Module Title: Fundamental of Electrical Engineering (AC)

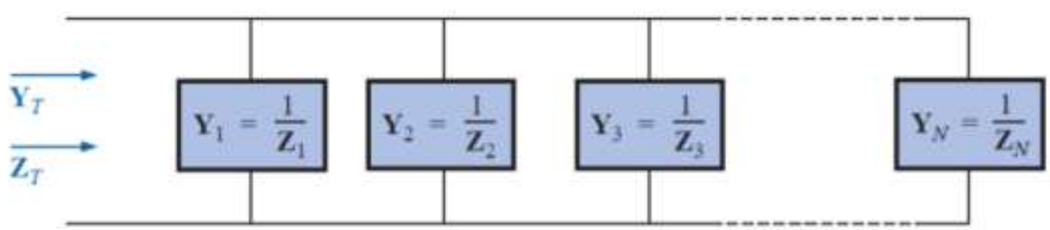
Module Code:	**UOMU024021**

Week 6

6. PARALLEL CONFIGURATION IN AC CIRCUITS

The overall properties of parallel AC circuits are the same as those for DC circuits A Parallel RLC AC Circuit is one where the resistor, inductor, and capacitor are connected in parallel to each other and the AC source. In this configuration, the voltage across each component is the same, but the currents through them different.

Example: Draw the impedance diagram for the circuit shown below, and find the total impedance.



$$\boxed{\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N}$$

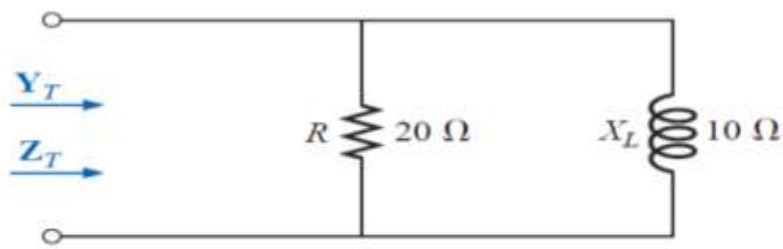
$$\boxed{\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}}$$

For two impedances in parallel

$$\boxed{\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}}$$

EXAMPLE: For the network

a. Find the admittance of each parallel branch.
b. Determine the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.



Solutions:

a. $\mathbf{Y}_R = G \angle 0° = \dfrac{1}{R} \angle 0° = \dfrac{1}{20\ \Omega} \angle 0°$

$= \mathbf{0.05\ S \angle 0° = 0.05\ S} + j\,\mathbf{0}$

$\mathbf{Y}_L = B_L \angle -90° = \dfrac{1}{X_L} \angle -90° = \dfrac{1}{10\ \Omega} \angle -90°$

$= \mathbf{0.1\ S \angle -90° = 0} - j\,\mathbf{0.1\ S}$

b. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\ S + j\,0) + (0 - j\,0.1\ S)$

$= \mathbf{0.05\ S} - j\,\mathbf{0.1\ S} = G - jB_L$

c. $\mathbf{Z}_T = \dfrac{1}{\mathbf{Y}_T} = \dfrac{1}{0.05\ S - j\,0.1\ S} = \dfrac{1}{0.112\ S \angle -63.43°}$

$= \mathbf{8.93\ \Omega \angle 63.43°}$

Or

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(20\ \Omega \angle 0°)(10\ \Omega \angle 90°)}{20\ \Omega + j\,10\ \Omega}$$

$$= \frac{200\ \Omega \angle 90°}{22.36 \angle 26.57°} = \mathbf{8.93\ \Omega \angle 63.43°}$$

