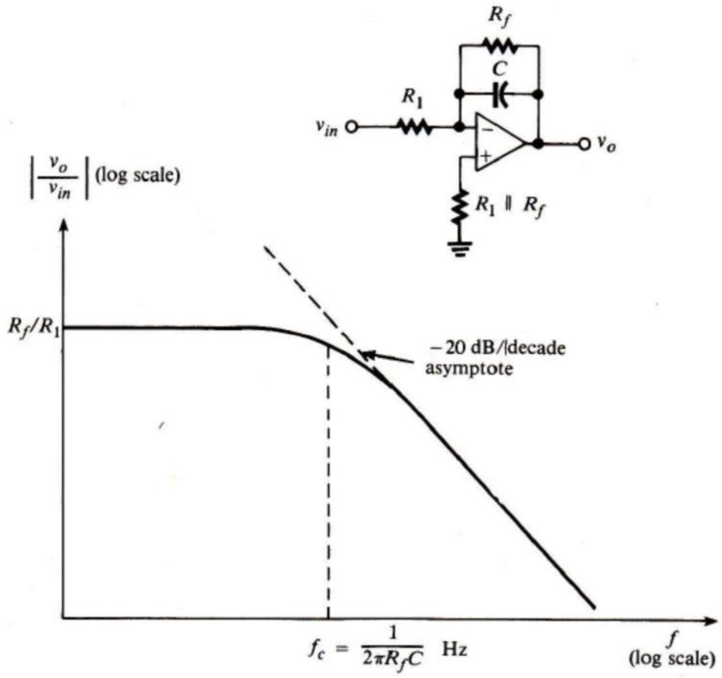


Fig. 2-18

In closing our discussion of integrators, we should note that it is possible to scale and integrate several input signals simultaneously, using an arrangement similar to the linear combination circuit studied earlier. Fig. 2-19 shows a practical, three-input integrator that performs the following operation at frequencies above f_c:

$$v_o = -\int \left(\frac{1}{C} v_1 + \frac{1}{{}_zC} v_2 + \frac{1}{{}_3C} v_3 \right) dt$$

$$v_o = -\frac{1}{C}\int v_1 dt - \frac{1}{{}_zC}\int v_2 dt - \frac{1}{{}_3C}\int v_3 dt \qquad [2\text{-}25]$$

If $R_1 = R_2 = R_3 = R$, then

$$v_o = \frac{-1}{C}\int (v_1 + v_2 + v_3) dt \qquad [2\text{-}26]$$

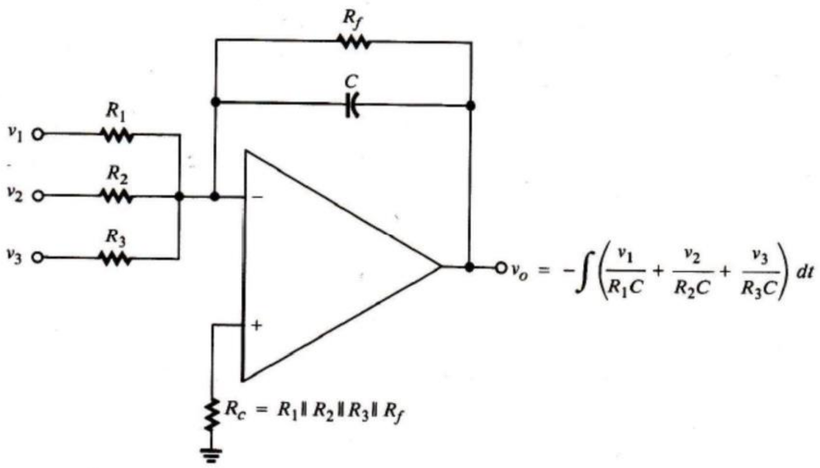


Fig. 2-19

Exercise 2-9:

(a) Find the peak value of the output of the ideal integrator shown in Fig. 2-20. The input is $v_{in} = 0.5 \sin(100t)$ V.

(b) Repeat, when $v_{in} = 0.5 \sin(10^3 t)$ V.

