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College of Engineering & Technology

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Lecture No.5

Lecture Title: [Dynamics]

Dynamics

Vibration and dynamics are fundamental subjects in engineering. Serious problems may arise from vibration when a structure is not carefully designed for its dynamic integrity. Vibration can cause malfunction or break down of machines that exhibit unbalance or mis alignment. It can also lead to massive engineering failures such as the collapse of a bridge. Simulation plays an important role in our ability to understand a structure’s dynamic behavior. Through modeling, the dynamic characteristics of a structure can be captured and improved before being put into actual use.

FIGURE
A dynamic force applied to the structure.

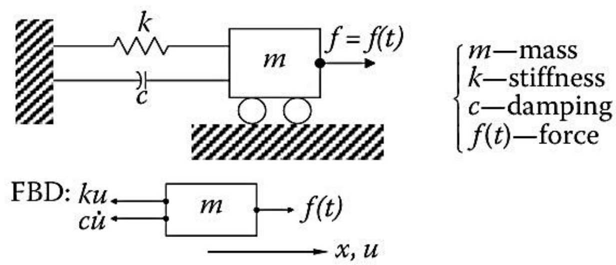

FIGURE
A single DOF system with damping.

A Single DOF System

From the free-body diagram (FBD) and Newton’s law of motion ($ma = f$), we have:

$$m\ddot{u} = f(t) - ku - c\dot{u}$$

that is

$$m\ddot{u} + c\dot{u} + ku = f(t)$$

where u is the displacement, $\dot{u} = du/dt$ the velocity, and $\ddot{u} = d^2u/dt^2$ the acceleration.

Free Vibration (no applied force to the mass or $f(t) = 0$):

Free vibration occurs when a mass is moved away from its rest position due to initial conditions.

Assuming zero damping ($c = 0$) in a free vibration, Equation 8.1 becomes:

$$m\ddot{u} + ku = 0$$

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To solve for such nontrivial solutions, we assume:

$$u(t) = U \sin \omega t$$

where ω is the circular frequency of oscillation and U the amplitude.

$$-U\omega^2.m \sin \omega t + kU \sin \omega t = 0$$

that is $$[-\omega^2 m + k]U = 0$$

For nontrivial solutions for U, we must have:

$$[-\omega^2 m + k] = 0$$

which yields

$$\omega = \sqrt{\frac{k}{m}}$$

This is the circular *natural frequency* of the single DOF system (rad/s). The cyclic frequency ($1/s = \text{Hz}$) is $\omega/2\pi$.

The typical response of the system in undamped free vibration is sketched in Figure 3.

For nonzero damping c, where

$$0 < c < c_c = 2m\omega = 2\sqrt{km} \quad (c_c = \text{critical damping})$$

we have the *damped natural frequency*:

$$\omega_d = \omega\sqrt{1-\xi^2}$$

where

$$\xi = c/c_c$$

is called the damping ratio.

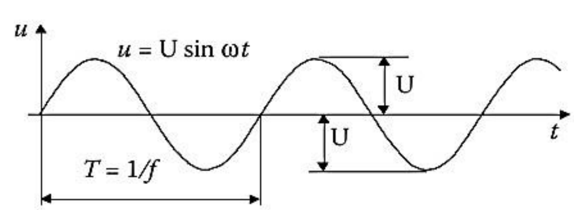

FIGURE .3
Typical response in an undamped free vibration.

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A Multi-DOF System

For a multi-DOF system, the equation of motion can be written as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t)$$

in which:

u—nodal displacement vector;
M—mass matrix;
C—damping matrix;
K—stiffness matrix;
f—forcing vector.

The physical meaning of Equation is

Inertia forces + Damping forces + Elastic forces = Applied forces

We already know how to determine the stiffness matrix **K** for a structure, as discussed in previous chapters. In vibration analysis, we also need to determine the mass matrix and damping matrix for the structure.