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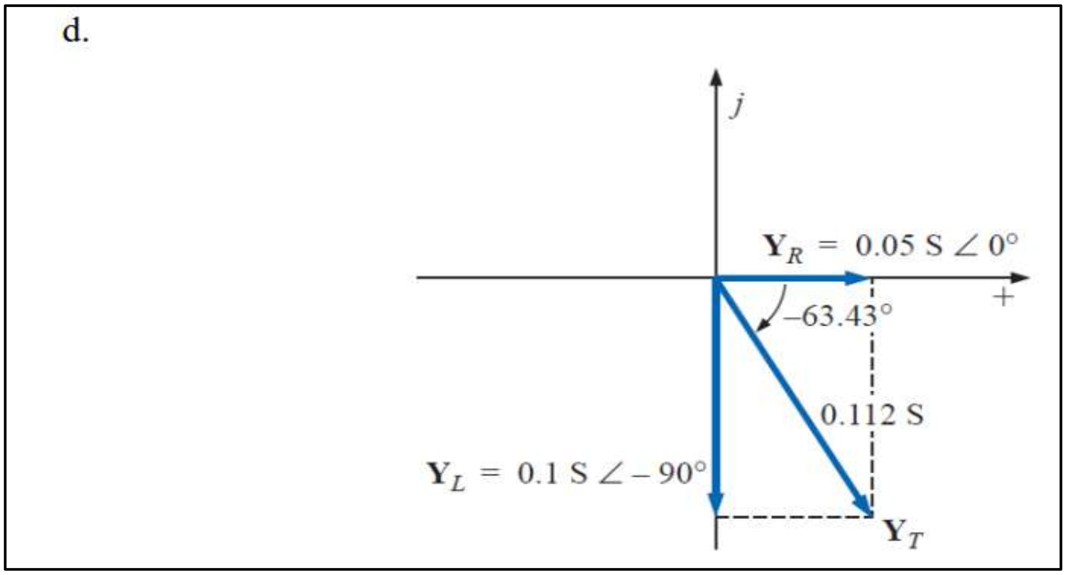
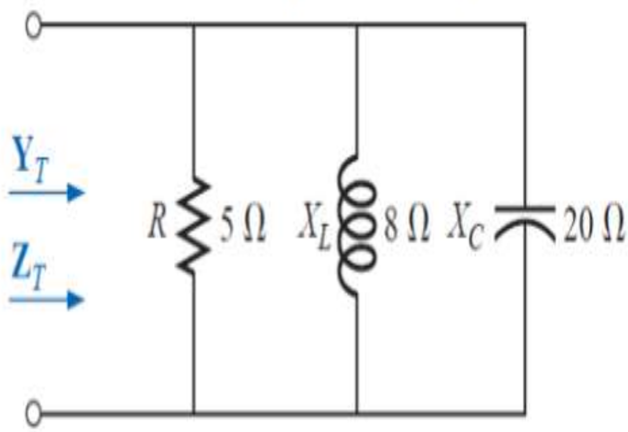
d.



EXAMPLE: Repeat the above Example for the parallel network.



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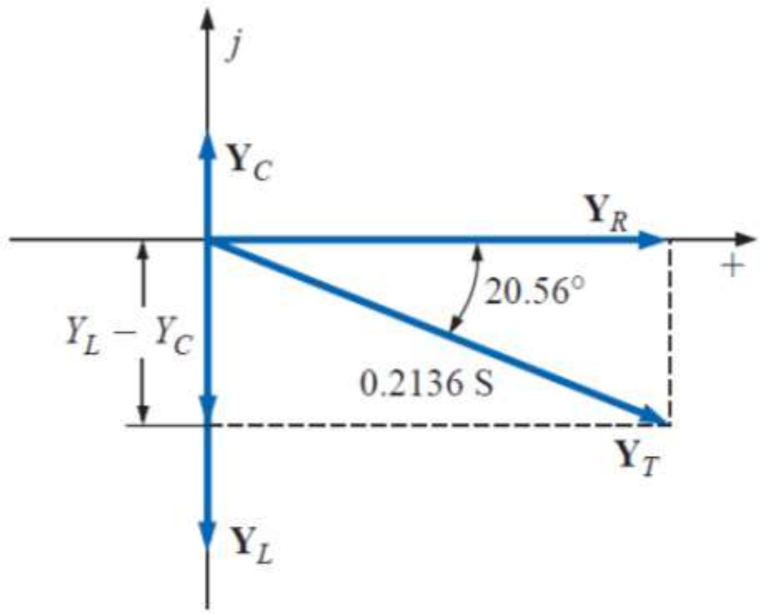
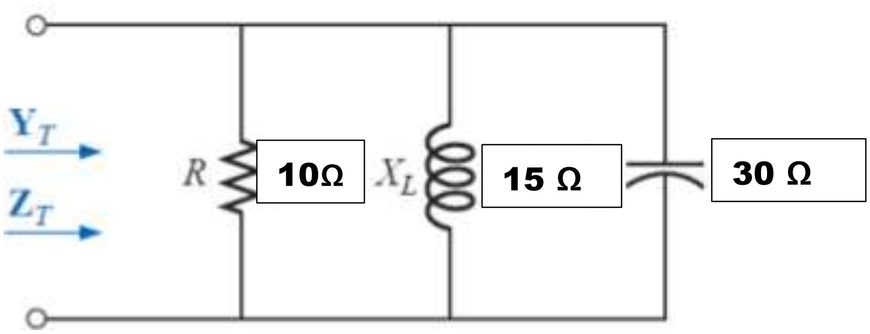


EXAMPLE: Repeat the above Example for the parallel network.