[Answers: (a) $v_o = 5 \cos(100t)$ V => peak value = 5 V,
(b) $v_o = 0.5 \cos(1000t)$ V => peak value = 0.5 V]

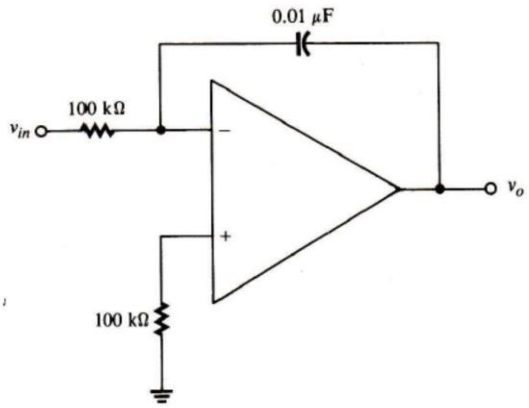


Fig. 2-20

Exercise 2-10:

Design a practical integrator that

(a) integrates signals with frequencies down to 100 Hz, and

(b) produces a peak output of 0.1 V when v_{in} is a 10 V peak sine wave at frequency 10 kHz. Choose $C = 0.01$ μF.

Find the dc component in the output when there is a +50 mV dc input.

[Answer: $R_f = 1.59$ MΩ, $R_1 = 159$ kΩ, $R_c = 145$ kΩ, $v_o = -0.5$ V, Fig. 2-21]

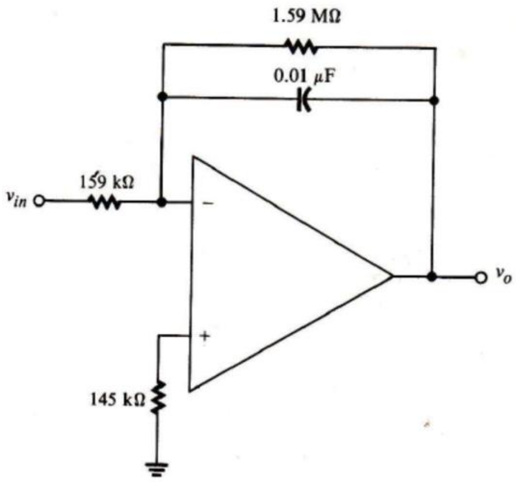
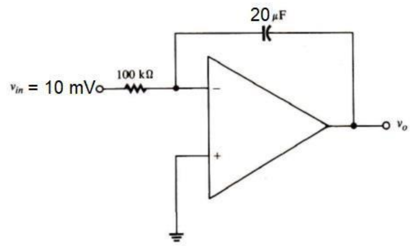


Fig. 2-21

Ex: An integrator circuit shown below, has R=100K and C 20 Micro F. Determine the output voltage when the input DC voltage of 10mv is applied.

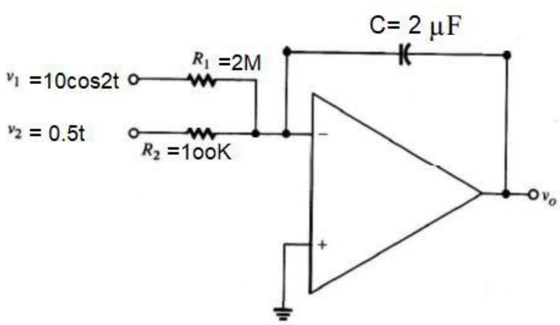


Ans:

$$V_0 = -\frac{1}{RC}\int_0^t V_{in}\,dt$$

$$V_0 = -\frac{1}{100 \times 10^3 \times 20 \times 10^{-6}}\int_0^t 5 \times 10^{-3}\,dt = -5t\ mV$$

Ex: for the following circuit, find V_o



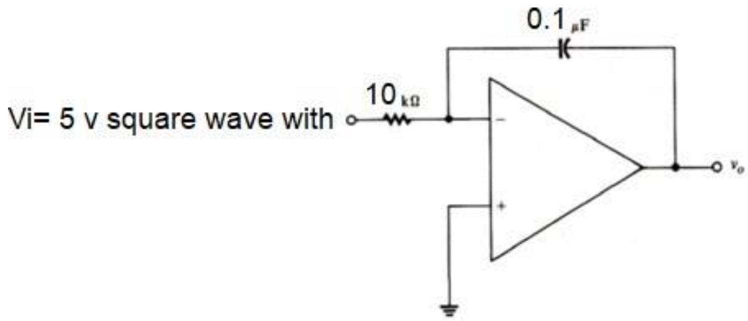
Ans:

$$V_0 = -\frac{1}{R_1C}\int_0^t V_1\,dt + \left(-\frac{1}{R_2C}\int_0^t V_2\,dt\right)$$

