Lecture Two

Series and Parallel DC Circuits

1.1 Introduction

This lecture introduced the basic concepts of electric circuits. To determine the values of variables in a given circuit, it is essential to understand fundamental laws that govern electrical behavior. These laws, known as Ohm's Law and Kirchhoff's Laws, form the foundation of electric circuit analysis. In addition to these laws, we will discuss various techniques commonly used in circuit design and analysis.

In network topology, we examine the properties associated with the arrangement of elements within a network and its geometric structure. These elements include **branches**, **nodes**, and **loops**.

A **branch** represents a single element such as a voltage source or a resistor. In other words, a branch represents any two-terminal element. The circuit in Fig. 1 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

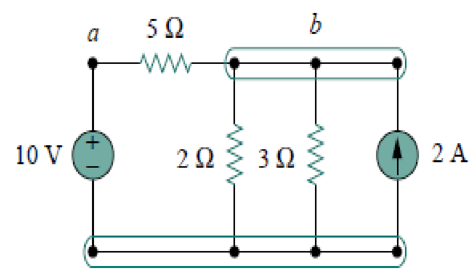

Figure 1

A node is the point of connection between two or more branches. A node is usually indicated by a **dot** in a circuit. If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node. The circuit in Fig.1 has three nodes *a*, *b*, and *c*.

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node.

Elements are **in series** when connected end to end, sharing a single common node with no other connections. They are **in parallel** if they share the same pair of terminals

Example (1): Determine the number of **branches** and **nodes** in the circuit shown in Fig. 2. Identify which elements are in series and which are in parallel.

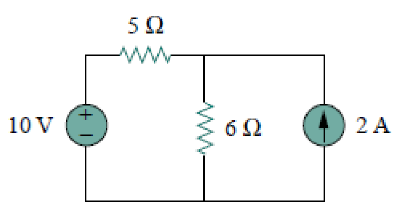

Figure 2

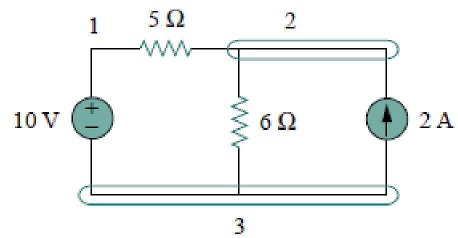

Figure 3

Solution

- Since there are four elements in the circuit, the circuit has four branches:

  10 V, 5 *Ω,* 6 *Ω,* and 2 A.

- The circuit has three nodes as shown in Fig. 3
- The 5*Ω* resistor is in series with the 10-V voltage source. The 6 *Ω* resistor is in parallel with the 2-A current source

Practical Problem

How many branches and nodes does the circuit in Fig. 4 have? Identify the elements that are in series and parallel.

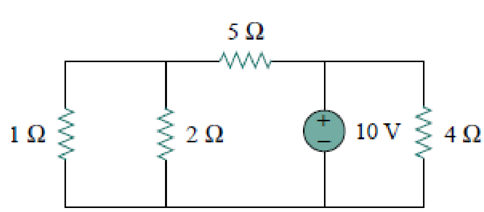

Figure 4

KIRCHHOFF'S LAWS

Ohm's law by itself is not sufficient to analyze circuits. However, when coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits.

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node is zero.

Mathematically, KCL implies that $\sum_{n=1}^{N} i_n = 0$

Where N is the number of branches connected to the node

and i_n is the *n*th current entering (or leaving) the node.

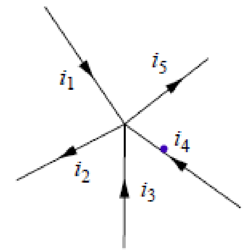

Figure 5

since currents i1, i3, and i4 are entering the node, while currents i2 and i5 are leaving it. By rearranging the terms, we get $i_1 + i_3 + i_4 = i_2 + i_5$

The sum of the currents entering a node equals the sum of the currents leaving the node.

$I_T = I_1 - I_2 + I_3$

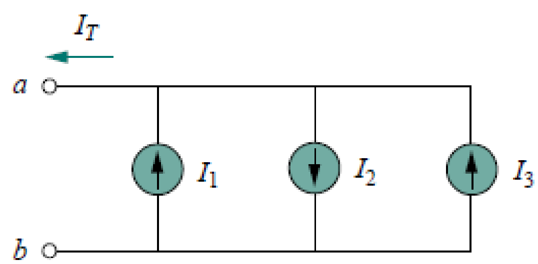

Figure 6

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\sum_{m=1}^{M} v_m = 0$$

Where *M* is the number of voltages in the loop (or the number of branches in the loop) and v_m is the *m*th voltage.