- Mass Matrices

Mass Matrices There are two types of mass matrices: lumped mass matrices and consistent mass matrices. The former is empirical and easier to determine, and the latter is analytical and more involved in their computing. We use a bar element to illustrate the lumped mass matrix (Figure 5). For this bar element, the lumped mass matrix for the element is found to be

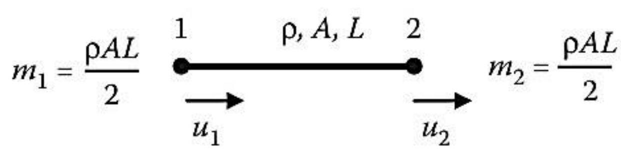

FIGURE.5
The lumped mass for a 1-D bar element.

$$\mathbf{m} = \begin{bmatrix} \frac{\rho AL}{2} & 0 \\ 0 & \frac{\rho AL}{2} \end{bmatrix}$$

which is a diagonal matrix and thus is easier to compute.

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In general, we apply the following element *consistent mass matrix*:

$$\mathbf{m} = \int_V \rho \mathbf{N}^T \mathbf{N} \, dV$$

where **N** is the same shape function matrix as used for the displacement field and V is the volume of the element.

Equation is obtained by considering the kinetic energy within an element:

$$\begin{aligned} K &= \frac{1}{2}\dot{\mathbf{u}}^T \mathbf{m} \dot{\mathbf{u}} \qquad\qquad (\text{cf. } \frac{1}{2} m v^2) \\ &= \frac{1}{2}\int_V \rho \dot{u}^2 \, dV = \frac{1}{2}\int_V \rho (\dot{u})^T \dot{u} \, dV \\ &= \frac{1}{2}\int_V \rho (\mathbf{N}\dot{\mathbf{u}})^T (\mathbf{N}\dot{\mathbf{u}}) \, dV \\ &= \frac{1}{2}\dot{\mathbf{u}}^T \underbrace{\int_V \rho \mathbf{N}^T \mathbf{N} \, dV}_{\mathbf{m}} \dot{\mathbf{u}} \end{aligned}$$

For the bar element (linear shape function), the *consistent mass matrix* is

$$\begin{aligned} \mathbf{m} &= \int_V \rho \mathbf{N}^T \mathbf{N} \, dV = \int_V \rho \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} \begin{bmatrix} 1-\xi & \xi \end{bmatrix} AL \, d\xi \\ &= \rho AL \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \begin{matrix} \ddot{u}_1 \\ \ddot{u}_2 \end{matrix} \end{aligned}$$

which is a nondiagonal matrix.

Similar to the formation of the global stiffness matrix **K**, element mass matrices are established in local coordinates first, then transformed to global coordinates, and finally assembled together to form the global structure mass matrix **M**.

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Damping

There are two commonly used models for viscous damping: *proportional damping* (also called Rayleigh damping) and *modal damping.*

In the *proportional damping* model, the damping matrix **C** is assumed to be proportional to the stiffness and mass matrices in the following fashion:

$$\mathbf{C} = \alpha\mathbf{K} + \beta\mathbf{M}$$

where the constants α and β are found from the following two equations:

$$\xi_1 = \frac{\alpha\omega_1}{2} + \frac{\beta}{2\omega_1}, \quad \xi_2 = \frac{\alpha\omega_2}{2} + \frac{\beta}{2\omega_2}$$

with ω_1, ω_2, ξ_1 and ξ_2 (damping ratios) being specified by the user. The plots of the above two equations are shown in Figure 8.7.

In the *modal damping* model, the viscous damping is incorporated in the modal equations. The *modal damping* can be introduced as

$$\mathbf{C}_\phi = \begin{bmatrix} 2\xi_1\omega_1 & 0 & \cdots & 0 \\ 0 & 2\xi_2\omega_2 & & \\ \vdots & & \ddots & \vdots \\ 0 & & \cdots & 2\xi_n\omega_n \end{bmatrix}$$

where ξ_i is the damping ratio at mode *i* of a *n*-DOF system.

