



Module Title: Fundamental of Electrical Engineering (AC)

Module Code:	UOMU024021
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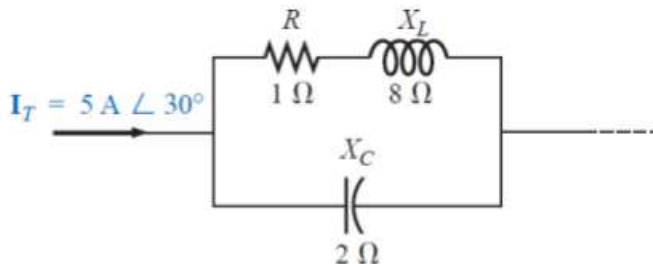
Week 7, and Week 8

Week 7:

Series-Parallel AC Network

The combination of RLC circuits contains current divider or voltage divider. The current through each parallel branch depends on the load. The voltage through each parallel branch is equal potential.

EXAMPLE 1 : Using the current divider rule, find the current through each parallel branch.



Solutions:

$$\begin{aligned} I_{R-L} &= \frac{Z_C I_T}{Z_C + Z_{R-L}} = \frac{(2 \Omega \angle -90^\circ)(5 \text{ A} \angle 30^\circ)}{-j 2 \Omega + 1 \Omega + j 8 \Omega} = \frac{10 \text{ A} \angle -60^\circ}{1 + j 6} \\ &= \frac{10 \text{ A} \angle -60^\circ}{6.083 \angle 80.54^\circ} \cong 1.644 \text{ A} \angle -140.54^\circ \end{aligned}$$

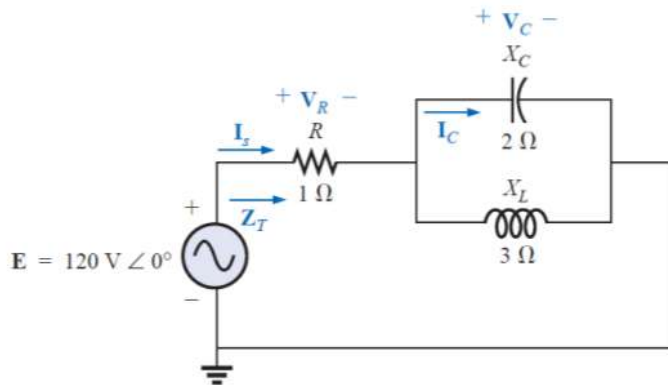
$$\begin{aligned} I_C &= \frac{Z_{R-L} I_T}{Z_{R-L} + Z_C} = \frac{(1 \Omega + j 8 \Omega)(5 \text{ A} \angle 30^\circ)}{6.08 \Omega \angle 80.54^\circ} \\ &= \frac{(8.06 \angle 82.87^\circ)(5 \text{ A} \angle 30^\circ)}{6.08 \angle 80.54^\circ} = \frac{40.30 \text{ A} \angle 112.87^\circ}{6.083 \angle 80.54^\circ} \\ &= 6.625 \text{ A} \angle 32.33^\circ \end{aligned}$$



Series-Parallel AC Networks examples:

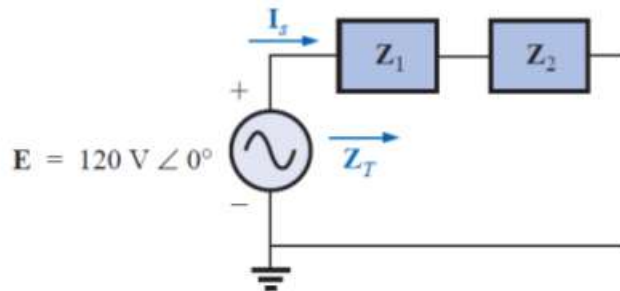
EXAMPLE 1: For the network shown below.

- Calculate Z_T .
- Determine I_s .
- Calculate V_R and V_C .
- Find I_C .