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To illustrate KVL, consider the circuit in Fig. 5.

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Rearranging terms gives

$$+ v_2 + v_3 + v_5 = v_1 + v_4$$

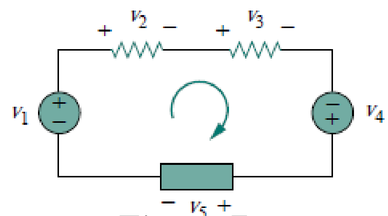


Figure 7

which may be interpreted as

Sum of voltage drops = Sum of voltage rises

Example (3): For the circuit of Fig. 8

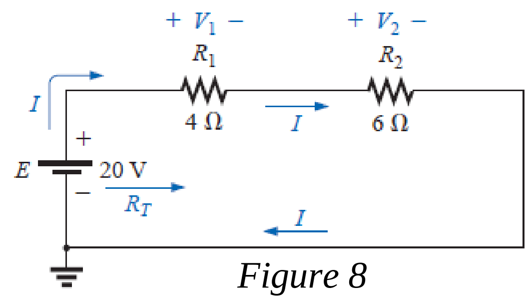


Figure 8

a. Find *RT*.

b. Find *I*.

c. Find V_1 and V_2.

d. Find the power to the 4Ω and 6Ω resistors.

e. Find the power delivered by the battery, and compare it to that dissipated by the 4Ω and 6Ω resistors combined.

f. Verify Kirchhoff's voltage law (clockwise direction).

Solution

a. *RT*= *R*1 + *R*2 = 4 Ω + 6 Ω =**10**Ω

b. $I = \frac{E}{R_T} = \frac{20V}{10\Omega} = 2A$

c. $V_1 = IR_1 = 2A \times 4\Omega = 8V$ $\quad V_2 = IR_2 = 2A \times 6\Omega = 12V$

d. $P_1 = \frac{V^2}{R_1} = \frac{8^2}{4} = 16W$ $\quad P_2 = \frac{V^2}{R_2} = \frac{12^2}{6} = 24W$

e. $P_E = P_1 + P_2 = 16 + 24 = 40W$

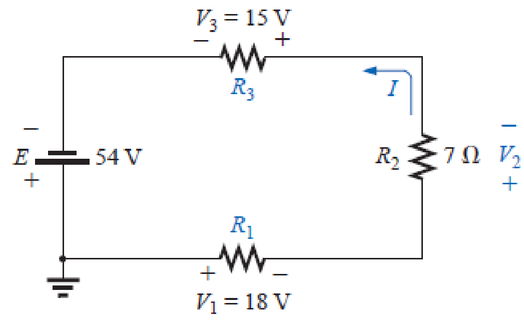
f. $-E + V_1 + V_2 = 0$ $\quad E = V_1 + V_2$ $\quad 20V = 8V + 12V$

Example (4): For the circuit of Fig. 9

a. Determine V_2 using Kirchhoff's voltage law.

b. Determine *I*.

c. Find *R*1 and *R*3.



Solution

a- Kirchhoff's voltage law (clockwise direction):

$-E + V3 + V2 + V1 = 0$

or $E = V1 + V2 + V3$

and $V2 = E - V1 - V3 = 54\text{ V} - 18\text{ V} - 15\text{ V} = \mathbf{21\ V}$

b- $I = \frac{V_2}{R_2} = \frac{21}{7} = 3A$

c- $R_1 = \frac{V_1}{I} = \frac{18}{3} = 6\Omega$

d- $R_3 = \frac{V_3}{I} = \frac{15}{3} = 5\Omega$

Example (5) Find the currents and voltages in the circuit shown in the figure below

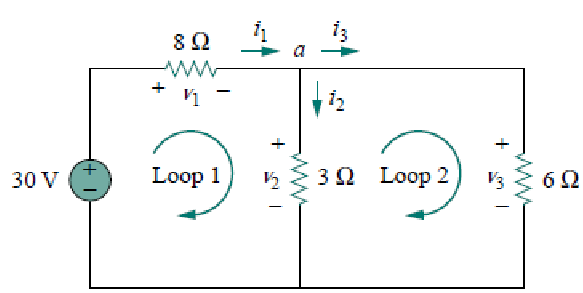
Solution

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$v1 = 8i_1, \quad v2 = 3i_2, \quad v3 = 6i_3$