But R = 10 Ω , X_L= 15 Ω and X_C= 30 Ω



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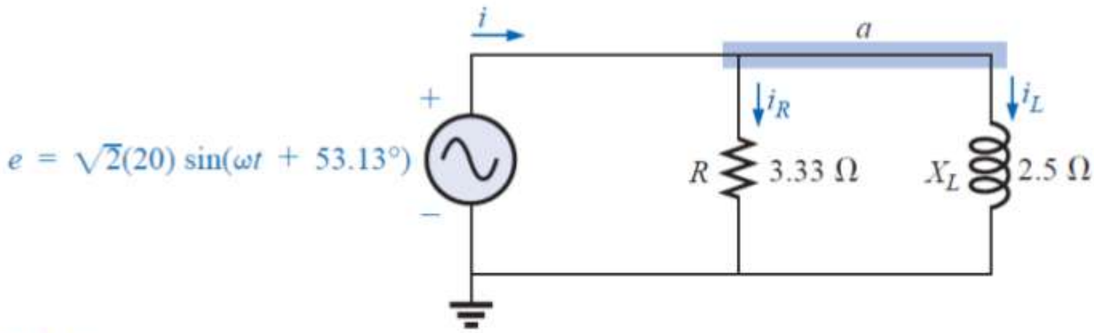
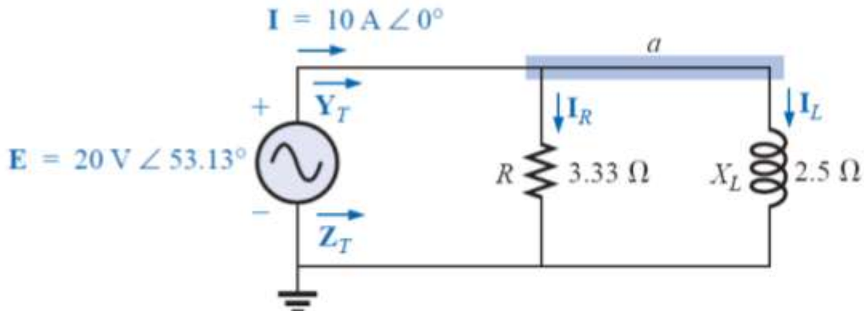
6.1 PARALLEL AC NETWORKS

(1) R-L PARALLEL AC

EXAMPLE: find the total impedance and the current in each branch for the network.



Solutions:



$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L$$
$$= G\angle 0° + B_L\angle -90° = \frac{1}{3.33\ \Omega}\angle 0° + \frac{1}{2.5\ \Omega}\angle -90°$$
$$= 0.3\ \text{S}\angle 0° + 0.4\ \text{S}\angle -90° = 0.3\ \text{S} - j\,0.4\ \text{S}$$
$$= \mathbf{0.5\ S\angle -53.13°}$$
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ \text{S}\angle -53.13°} = \mathbf{2\ \Omega\angle 53.13°}$$

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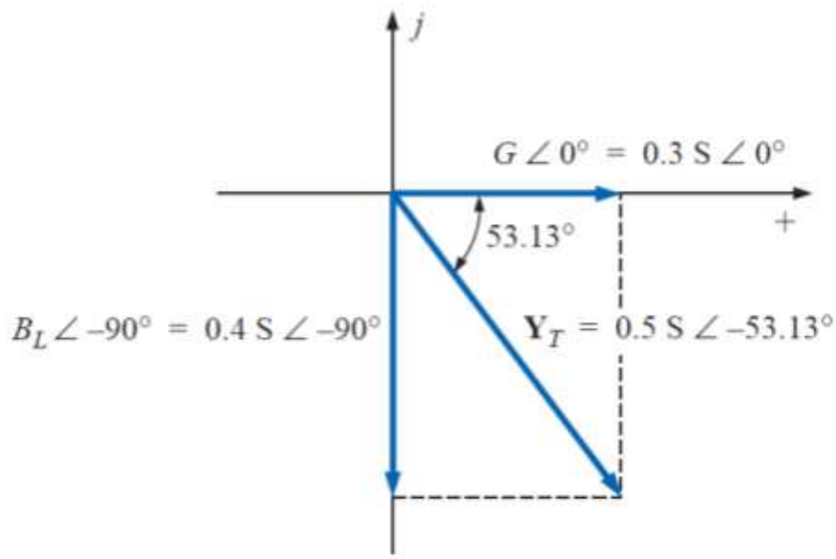