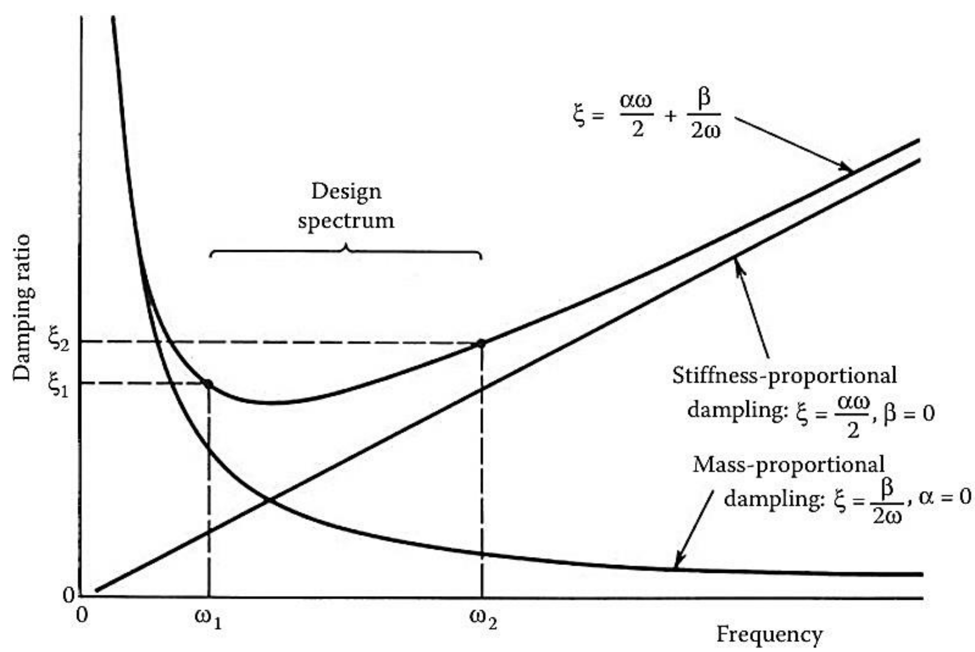

FIGURE .7
Two equations for determining the proportional damping coefficients.

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“Dynamic behavior” may be one or more of the following: –

– Vibration characteristics

• how the structure vibrates and at what frequencies

– Effect of harmonic loads.

– Effect of seismic or shock loads.

– Effect of random loads. Effect of random loads.

– Effect of time-varying loads

- **The linear general equation of motion, which will be referred to throughout this course, is as follows (matrix form):**

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F\}$$

$[M]$ = structural mass matrix $\quad\{\ddot{u}\}$ = nodal acceleration vector

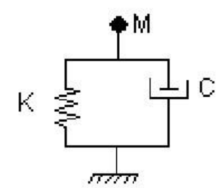
$[C]$ = structural damping matrix $\quad\{\dot{u}\}$ = nodal velocity vector

$[K]$ = structural stiffness matrix $\quad\{u\}$ = nodal displacement vector

$\{F\}$ = applied load vector

- **Note that this is simply a force balance:**

$$\overbrace{[M]\{\ddot{u}\}}^{F_{inertial}}+\overbrace{[C]\{\dot{u}\}}^{F_{damping}}+\overbrace{[K]\{u\}}^{F_{stiffness}}=\overbrace{\{F\}}^{F_{applied}}$$



- **Different analysis types solve different forms of this equation.**
  - **Modal**
    - **F(t) set to zero; [C] usually ignored.**
  - **Harmonic Response**
    - **F(t) and u(t) assumed to be sinusoidal.**
  - **Response Spectrum**
    - **Input is a known spectrum of response magnitudes at varying frequencies in known directions.**
  - **Random Vibration**
    - **Input is a probabilistic spectrum of input magnitudes at varying frequencies in known directions.**
  - **Transient**
    - **The complete, general form of the equation is solved.**