Solutions:

a)





$$Z_T = Z_1 + Z_2$$

$$Z_1 = R \angle 0^\circ = 1 \Omega \angle 0^\circ$$

$$Z_2 = Z_C \parallel Z_L = \frac{(X_C \angle -90^\circ)(X_L \angle 90^\circ)}{-jX_C + jX_L} = \frac{(2 \Omega \angle -90^\circ)(3 \Omega \angle 90^\circ)}{-j2 \Omega + j3 \Omega}$$

$$= \frac{6 \Omega \angle 0^\circ}{j1} = \frac{6 \Omega \angle 0^\circ}{1 \angle 90^\circ} = 6 \Omega \angle -90^\circ$$

$$Z_T = Z_1 + Z_2 = 1 \Omega - j6 \Omega = 6.08 \Omega \angle -80.54^\circ$$

b)

$$I_s = \frac{E}{Z_T} = \frac{120 \text{ V} \angle 0^\circ}{6.08 \Omega \angle -80.54^\circ} = 19.74 \text{ A} \angle 80.54^\circ$$

c)

$$V_R = I_s Z_1 = (19.74 \text{ A} \angle 80.54^\circ)(1 \Omega \angle 0^\circ) = 19.74 \text{ V} \angle 80.54^\circ$$

$$V_C = I_s Z_2 = (19.74 \text{ A} \angle 80.54^\circ)(6 \Omega \angle -90^\circ)$$

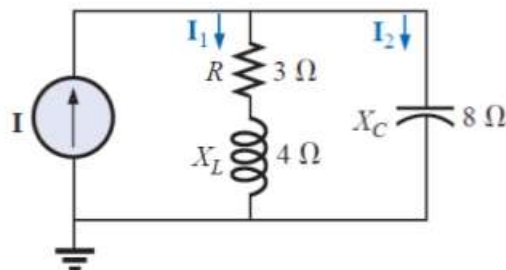
$$= 118.44 \text{ V} \angle -9.46^\circ$$

d)

$$I_C = \frac{V_C}{Z_C} = \frac{118.44 \text{ V} \angle -9.46^\circ}{2 \Omega \angle -90^\circ} = 59.22 \text{ A} \angle 80.54^\circ$$

EXAMPLE 2: For the network shown below.

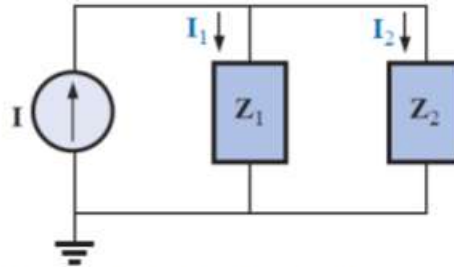
- If I is $50 \text{ A} \angle 30^\circ$, calculate I_1 using the current divider rule.
- Repeat part (a) for I_2 .
- Verify Kirchhoff's current law at one node.





Solutions:

a)



$$Z_1 = R + jX_L = 3 \Omega + j4 \Omega = 5 \Omega \angle 53.13^\circ$$

$$Z_2 = -jX_C = -j8 \Omega = 8 \Omega \angle -90^\circ$$

$$\begin{aligned} I_1 &= \frac{Z_2 I}{Z_2 + Z_1} = \frac{(8 \Omega \angle -90^\circ)(50 \text{ A} \angle 30^\circ)}{(-j8 \Omega) + (3 \Omega + j4 \Omega)} = \frac{400 \angle -60^\circ}{3 - j4} \\ &= \frac{400 \angle -60^\circ}{5 \angle -53.13^\circ} = 80 \text{ A} \angle -6.87^\circ \end{aligned}$$

b)

$$\begin{aligned} I_2 &= \frac{Z_1 I}{Z_2 + Z_1} = \frac{(5 \Omega \angle 53.13^\circ)(50 \text{ A} \angle 30^\circ)}{5 \Omega \angle -53.13^\circ} = \frac{250 \angle 83.13^\circ}{5 \angle -53.13^\circ} \\ &= 50 \text{ A} \angle 136.26^\circ \end{aligned}$$

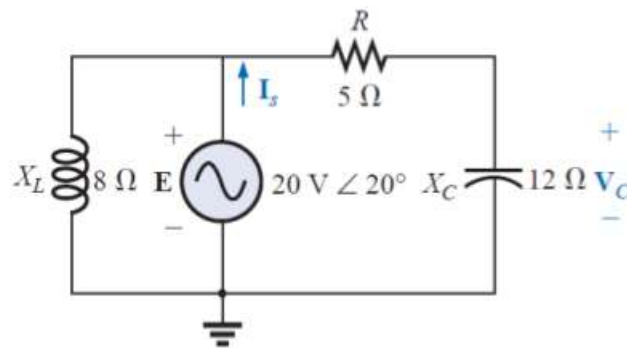
c)

$$\begin{aligned} I &= I_1 + I_2 \\ 50 \text{ A} \angle 30^\circ &= 80 \text{ A} \angle -6.87^\circ + 50 \text{ A} \angle 136.26^\circ \\ &= (79.43 - j9.57) + (-36.12 + j34.57) \\ &= 43.31 + j25.0 \\ 50 \text{ A} \angle 30^\circ &= 50 \text{ A} \angle 30^\circ \quad (\text{checks}) \end{aligned}$$



