Module Title: Fundamental of Electrical Engineering (AC)

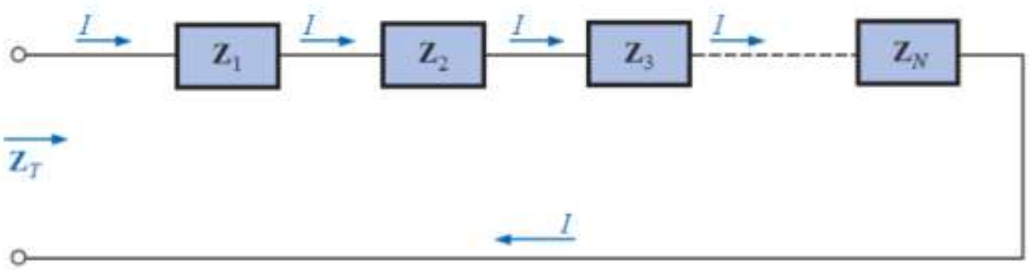
Module Code:	**UOMU024021**

Week 5

5. SERIES CONFIGURATION IN AC CIRCUITS

The overall properties of series ac circuits are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances:

Example: Draw the impedance diagram for the circuit shown below, and find the total impedance.



$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N$$

5.1 Series and Parallel Waveforms AC Circuits Resistive Elements

Resistive Elements:

$$I_m = \frac{V_m}{R}$$
$$V_m = I_m R$$

In phaser form,

$$v = V_m sin\omega t = V\angle 0$$

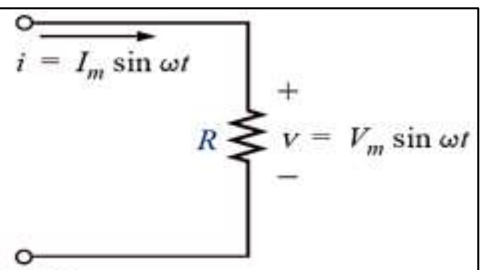
Where $V = 0.707V_m$

Applying Ohm's law and using phaser algebra, we have

$$I = \frac{V\angle 0}{R\angle 0}$$

So that in the time domain,

$$i = \sqrt{2}\frac{V}{R}sin\omega t$$



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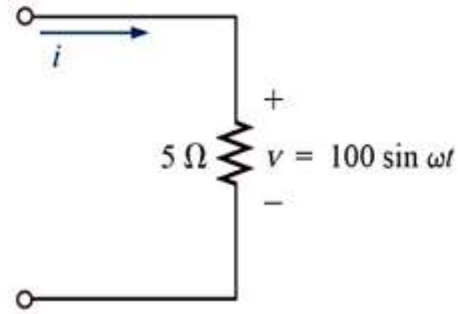
Example: 1

Using complex algebra, find the current i for the circuit shown below. Sketch the waveforms of v and i.

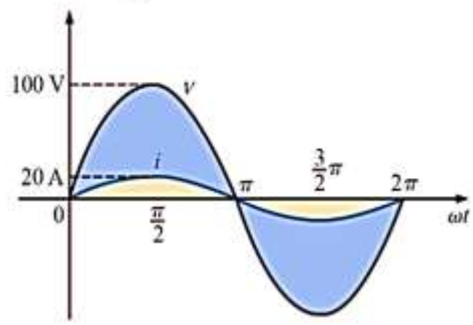


Solution

$v = 100 sin\omega t = 70.7\angle 0$

$I = \frac{V\angle 0}{Z_R\angle 0} = \frac{70.7\angle 0}{5\angle 0} = 14.14\angle 0\ A$

$i = \sqrt{2} \times 14.14\ sin\omega t = 20\ sin\omega t\ A$



Inductive Reactance

The voltage leads the current by 90° and that the reactance of the coil XL is determined by ωL.