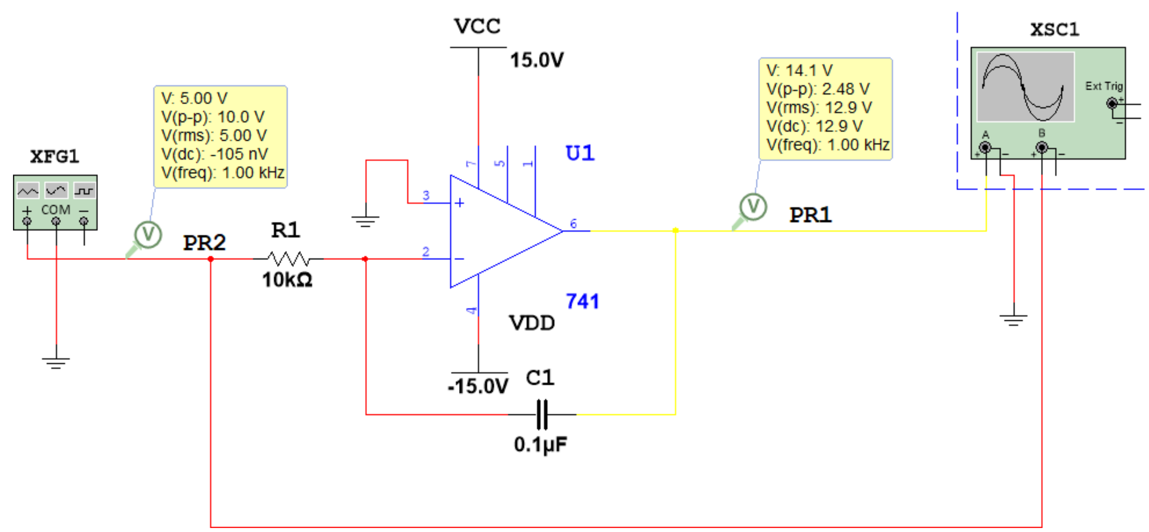
$$V_0 = -\frac{1}{R_1C}\int_0^t 10cos2t\,dt - \frac{1}{R_2C}\int_0^t 0.5t\,dt$$

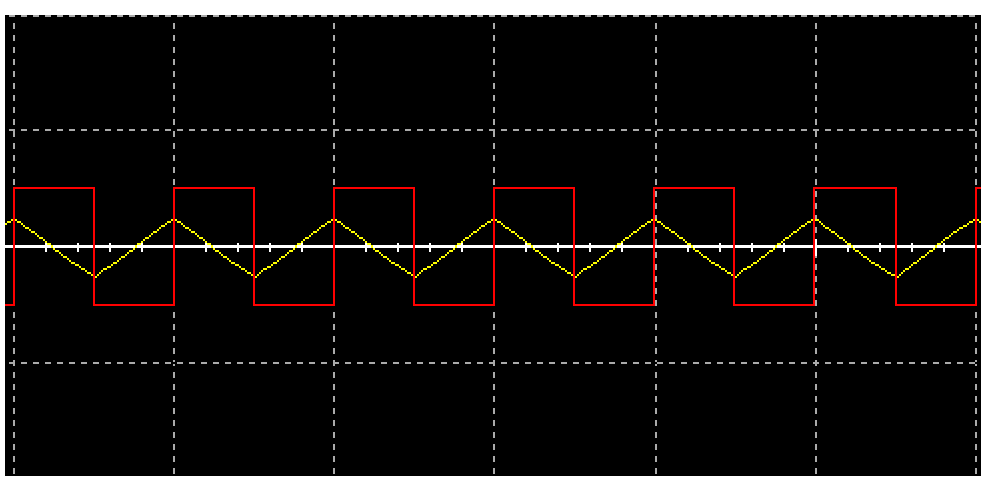
$$V_0 = -\frac{5}{6}sin2t - \frac{5}{4}t^2\ \ \text{mV}$$