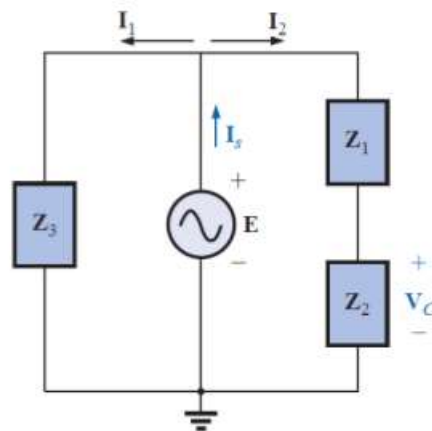
EXAMPLE 3: For the network shown below .

- Calculate the voltage V_C using the voltage divider rule.
- Calculate the current I_s .



Solutions:

a)



$$\begin{aligned} V_C &= \frac{Z_2 E}{Z_1 + Z_2} = \frac{(12 \Omega \angle -90^\circ)(20 \text{ V} \angle 20^\circ)}{5 \Omega - j 12 \Omega} = \frac{240 \text{ V} \angle -70^\circ}{13 \angle -67.38^\circ} \\ &= 18.46 \text{ V} \angle -2.62^\circ \end{aligned}$$



b)

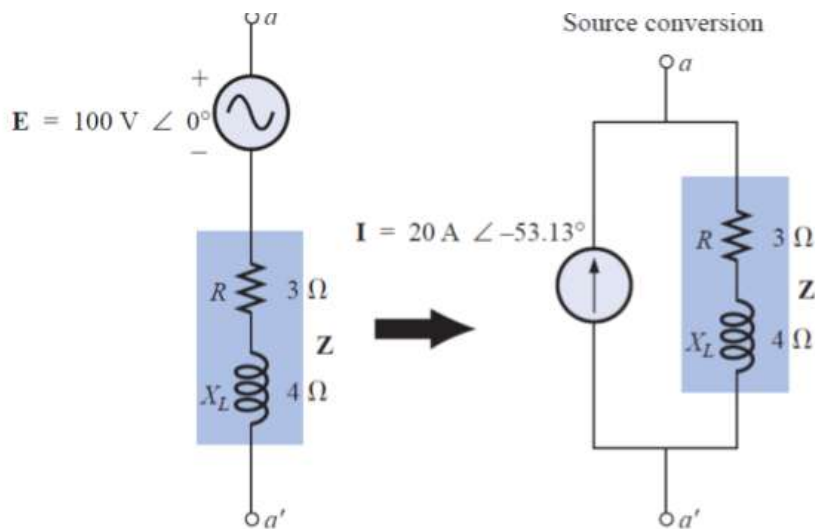
$$I_1 = \frac{E}{Z_3} = \frac{20 \text{ V } \angle 20^\circ}{8 \Omega \angle 90^\circ} = 2.5 \text{ A } \angle -70^\circ$$

$$I_2 = \frac{E}{Z_1 + Z_2} = \frac{20 \text{ V } \angle 20^\circ}{13 \Omega \angle -67.38^\circ} = 1.54 \text{ A } \angle 87.38^\circ$$

$$\begin{aligned} I_s &= I_1 + I_2 \\ &= 2.5 \text{ A } \angle -70^\circ + 1.54 \text{ A } \angle 87.38^\circ \\ &= (0.86 - j 2.35) + (0.07 + j 1.54) \\ I_s &= 0.93 - j 0.81 = 1.23 \text{ A } \angle -41.05^\circ \end{aligned}$$

Methods of Analysis (AC):

EXAMPLE 1 : Convert the voltage source to a current source.