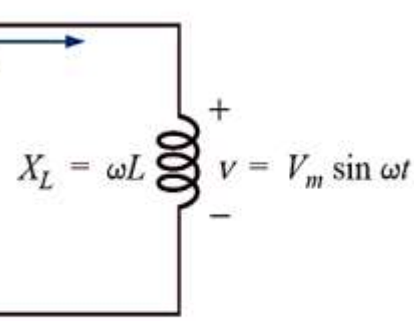
$v = V_m sin\omega t = V\angle 0$

By Ohm's law,

$\mathbf{I} = \frac{V\angle 0}{X_L\angle 90} = \frac{V}{X_L}\angle -90$

so that in the time domain,

$i = \sqrt{2}\frac{V}{X_L}sin(\omega t - 90)$



$Z_L = X_L\angle 90$

Example:

Using complex algebra, find the current i for the circuit shown below. Sketch the v and i curves.

Solution:

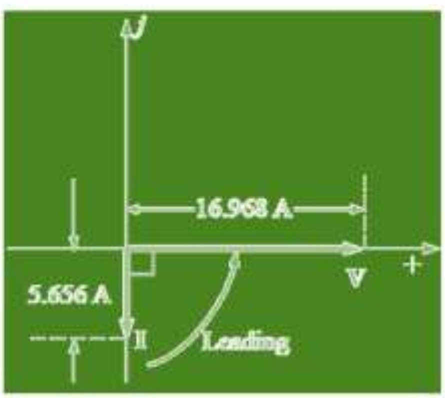
$v = 24 sin\omega t$

In polar form

$\mathbf{V} = 16.968\angle 0$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_L}} = \frac{V\angle 0}{X_L\angle 90} = \frac{16.968\angle 0}{3\angle 90} = 5.656\ \text{A}\angle -90$$

$$i = \sqrt{2}(5.656)\ sin(\omega t - 90) = 8\ sin(\omega t - 90)$$



Capacitive Reactance

The current leads the voltage by 90° and that the reactance of the capacitor X_C is determined by $\frac{1}{\omega C}$.

$v = V_m sin\omega t$

In polar form

$\mathbf{V} = V\angle 0$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_C}} = \frac{V\angle 0}{X_C\angle -90} = \frac{V}{X_C}\angle 90$$

$$i = \sqrt{2}\frac{V}{X_C} sin(\omega t + 90)$$

$\mathbf{Z_C} = X_C\angle -90$

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Example:
Using complex algebra, find the current i for the circuit shown below. Sketch the v and i curves.

solution:

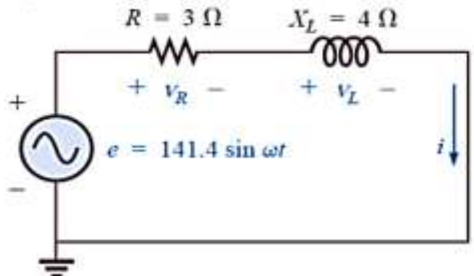
$v = 15\ sin\omega t$
In polar form
$\mathbf{V} = 10.605\ \angle 0$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_C}} = \frac{V\angle 0}{X_C\angle -90} = \frac{10.605\ \angle 0}{2\angle -90} = 5.303\ \text{A}\angle 90$$

$$i = \sqrt{2}\frac{V}{X_C} sin(\omega t + 90) = 7.5\ sin(\omega t + 90)$$

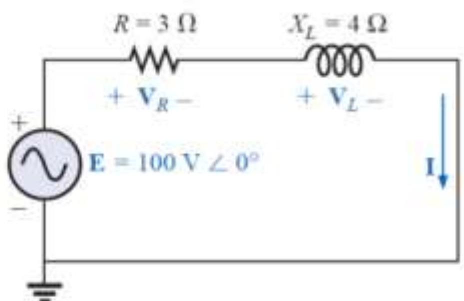
(1) R-L Circuits:

EXAMPLE: Determine the input impedance to the series network and find i , Vr,VL. Draw the impedance diagram.