At node *a*, KCL gives

$i_1 - i_2 - i_3 = 0$



Applying KVL to loop 1

$-30 + v_1 + v_2 = 0$

We express this in terms of i_1 and i_2

$-30 + 8i_1 + 3i_2 = 0$

$$i_1 = \frac{(30 - 3i_2)}{8}$$

Applying KVL to loop 2,

$-v_2 + v_3 = 0 \;\Rightarrow v_3 = v_2$

$$6i_3 = 3i_2 \quad \Rightarrow \quad i_3 = \frac{3i_2}{6} = \frac{i_2}{2} \;\Rightarrow$$

$$i_1 - i_2 - i_3 = 0$$

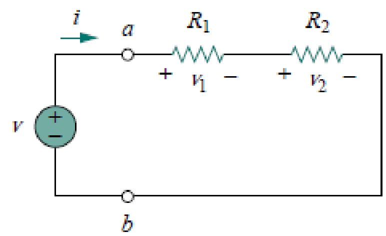
$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

$i_2 = 2$ A

$$i_1 = 3\text{ A}, \qquad i_3 = 1\text{ A}, \qquad v_1 = 24\text{ V}, \qquad v_2 = 6\text{ V}, \qquad v_3 = 6\text{ V}$$

Series resistors and voltage division

The two resistors are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain



$$v_1 = iR_1, \qquad v_2 = iR_2$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0$$

$$v = v_1 + v_2 = i(R_1 + R_2) \Rightarrow i = \frac{v}{R_1 + R_2}, \quad R_{eq} = R_1 + R_2$$

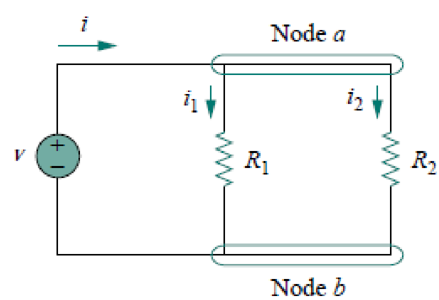
For N resistors in series then

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

Parallel resistors and current division

Where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,



$$v = i_1R_1 = i_2R_2 \Rightarrow i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

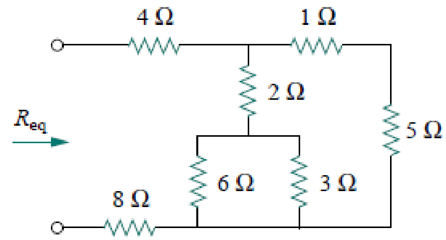
Applying KCL at node a gives the total current i as

$$i = i_1 + i_2$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}, \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

Example (5): Find R_{eq} for the circuit shown in the figure below



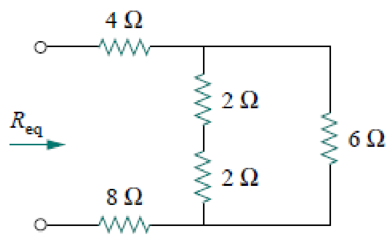
Solution:

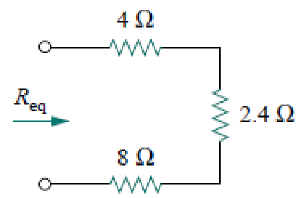
$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6+3} = 2\ \Omega$$

$1\ \Omega + 5\ \Omega = 6\ \Omega$

$2\ \Omega + 2\ \Omega = 6\ \Omega$

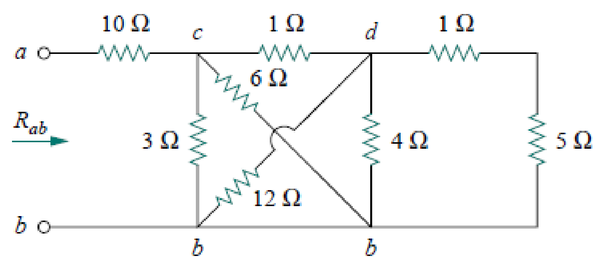
$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4+6} = 2.4\ \Omega$$





Req = 4 Ω + 2.4 Ω + 8 Ω = 14.4 Ω

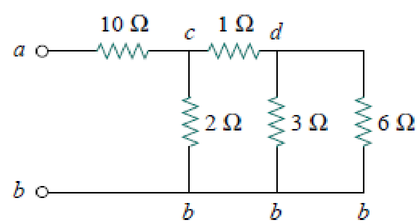
Example (6): Calculate the equivalent resistance *Rab* in the figure below



Solution:

The 3 Ω and 6 Ω resistors are in parallel because they are connected to the same two nodes *c* and *b*.