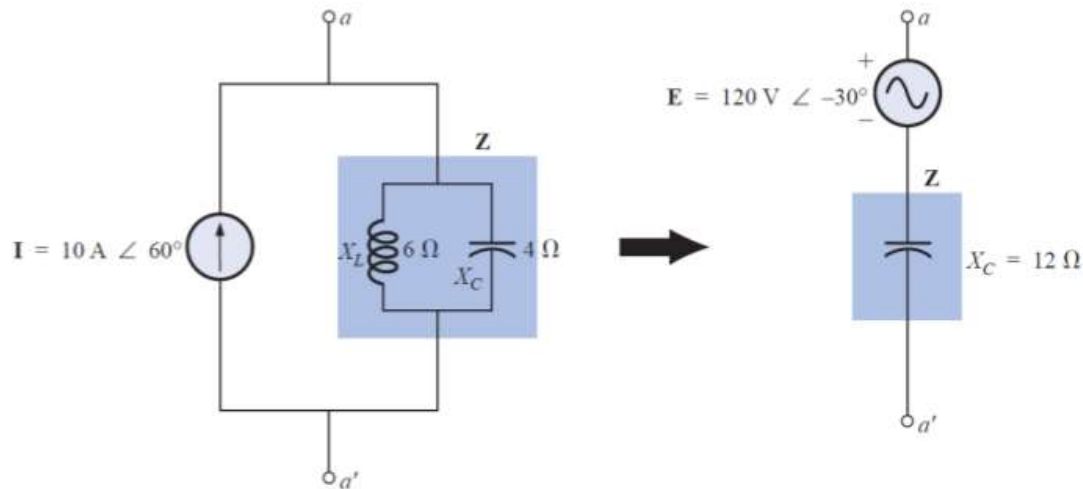


Solutions:

$$\begin{aligned} I &= \frac{E}{Z} = \frac{100 \text{ V } \angle 0^\circ}{5 \Omega \angle 53.13^\circ} \\ &= 20 \text{ A } \angle -53.13^\circ \end{aligned}$$



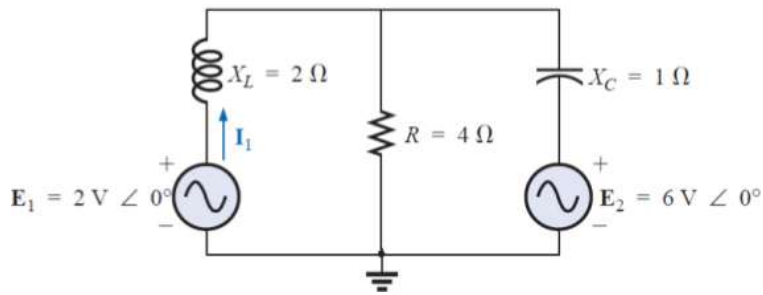
EXAMPLE 2: Convert the current source to a voltage source.



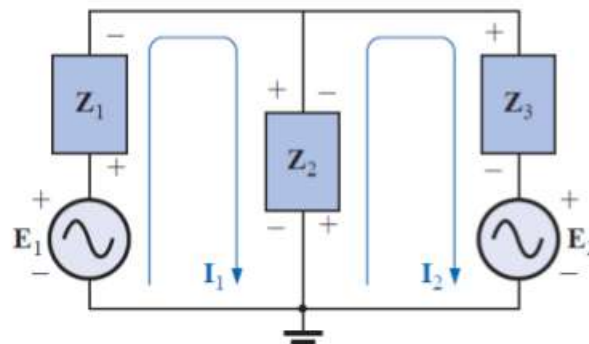
Solutions:

$$\begin{aligned} Z &= \frac{Z_C Z_L}{Z_C + Z_L} = \frac{(X_C \angle -90^\circ)(X_L \angle 90^\circ)}{-j X_C + j X_L} \\ &= \frac{(4 \Omega \angle -90^\circ)(6 \Omega \angle 90^\circ)}{-j 4 \Omega + j 6 \Omega} = \frac{24 \Omega \angle 0^\circ}{2 \angle 90^\circ} \\ &= 12 \Omega \angle -90^\circ \\ E &= IZ = (10 \text{ A} \angle 60^\circ)(12 \Omega \angle -90^\circ) \\ &= 120 \text{ V} \angle -30^\circ \end{aligned}$$

EXAMPLE 3: Using the general approach to mesh analysis, find the current I_1 in the network shown below .