Ex: For the following circuit, draw the waveforms



Answer in Multisim:





2.5 Op-Amp differentiators:

An electronic differentiator produces an output waveform whose value at any instant of time is equal to the rate of change of the input at that point in time. Fig. 2-22 demonstrates the operation of an ideal electronic differentiator. The input is the ramp voltage $v_{in} = Et$. The rate of change, or slope, of this ramp is a constant E volts/second. Since the rate of change of the input is constant, we see that the output of the differentiator is the constant dc level E volts. We would write

$$\frac{dv_{in}}{dt} = \frac{d(Et)}{dt} = E \qquad [2\text{-}27]$$

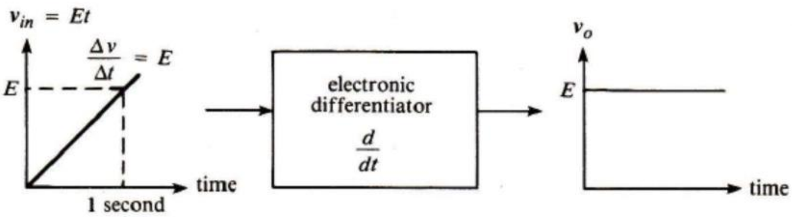


Fig. 2-22

Fig. 2-23 shows how an ideal differentiator is constructed using an operational amplifier. Note that we no w have a capacitive input and a resistive feedback-again, just the opposite of an integrator. It can be shown that the output of this differentiator is

$$v_o = -R_f C \frac{dvin}{dt} \qquad [2\text{-}28]$$

Now, we can show how the circuit of Fig. 2-23 performs differentiation. Since the current into the − terminal is 0, we have, from Kirchhoff's current law, $i_C + i_f = 0$.

Since $v^- = 0$, $v_C = v_{in}$ and $i_C = C\frac{dvin}{dt}$.

Also, $i_f = \frac{vo}{R_f}$, so $C\frac{dvin}{dt} + \frac{vo}{R_f} = 0$ or $v_o = -R_f C\frac{dvin}{dt}$.

If the input to the inverting integrator in Fig. 2-23 is $v_{in} = A\sin wt$, the output is

$$v_o = -R_f C\frac{d(A\sin wt)}{dt} = -AwR_f C\cos(wt) = AwR_f C\sin(wt - 90^\circ) \qquad [2\text{-}29]$$

Eqn. [2-29] shows that when the input is sinusoidal, the amplitude of the output of a differentiator is directly proportional to frequency. Also the output lags the input by 90°, regardless of frequency. The gain of the differentiator is

$$\left|\frac{v_o}{v_{in}}\right| = \frac{AwR_f C}{A} = wR_f C \qquad [2\text{-}30]$$

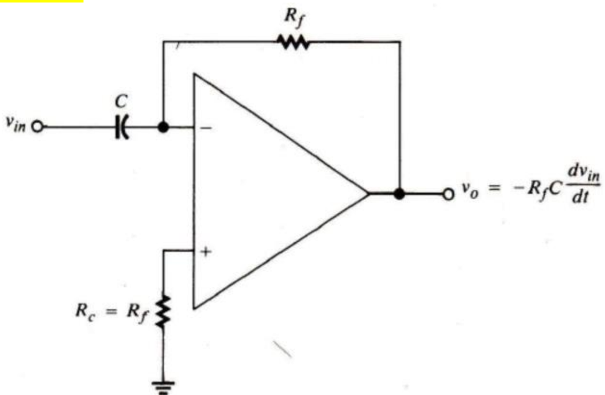


Fig. 2-23

In a *practical differentiator*, the amplification of signals in direct proportion to their frequencies cannot continue indefinitely as frequency increases, because the amplifier has a finite bandwidth. As we have already known, there is some frequency at which the output amplitude must begin to fall off. Nevertheless, it is often desirable to design a practical differentiator so that it will have a break frequency even lower than that determined by the upper cutoff frequency of the amplifier, that is, to roll off its gain characteristic at some relatively low frequency. This action is accomplished in a practical differentiator by connecting a resistor in series with the input capacitor, as shown in Fig. 2-24. We can understand how this modification achieves the stated goal by considering the net impedance of the R_1C combination at low and high frequencies:

$$Z_{in} = R_1 - j/\omega C \Rightarrow$$

$$|Z_{in}| = \sqrt{R_1^2 + (1/\omega C)^2}\ .$$

At very small values of ω, Z_{in}, is dominated by the capacitive reactance component, so the combination is essentially the same as C alone, and differentiator action occurs. At very high values of ω, $1/\omega C$ is negligible, so Z_{in} is essentially the resistance R_1, and the circuit behaves like an ordinary inverting amplifier.

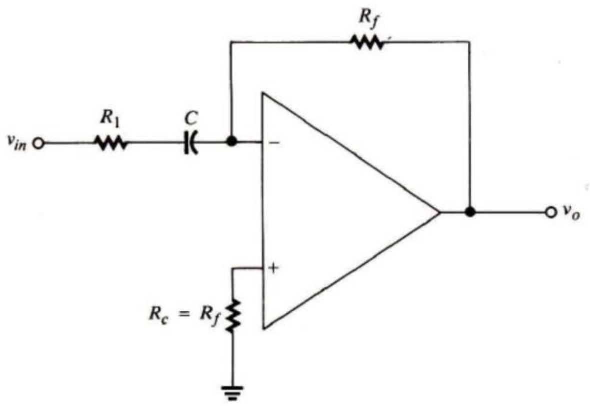


Fig. 2-24

The break frequency f_b beyond which differentiation no longer occurs in Fig. 2-24 is the frequency at which $X_C = R_1$:

$$X_C = \frac{1}{2\pi f_b C} = R_1 \Rightarrow f_b = \frac{1}{2\pi R_1 C} \qquad [2\text{-}31]$$

In designing a practical differentiator, the break frequency should be set well above the highest frequency at which accurate differentiation is desired:

$$f_b >> f_h \qquad [2\text{-}32]$$

where f_h is the highest differentiation frequency. Fig. 2-25 shows Bode plots for the gain of the ideal and practical differentiators. In the low-frequency region where differentiation occurs, note that the gain rises with frequency at the rate of 20 dB/decade. The plot shows that the gain levels off beyond the break frequency f_b and then falls off at −20 dB/decade beyond the amplifier's upper cutoff frequency. Recall that the closed-loop bandwidth, or upper cutoff frequency of the amplifier, is given by

$$f_2 = \beta f_t \qquad [2\text{-}33]$$

where β in this case is $R_1/(R_1 + R_f)$.

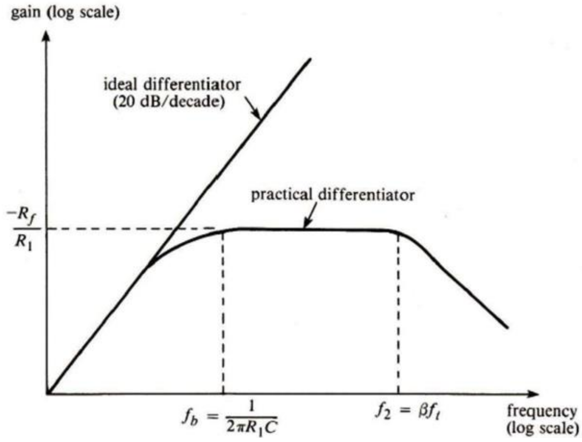


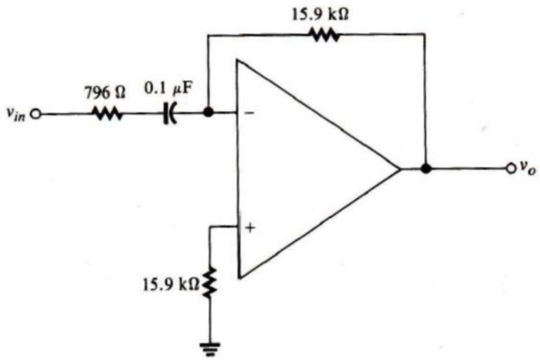
Fig. 2-25

Exercise 2-11:

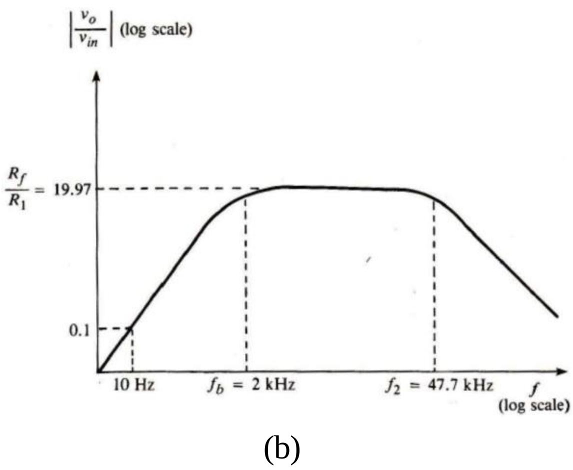
(a) Design a practical differentiator that will differentiate signals with frequencies up to 200 Hz. The gain at 10 Hz should be 0.1. Choose $f_b = 10\ f_h$, and $C = 0.1\ \mu F$.

(b) If the operational amplifier used in the design has a unity-gain frequency of 1 MHz, what is the upper cutoff frequency of the differentiator?

[Answer: (a) $R_1 = 796\ \Omega$, $R_f = 15.9\ k\Omega$, Fig. 2-26(a)
(b) $f_2 = 47.7$ kHz, Fig. 2-26(b)]

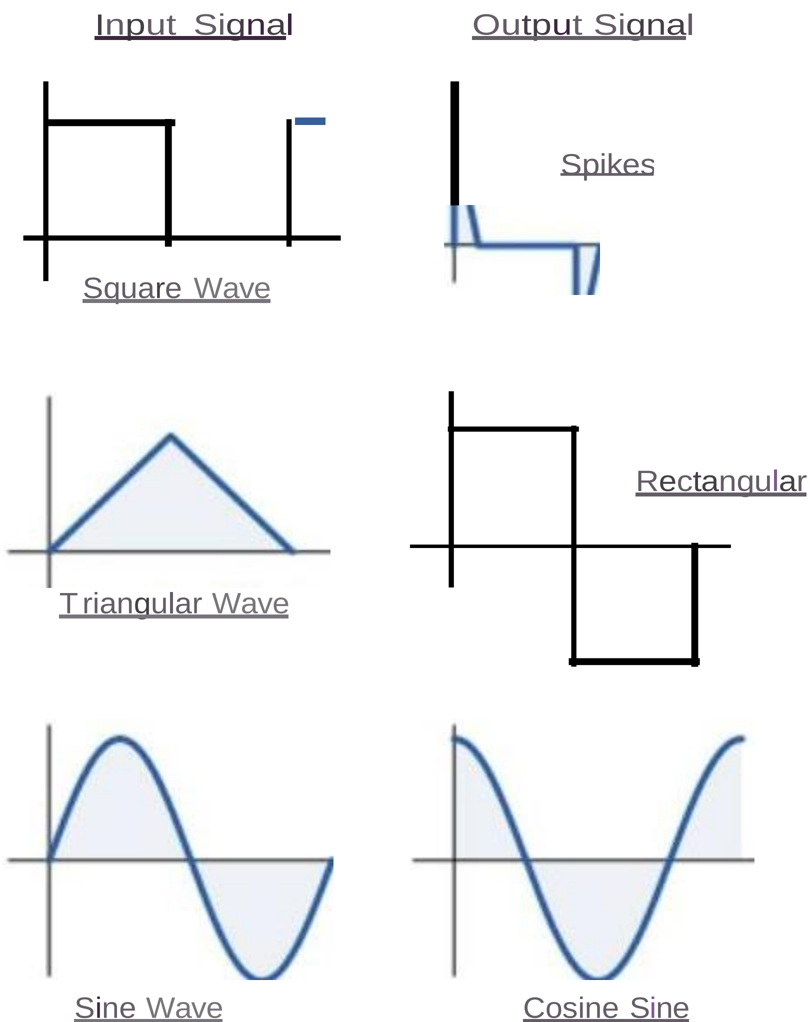


(a)



(b)

Fig. 2-26



Op-amp Differentiator Amplifier