Solutions:

$e = 141.4\ \sin \omega t \Rightarrow \mathbf{E} = 100\ \text{V}\ \angle 0^\circ$



$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 3\ \Omega\ \angle 0^\circ + 4\ \Omega\ \angle 90^\circ = 3\ \Omega + j4\ \Omega$$

$$\mathbf{Z}_T = \mathbf{5\ \Omega\ \angle 53.13^\circ}$$

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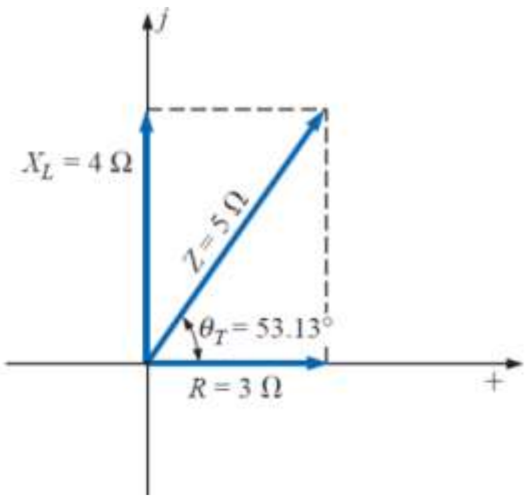
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\text{ V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ} = \mathbf{20\ A\angle -53.13^\circ}$$

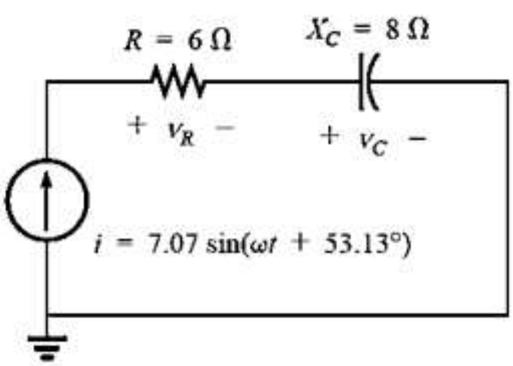
$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (20\text{ A}\angle -53.13^\circ)(3\ \Omega\angle 0^\circ) = \mathbf{60\ V\angle -53.13^\circ}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (20\text{ A}\angle -53.13^\circ)(4\ \Omega\angle 90^\circ) = \mathbf{80\ V\angle 36.87^\circ}$$



(2) R-C Circuits

EXAMPLE: Determine the input impedance to the series network and find E , Vr,Vc. Draw the impedance diagram.



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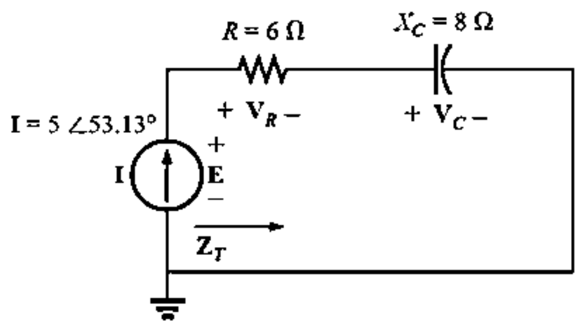
Solutions:

$i = 7.07 \sin(\omega t + 53.13°) \Rightarrow \mathbf{I} = 5 \text{ A} \angle 53.13°$



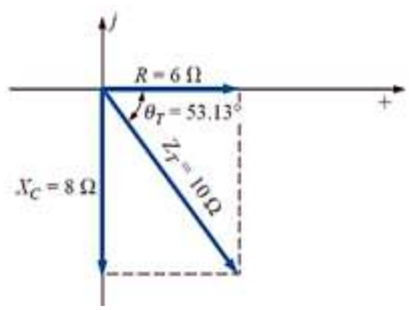
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 6\ \Omega \angle 0° + 8\ \Omega \angle -90° = 6\ \Omega - j8\ \Omega$$