Solutions:



$$\begin{aligned} Z_1 &= +j X_L = +j 2 \Omega & E_1 &= 2 \text{ V } \angle 0^\circ \\ Z_2 &= R = 4 \Omega & E_2 &= 6 \text{ V } \angle 0^\circ \\ Z_3 &= -j X_C = -j 1 \Omega \end{aligned}$$

$$\begin{aligned} I_1(Z_1 + Z_2) - I_2 Z_2 &= E_1 \\ I_2(Z_2 + Z_3) - I_1 Z_2 &= -E_2 \end{aligned}$$

$$\begin{aligned} I_1(Z_1 + Z_2) - I_2 Z_2 &= E_1 \\ -I_1 Z_2 + I_2(Z_2 + Z_3) &= -E_2 \end{aligned}$$



$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$

$$= \frac{\mathbf{E}_1(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{E}_2(\mathbf{Z}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - (\mathbf{Z}_2)^2}$$

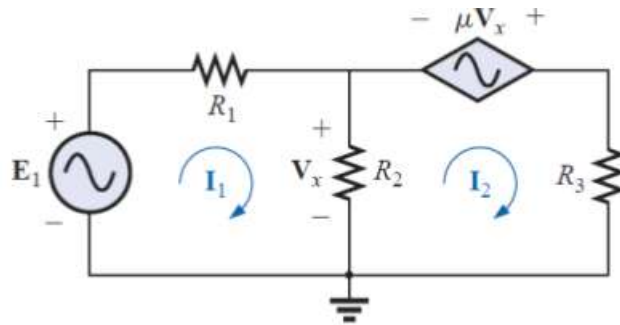
$$= \frac{(\mathbf{E}_1 - \mathbf{E}_2)\mathbf{Z}_2 + \mathbf{E}_1\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

$$\mathbf{I}_1 = \frac{(2 \text{ V} - 6 \text{ V})(4 \Omega) + (2 \text{ V})(-j 1 \Omega)}{(+j 2 \Omega)(4 \Omega) + (+j 2 \Omega)(-j 2 \Omega) + (4 \Omega)(-j 2 \Omega)}$$

$$= \frac{-16 - j 2}{j 8 - j^2 2 - j 4} = \frac{-16 - j 2}{2 + j 4} = \frac{16.12 \text{ A} \angle -172.87^\circ}{4.47 \angle 63.43^\circ}$$

$$= 3.61 \text{ A} \angle -236.30^\circ \text{ or } 3.61 \text{ A} \angle 123.70^\circ$$

EXAMPLE 4: Write the mesh currents for the network having a dependent voltage source.



Solutions:

$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu \mathbf{V}_x - \mathbf{I}_2 R_3 = 0 \quad \mathbf{V}_x = (\mathbf{I}_1 - \mathbf{I}_2) R_2$$

$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$\underline{R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu R_2(\mathbf{I}_1 - \mathbf{I}_2) - \mathbf{I}_2 R_3 = 0}$$

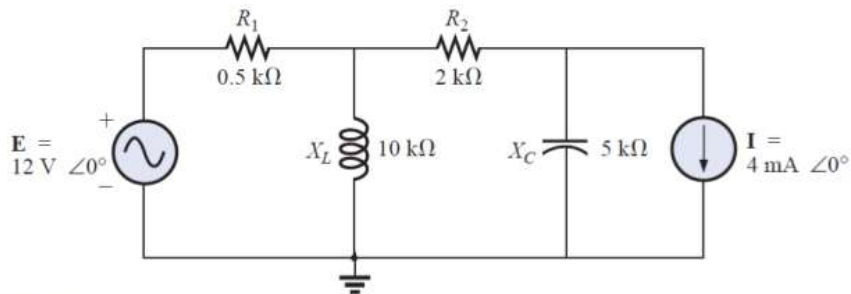


Week 8:

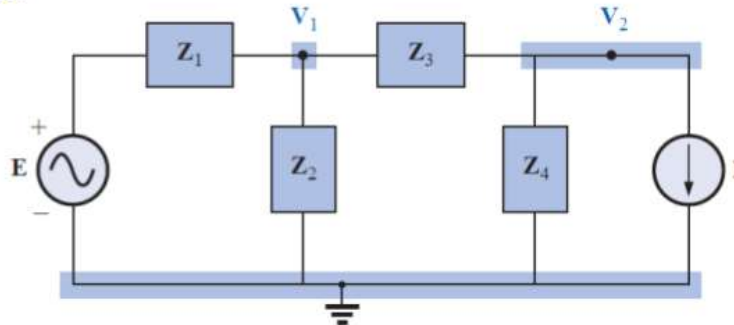
Methods of Analysis (AC) Network :

Different network “Nodal Equations” for the network having a dependent current source.