Their combined resistance is $3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3+6} = 2\ \Omega$



Similarly, the 12 Ω and 4 Ω resistors are in parallel since they are connected to the same two nodes *d* and *b*. *Hence* $12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12+4} = 3\ \Omega$

Also, the 1 Ω and 5 Ω resistors are in series, 1 Ω + 5 Ω = 6 Ω

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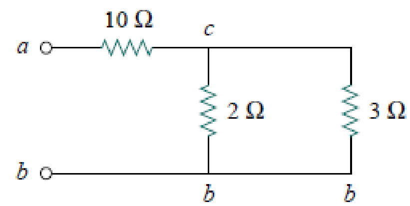


Also, the 1 Ω and 2 Ω resistors are in series, 1 Ω + 2 Ω = 3 Ω

In the last figure, the 3Ω in parallel with 2Ω

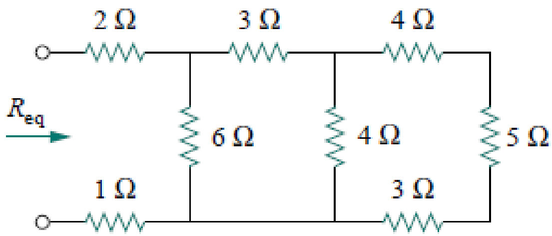
$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$

This 1.2 Ω resistor is in series with the 10 Ω resistor, so that

R_{ab} = 10 Ω + 1.2 Ω = 11.2 Ω

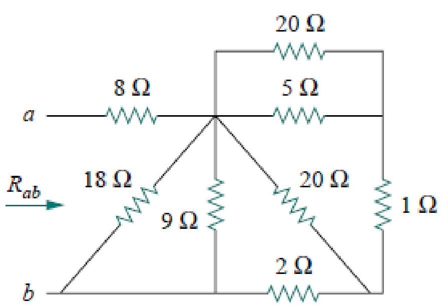
Practice Problem

- By combining the resistors in the figure below, find *R*eq.



Answer: 6 Ω

- Find *Rab* for the circuit in the figure below



Answer: 11 Ω

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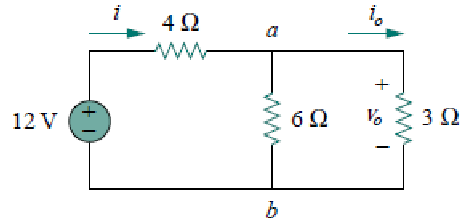
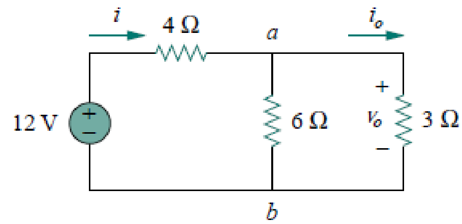
Example (7): Find i_o and v_o in the circuit shown in the figure below. Calculate the power dissipated in the 3Ω resistor.



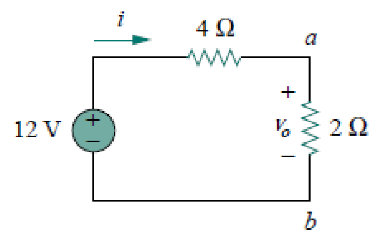
Solution

The 6 Ω and 3 Ω resistors are in parallel, so their combined resistance is

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$



Apply Ohm's law $\quad i = \dfrac{12}{4 + 2} = 2\ \text{A}$



Apply voltage division $\quad v_o = \dfrac{2}{2 + 4}(12\ \text{V}) = 4\ \text{V}$

$v_o = 3i_o = 4 \ \Rightarrow \ i_o = \dfrac{4}{3}\ \text{A}$

Another approach is to apply the current division $\quad i_o = \dfrac{6}{6 + 3}i = \dfrac{2}{3}(2\ \text{A}) = \dfrac{4}{3}\ \text{A}$