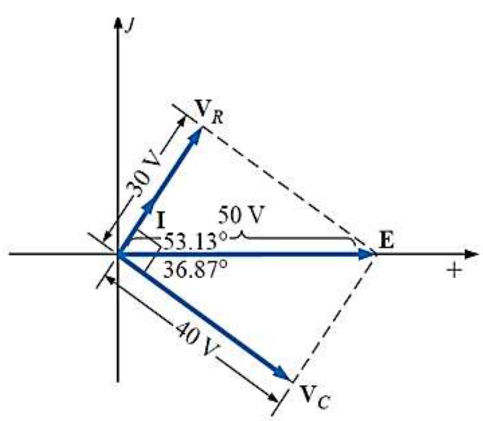
$$\mathbf{Z}_T = 10\ \Omega \angle -53.13°$$

$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = (5 \text{ A} \angle 53.13°)(10\ \Omega \angle -53.13°) = 50 \text{ V} \angle 0°$$

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0°) = (5 \text{ A} \angle 53.13°)(6\ \Omega \angle 0°) = 30 \text{ V} \angle 53.13°$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90°) = (5 \text{ A} \angle 53.13°)(8\ \Omega \angle -90°) = 40 \text{ V} \angle -36.87°$$





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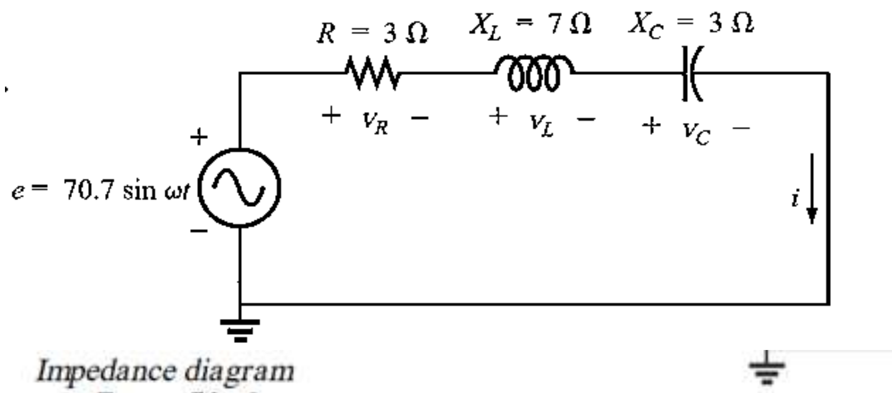
(3) R-L-C Circuits

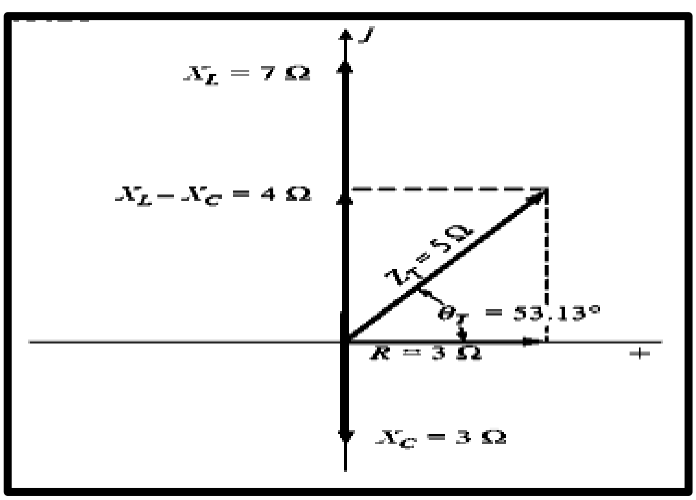
Example :