EXAMPLE 5: Write the nodal equations for the network having a dependent current source shown below.



Solutions:

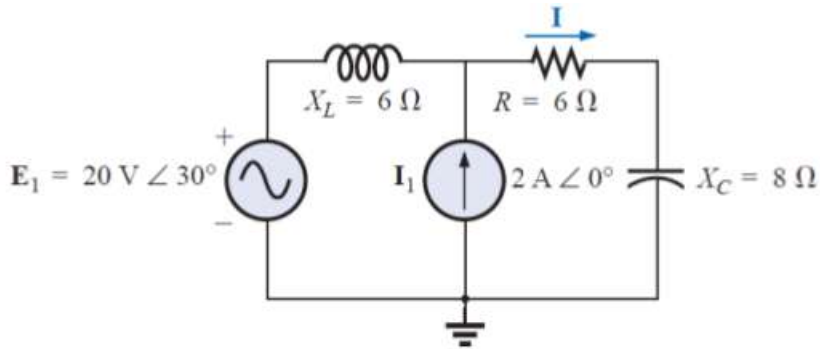


$$V_1 \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right] - V_2 \left[\frac{1}{Z_3} \right] = \frac{E_1}{Z_1}$$

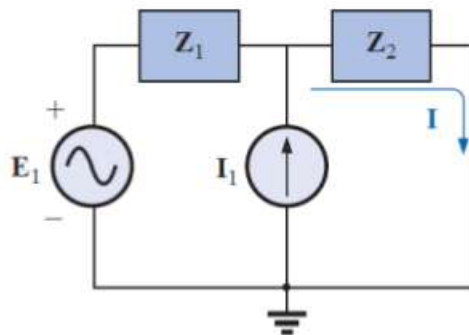
$$V_2 \left[\frac{1}{Z_3} + \frac{1}{Z_4} \right] - V_1 \left[\frac{1}{Z_3} \right] = -I$$



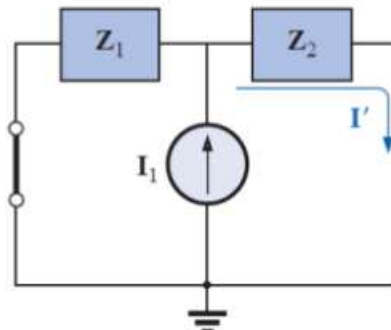
EXAMPLE 6 : Using superposition, find the current I through the $6\ \Omega$ resistor shown in the network below.



Solutions:

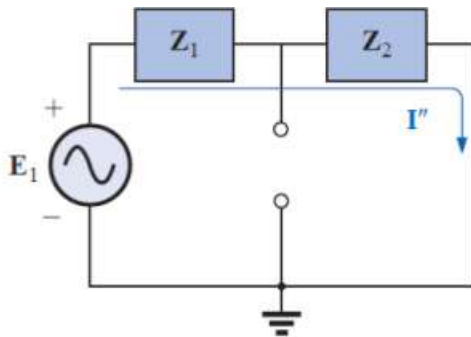


$$Z_1 = j6\ \Omega \quad Z_2 = 6 - j8\ \Omega$$





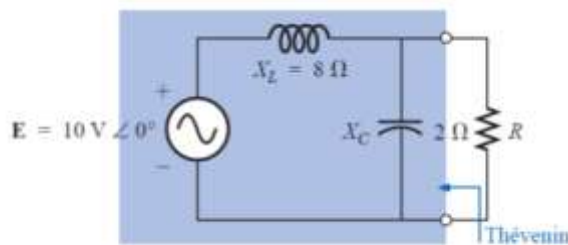
$$\begin{aligned} I' &= \frac{Z_1 I_1}{Z_1 + Z_2} = \frac{(j6 \Omega)(2 \text{ A})}{j6 \Omega + 6 \Omega - j8 \Omega} = \frac{j12 \text{ A}}{6 - j2} \\ &= \frac{12 \text{ A} \angle 90^\circ}{6.32 \angle -18.43^\circ} \\ I' &= 1.9 \text{ A} \angle 108.43^\circ \end{aligned}$$



$$\begin{aligned} I'' &= \frac{E_1}{Z_T} = \frac{E_1}{Z_1 + Z_2} = \frac{20 \text{ V} \angle 30^\circ}{6.32 \Omega \angle -18.43^\circ} \\ &= 3.16 \text{ A} \angle 48.43^\circ \end{aligned}$$

