**Al-Mustaqbal University**

**College of Engineering & Technology**

**Biomedical Engineering Department**

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## Lecturer: م.د علي كامل كريم

## Email: ali.kamil.kareem@uomus.edu.iq

## Lecture No.2

## Lecture Title: [Solving Methods of Engineering problem]

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# Numerical solution methods for PDEs

- Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations. Although there are many kinds of numerical methods, they have one common characteristic: they invariably involve large numbers of tedious arithmetic calculations.

-- Numerical methods for ODE can also be extended to solution of PDE. Methods discussed for treating initial value problems can be adopted for parabolic as well as hyperbolic equations. Similarly, methods that have been discussed for treating BVPs can be adopted for solution of elliptic PDEs which are also boundary value problems. However, the extension of the methods to solve PDE is not straightforward.

- Methods such as finite difference method (FDM), finite volume method (FVM), finite element method (FEM), boundary element method (BEM) etc are commonly used for treating PDE numerically. All numerical methods used to solve PDEs should have consistency, stability and convergence.

- A numerical method is said to be consistent if all the approximations (finite difference, finite element, finite volume etc) of the derivatives tend to the exact value as the step size ($\Delta t$, $\Delta x$ etc) tends to zero

## 1) Finite Element Method (FEM) :

The **finite element method** (**FEM**) is a numerical technique for finding approximate solutions to boundary value problems for partial differential equations. It is also referred to as **finite element** analysis (FEA).

## 2) Boundary Element Method (BEM) :

The **boundary element method (BEM)** is a technique for solving a range of engineering/physical problems. It is most often used as an engineering design aid - similar to the more common finite element method - but the **BEM** has the distinction and advantage that only the surfaces of the domain need to be meshed.

### 3) Finite Volume Method (FVM) :

Most **Computational Fluid Dynamics (CFD)** software is based on FVM. The unit volume is considered in Finite Volume Method (similar to element in finite element analysis). Variable properties at the nodes include pressure, velocity, area, mass, etc. It is based on the Navier - Stokes equations (Mass, Momentum, and Energy conservation equilibrium equations).

### 4) Finite Difference Method (FDM) :

Finite Element and **Finite Difference Methods** share many common things. In general the Finite Difference Method is described as a way to solve differential equation. It uses Taylor's series to convert a differential equation to an algebraic equation. In the conversion process higher order terms are neglected. It is used in combination with BEM or FVM to solve Thermal and CFD coupled problems.