The power dissipated in the 3 Ω resistor is

$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333\ \text{W}$$

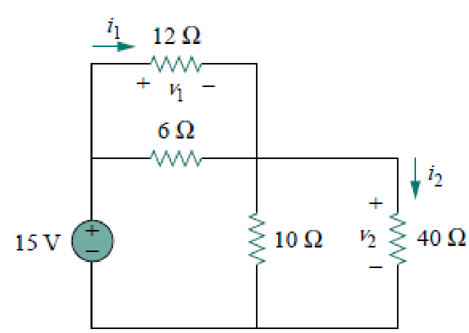
Practice Problem

Find v_1 and v_2 in the figure below. Also calculate i_1 and i_2 and the power dissipated in the 12 Ω and 40 Ω resistors.

Answer:

$V1 = 5$ V, $i_1 = 416.7$ mA, $p_1 = 2.083$ W, $v_2 = 10$ V,

$i_2 = 250$ mA, $p_2 = 2.5$ W.



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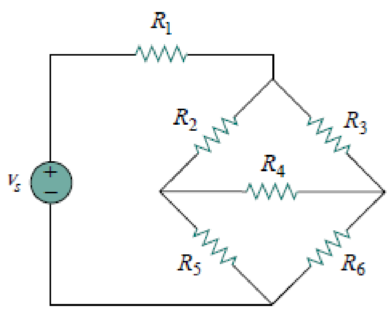
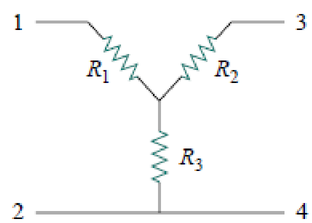
Wye-Delta transformations

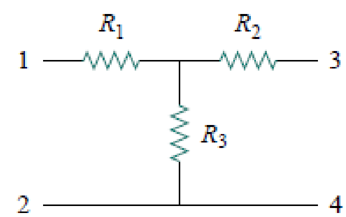
In circuit analysis, some resistor configurations are neither purely series nor parallel, such as the bridge circuit in Fig.1. To simplify these circuits, we use three-terminal equivalent networks: the wye (Y) or tee (T) network and the delta (Δ) or pi (π) network. These transformations are essential in three-phase systems, electrical filters, and matching networks. The key focus is identifying these configurations within a circuit and applying the wye-delta transformation for simplification.



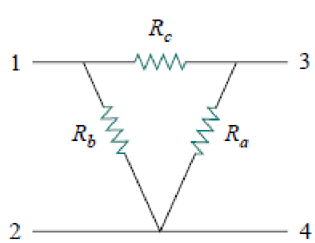
Bridge



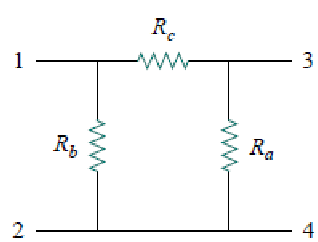
wye (Y)



tee (T)



delta (Δ)



pi (π)

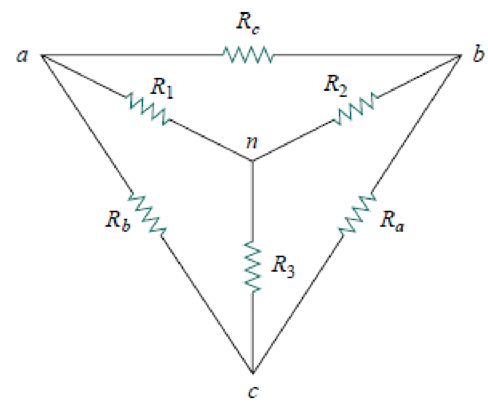
Delta to Wye Conversion

To replace a delta network with an equivalent wye network, we ensure that the resistances between corresponding node pairs remain the same. By superimposing a wye network onto the delta configuration, we derive the equivalent resistances.