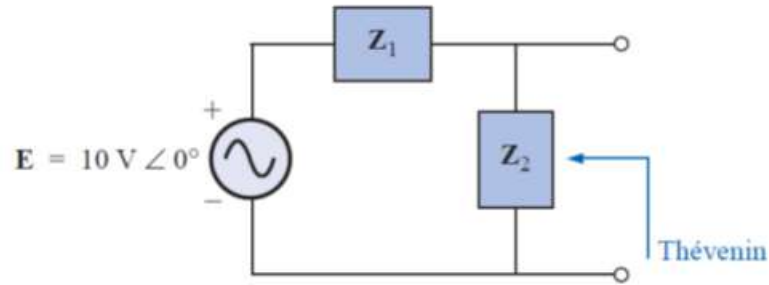
$$\begin{aligned} I &= I' + I'' \\ &= 1.9 \text{ A} \angle 108.43^\circ + 3.16 \text{ A} \angle 48.43^\circ \\ &= (-0.60 \text{ A} + j1.80 \text{ A}) + (2.10 \text{ A} + j2.36 \text{ A}) \\ &= 1.50 \text{ A} + j4.16 \text{ A} \\ I &= 4.42 \text{ A} \angle 70.2^\circ \end{aligned}$$

EXAMPLE 7 : Find the Thévenin equivalent circuit for the network external to resistor R for the network shown below.

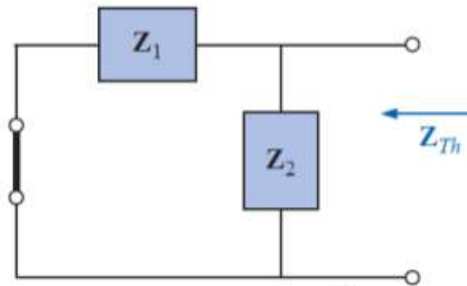




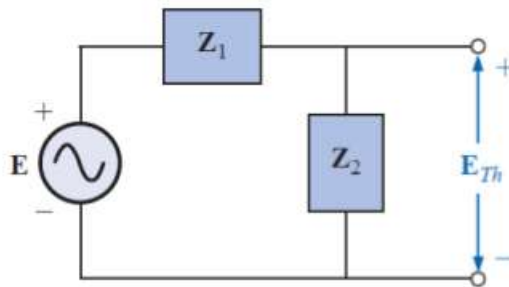
Solutions:



$$Z_1 = jX_L = j8\ \Omega \quad Z_2 = -jX_C = -j2\ \Omega$$

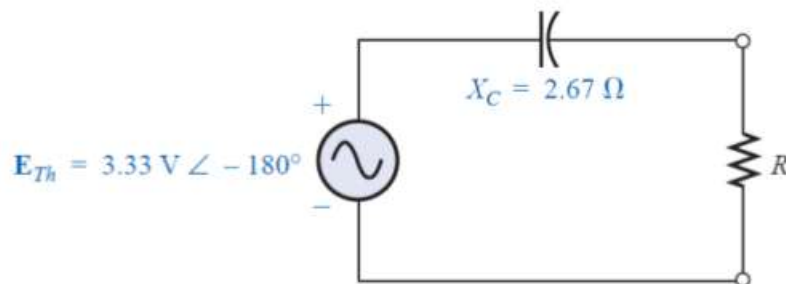
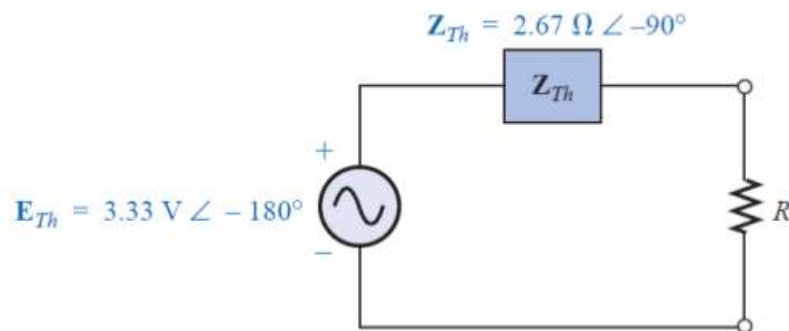