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (20\text{ V}\angle 53.13^\circ)(0.5\text{ S}\angle -53.13^\circ) = \mathbf{10\text{ A}\angle 0^\circ}$$

$$\mathbf{I}_R = \frac{E\angle\theta}{R\angle 0^\circ} = (E\angle\theta)(G\angle 0^\circ)$$
$$= (20\text{ V}\angle 53.13^\circ)(0.3\text{ S}\angle 0^\circ) = \mathbf{6\text{ A}\angle 53.13^\circ}$$
$$\mathbf{I}_L = \frac{E\angle\theta}{X_L\angle 90^\circ} = (E\angle\theta)(B_L\angle -90^\circ)$$
$$= (20\text{ V}\angle 53.13^\circ)(0.4\text{ S}\angle -90^\circ)$$
$$= \mathbf{8\text{ A}\angle -36.87^\circ}$$

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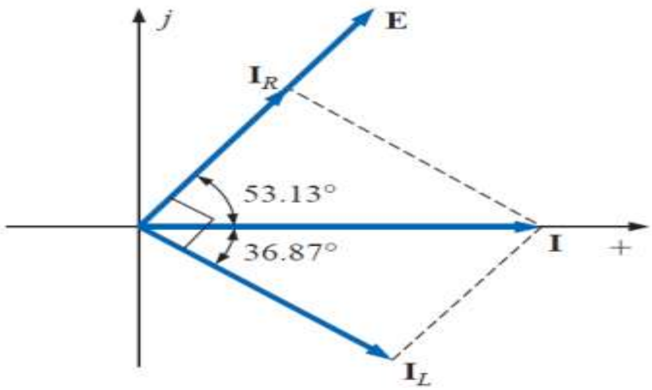
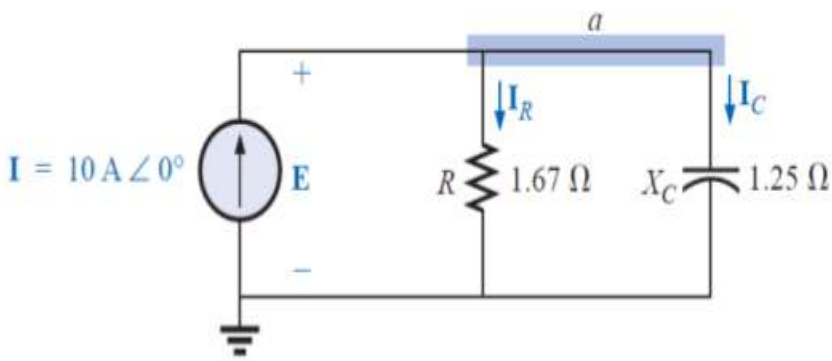


(2) R-C PARALLEL AC

EXAMPLE: find the total impedance and the current in each branch for the network.



Solutions:

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_C = G\angle 0^\circ + B_C\angle 90^\circ = \frac{1}{1.67\ \Omega}\angle 0^\circ + \frac{1}{1.25\ \Omega}\angle 90^\circ$$