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



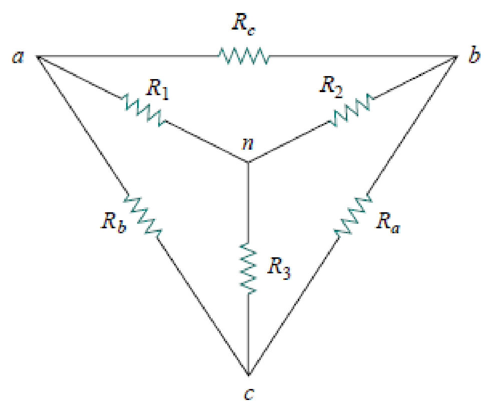
Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network into an equivalent delta network:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$



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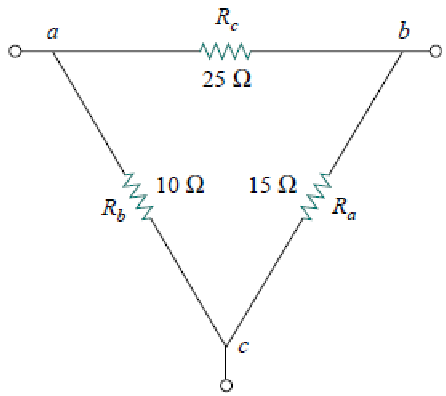
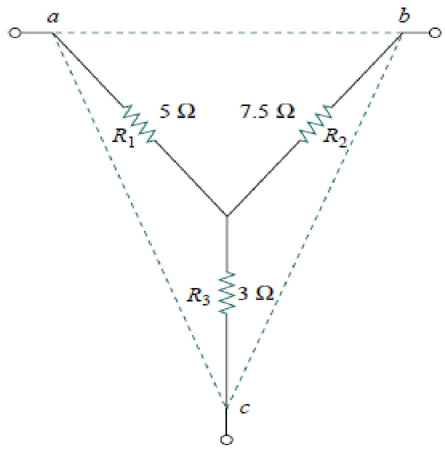
Example (8): Convert the network in the figure below to an equivalent Y network



Solution



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{25 \times 10}{25 + 10 + 15} = \frac{250}{50} = 5\ \Omega$$

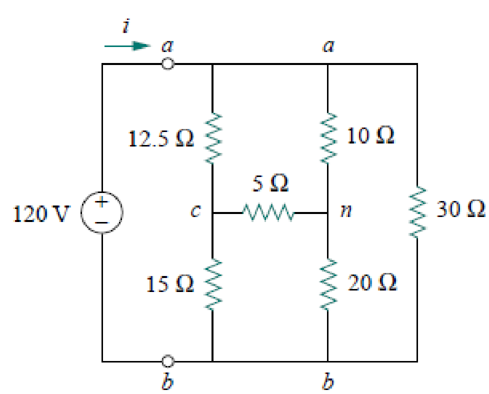
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

Example (8): Obtain the equivalent resistance R_{ab} for the circuit in the figure below and use it to find current i.

solution

If we convert the Y network comprising the 5Ω, 10Ω, and 20Ω resistors



$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

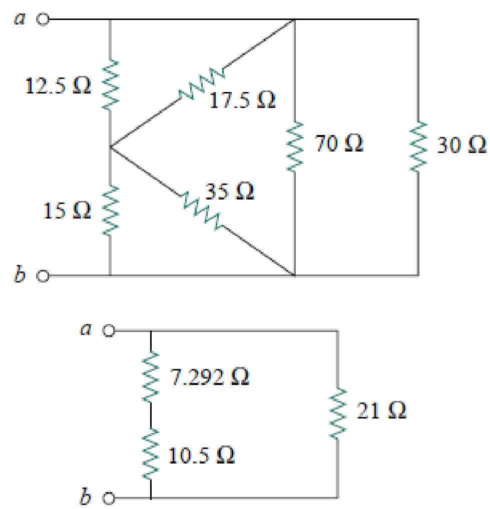
$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.2917\ \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

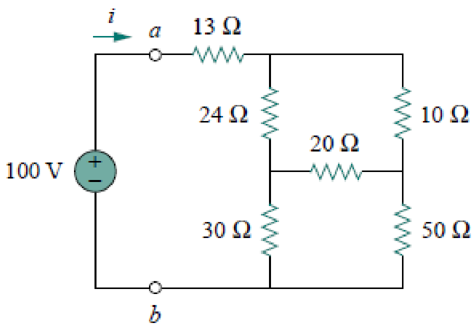
$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = 9.632\ \Omega$$

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458\ \text{A}$$



Practice Problem

For the bridge network in the figure below, find R_{ab} and i.



Answer: 40 Ω, 2.5 A.