# GENERAL PROCESS

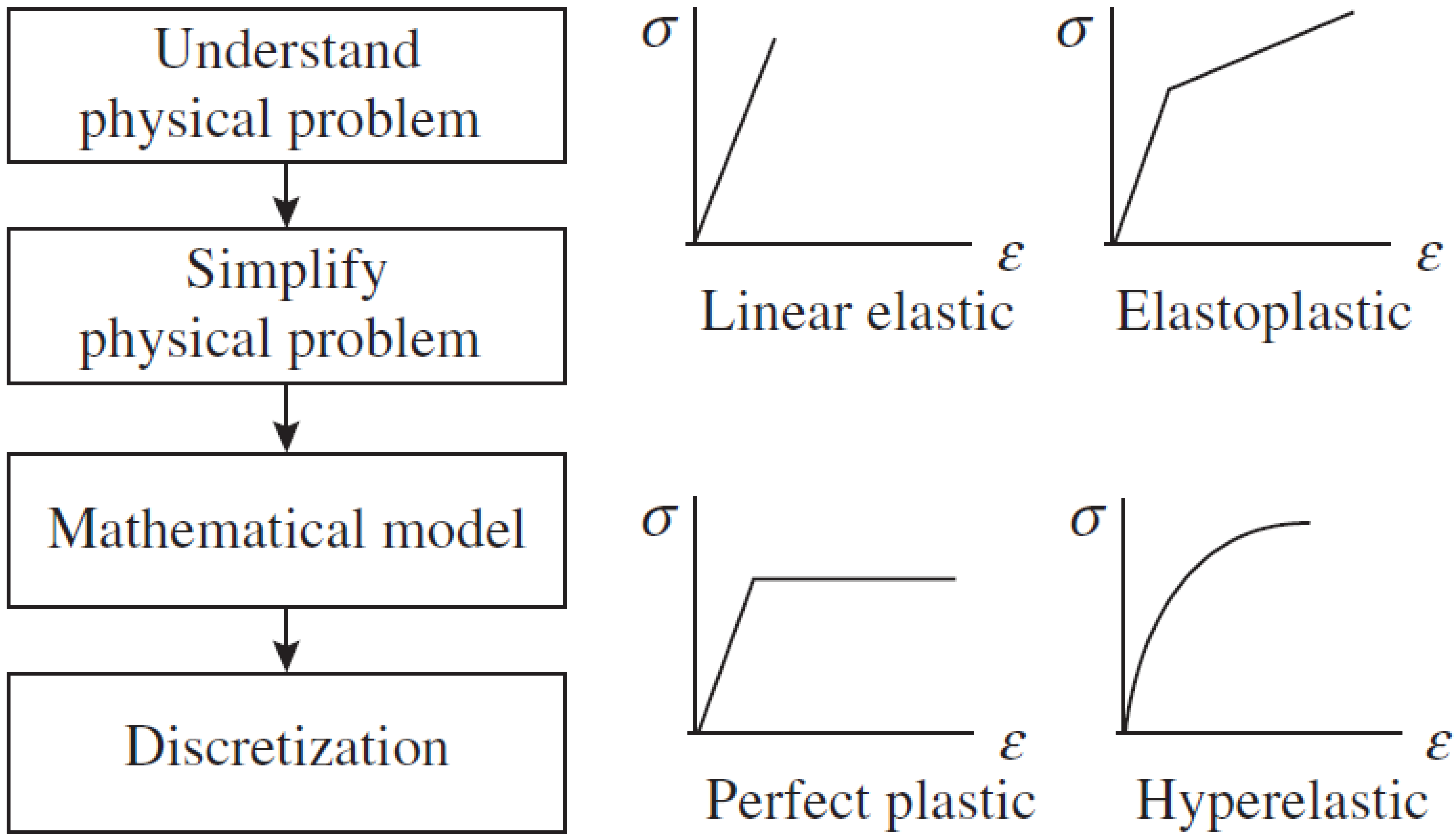
**<u>The first step</u>** in creating an FEA model is to fully understand the physical problem being considered. This includes the operating condition of the mechanical component or system, material properties, geometric shape and dimensions, boundary conditions, possible failure scenarios, and, most importantly, the questions FEA is to answer.

- **<u>The next step</u>** is about simplification and idealization of the structural problem. This is often necessary since the physical problems are usually too complex to solve as they are. A physical problem can be idealized by making adequate assumptions to reduce the complexity level to one that FEA is capable of solving, and yet with the numerical solutions closely resembling the behavior of the physical problem.

It is important to understand the physics of the structural problem to be solved and to identify the proper mathematical model that can be applied to solve the problem. There are at least three aspects to consider. The first is to understand in general how the deformation will mostly occur in the structure.

Is it going to be a small deformation within the linear elastic range? Or, if the deformation is hyperelastic or plastic, is it beyond the linear elastic range? The second aspect is the load: Is it static or time-dependent? Third, is it a multiphysics problem, such as aero-structure, fluid structure, acoustics or thermal structure? All of these must be well understood before creating finite element models and choosing an adequate FEA code for solution.

**The final step** involves finite element mesh generation. The questions to ask are: Does the problem require a very refined mesh, are the FEA software and computers able to handle a large finite element model? What type of elements will be used, linear or quadratic? Also, does the problem involve discontinuity in responses? These are essential questions to go over before creating the model. The more thought that is given to these questions, the better the preparation, which usually means a better FEA model and eventually time savings.