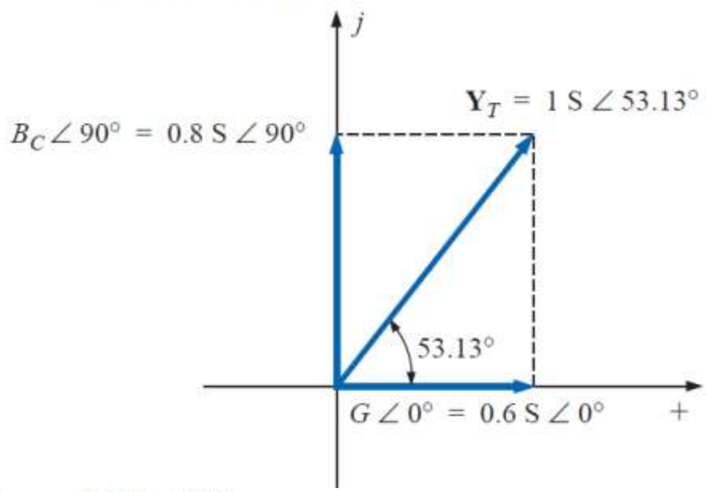
$$= 0.6\text{ S}\angle 0^\circ + 0.8\text{ S}\angle 90^\circ = 0.6\text{ S} + j\,0.8\text{ S} = \mathbf{1.0\ S\angle 53.13^\circ}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{1.0\text{ S}\angle 53.13^\circ} = \mathbf{1\ \Omega\angle -53.13^\circ}$$

Or

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_C}{\mathbf{Z}_R + \mathbf{Z}_C} = \frac{(1.67\ \Omega\angle 0^\circ)(1.25\ \Omega\angle -90^\circ)}{1.67\ \Omega\angle 0^\circ + 1.25\ \Omega\angle -90^\circ}$$

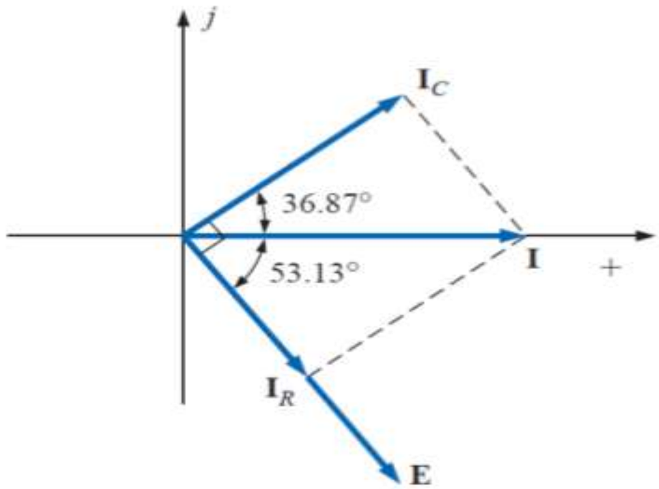
$$= \frac{2.09\angle -90^\circ}{2.09\angle -36.81^\circ} = \mathbf{1\ \Omega\angle -53.19^\circ}$$



$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \frac{\mathbf{I}}{\mathbf{Y}_T} = \frac{10\text{ A}\angle 0^\circ}{1\text{ S}\angle 53.13^\circ} = \mathbf{10\ V\angle -53.13^\circ}$$

$$\mathbf{I}_R = (E\angle\theta)(G\angle 0^\circ)$$
$$= (10\text{ V}\angle -53.13^\circ)(0.6\text{ S}\angle 0^\circ) = \mathbf{6\ A\angle -53.13^\circ}$$

$$\mathbf{I}_C = (E\angle\theta)(B_C\angle 90^\circ)$$
$$= (10\text{ V}\angle -53.13^\circ)(0.8\text{ S}\angle 90^\circ) = \mathbf{8\ A\angle 36.87^\circ}$$



(3) R-L-C PARALLEL AC

A Parallel RLC AC Circuit is one where the resistor, inductor, and capacitor are connected in parallel to each other and the AC source. In this configuration, the voltage across each component is the same, but the currents through them different.

Applied alternating voltage: **E = Emax cos ωt** Resulting alternating current: **I = Imax cos(ωt − δ)** Goals: • Find Imax, **δ** for given **Emax, ω**. • Find currents IR, IL, IC through devices. Junction rule:

I = IR + IL + IC

Note: • All currents are time-dependent. • In general, each current has a different phase • IR has the same phase as E.