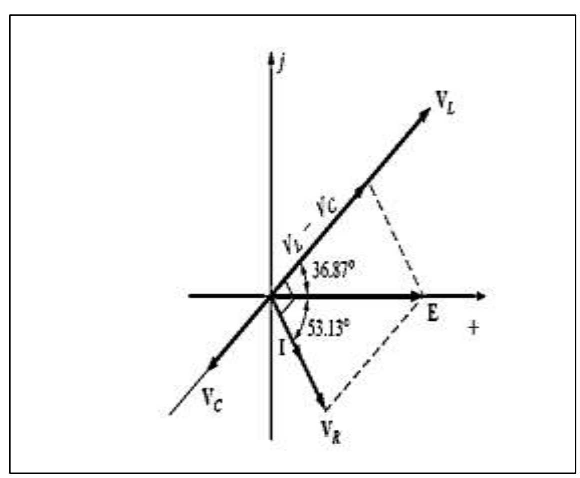
$$\mathbf{Z_T} = \mathbf{Z_R} + \mathbf{Z_L} + \mathbf{Z_C} = R + jX_L - jX_C$$
$$\mathbf{Z_T} = 3 + j7 - j3 = 3 + j4 = 5\angle 53.13^\circ$$



Impedance diagram

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z_T}} = \frac{50\angle 0}{5\angle 53.13^\circ} = 10\angle -53.13^\circ \text{ A}$$
$$\mathbf{V}_R = \mathbf{I}\mathbf{Z_R} = 3 \times 10\angle -53.13^\circ = 30\angle -53.13^\circ \text{ V}$$
$$\mathbf{V}_L = \mathbf{I}\mathbf{Z_L} = 7\angle 90 \times 10\angle -53.13^\circ = 70\angle 36.87^\circ \text{ V}$$
$$\mathbf{V}_C = \mathbf{I}\mathbf{Z_C} = 3\angle -90 \times 10\angle -53.13^\circ = 30\angle -143.13^\circ \text{ V}$$





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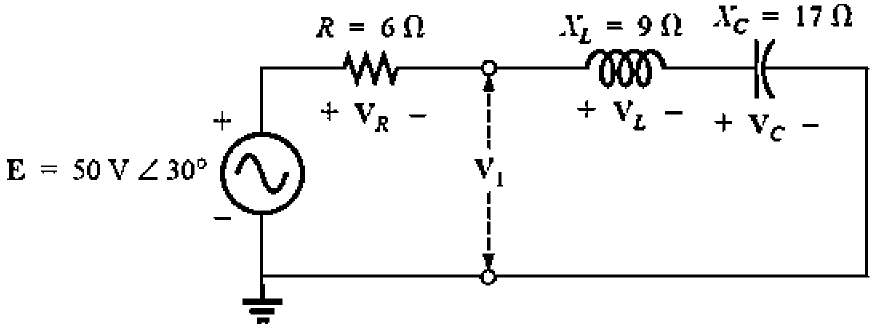
EXAMPLE: Using the voltage divider rule, find the unknown voltages VR, VL, VC, and V1 for the circuit.



Solutions:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega\ \angle 0^\circ)(50\ \text{V}\ \angle 30^\circ)}{6\ \Omega\ \angle 0^\circ + 9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ}$$

$$= \frac{300\ \angle 30^\circ}{6 + j\,9 - j\,17} = \frac{300\ \angle 30^\circ}{6 - j\,8}$$

$$= \frac{300\ \angle 30^\circ}{10\ \angle -53.13^\circ} = \mathbf{30\ V\ \angle 83.13^\circ}$$

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{450\ \text{V}\angle 120^\circ}{10\ \angle -53.13^\circ}$$

$$= \mathbf{45\ V\ \angle 173.13^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ} = \frac{850\ \text{V}\angle -60^\circ}{10\ \angle -53^\circ}$$

$$= \mathbf{85\ V\ \angle -6.87^\circ}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ)(50\ \text{V}\ \angle 30^\circ)}{10\ \Omega\ \angle -53.13^\circ}$$

$$= \frac{(8\ \angle -90^\circ)(50\ \angle 30^\circ)}{10\ \angle -53.13^\circ}$$

$$= \frac{400\ \angle -60^\circ}{10\ \angle -53.13^\circ} = \mathbf{40\ V\ \angle -6.87^\circ}$$

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