Understand physical problem
Simplify physical problem
Mathematical model
Discretization
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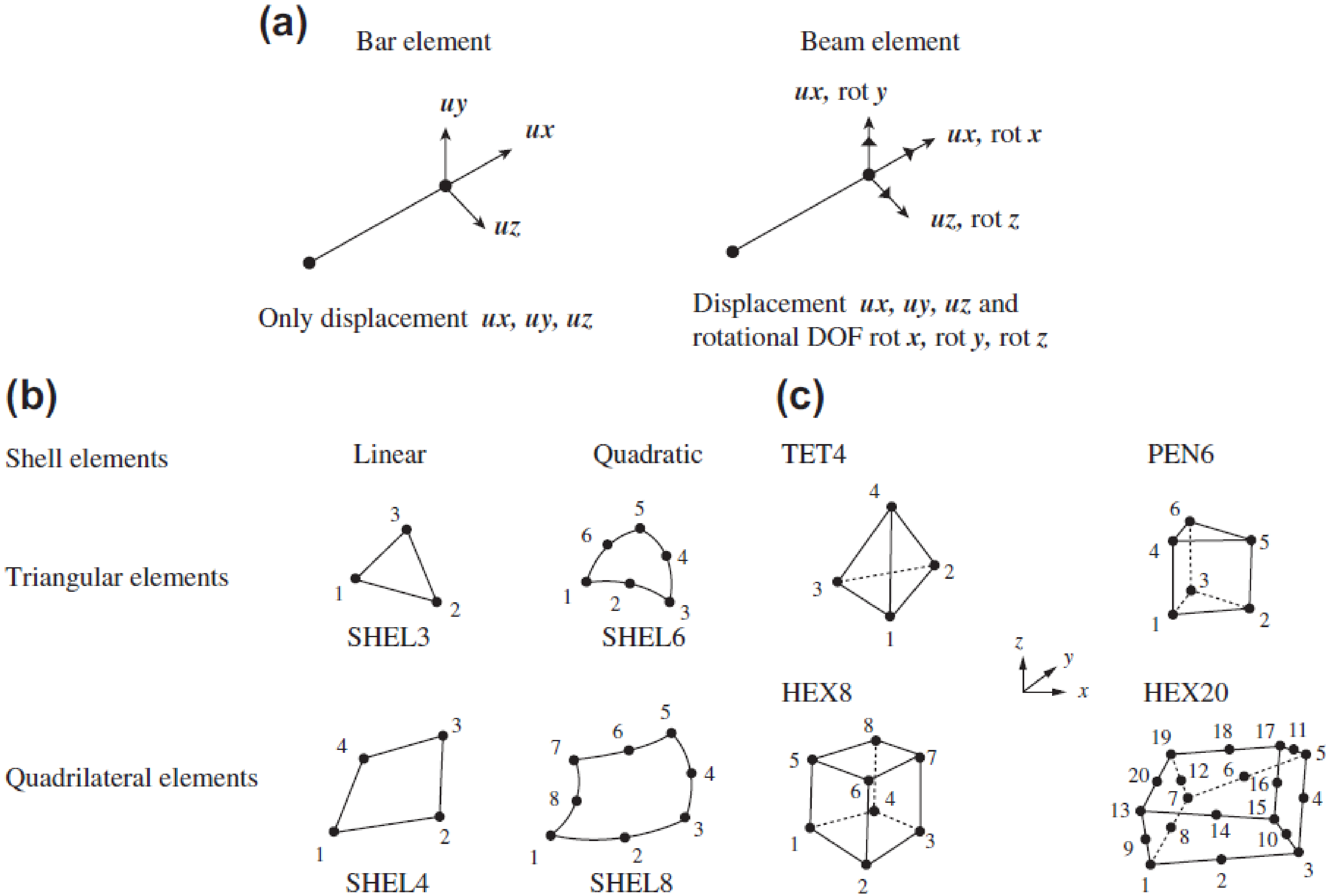


Typical finite element types: (a) bars and beams; (b) 2D shell elements; (c) 3D solid elements.