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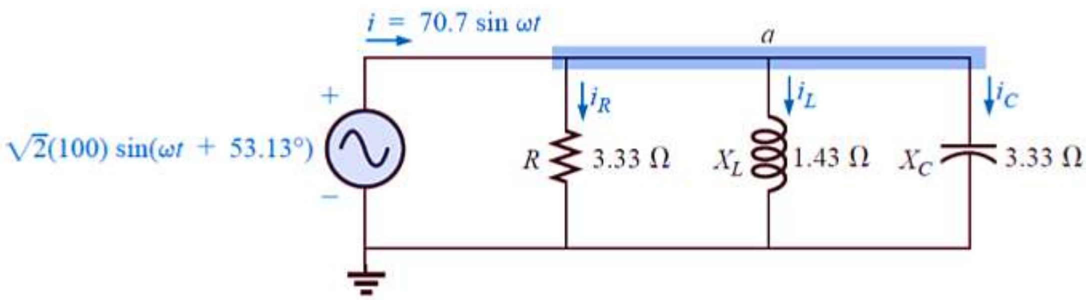
EXAMPLE: find the total impedance and the current in each branch for the network.



Solutions:

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = G\,\angle 0^\circ + B_L\,\angle -90^\circ + B_C\,\angle 90^\circ$$
$$= \frac{1}{3.33\ \Omega}\angle 0^\circ + \frac{1}{1.43\ \Omega}\angle -90^\circ + \frac{1}{3.33\ \Omega}\angle 90^\circ$$
$$= 0.3\ \text{S}\,\angle 0^\circ + 0.7\ \text{S}\,\angle -90^\circ + 0.3\ \text{S}\,\angle 90^\circ$$
$$= 0.3\ \text{S} - j\,0.7\ \text{S} + j\,0.3\ \text{S}$$
$$= 0.3\ \text{S} - j\,0.4\ \text{S} = \mathbf{0.5\ S\,\angle -53.13^\circ}$$
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ \text{S}\,\angle -53.13^\circ} = \mathbf{2\ \Omega\,\angle 53.13^\circ}$$

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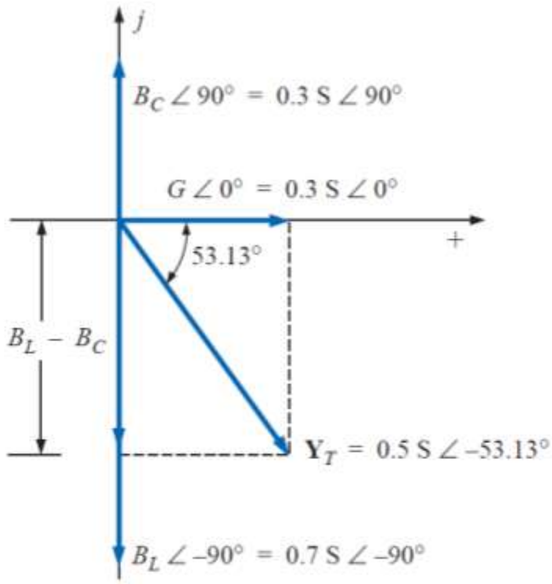
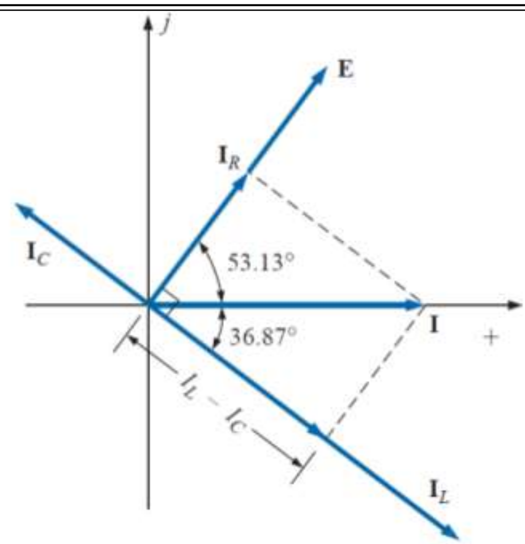


$$\mathbf{I}_R = (E \angle\theta)(G \angle 0°)$$
$$= (100\ \text{V} \angle 53.13°)(0.3\ \text{S} \angle 0°) = \mathbf{30\ A \angle 53.13°}$$
$$\mathbf{I}_L = (E \angle\theta)(B_L \angle -90°)$$
$$= (100\ \text{V} \angle 53.13°)(0.7\ \text{S} \angle -90°) = \mathbf{70\ A \angle -36.87°}$$
$$\mathbf{I}_C = (E \angle\theta)(B_C \angle 90°)$$
$$= (100\ \text{V} \angle 53.13°)(0.3\ \text{S} \angle +90°) = \mathbf{30\ A \angle 143.13°}$$



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