$$Z_{Th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(j8\ \Omega)(-j2\ \Omega)}{j8\ \Omega - j2\ \Omega} = \frac{-j^2 16\ \Omega}{j6} = \frac{16\ \Omega}{6 \angle 90^\circ} \\ = 2.67\ \Omega \angle -90^\circ$$

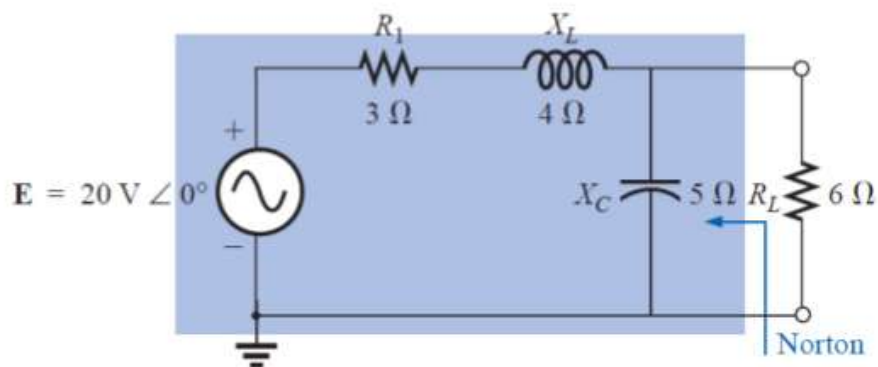




$$\begin{aligned} E_{Th} &= \frac{Z_2 E}{Z_1 + Z_2} \quad (\text{voltage divider rule}) \\ &= \frac{(-j 2 \Omega)(10 \text{ V})}{j 8 \Omega - j 2 \Omega} = \frac{-j 20 \text{ V}}{j 6} = 3.33 \text{ V} \angle -180^\circ \end{aligned}$$

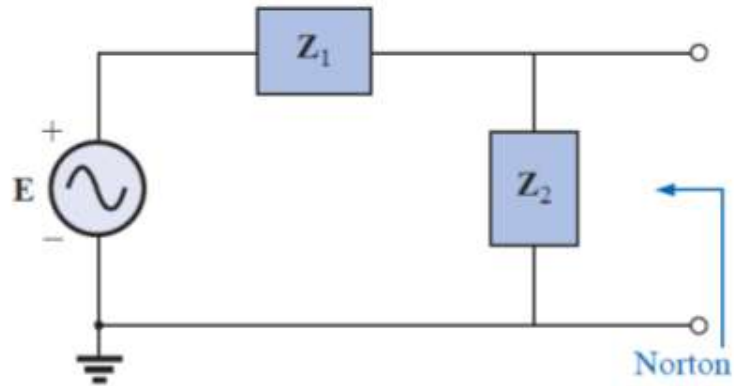


EXAMPLE 8: Determine the Norton equivalent circuit for the network external to the 6 Ω resistor for the network shown below.



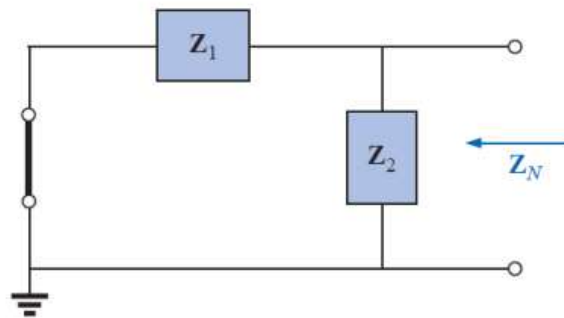


Solutions:



$$Z_1 = R_1 + jX_L = 3 \Omega + j4 \Omega = 5 \Omega \angle 53.13^\circ$$

$$Z_2 = -jX_C = -j5 \Omega$$



$$\begin{aligned}
 Z_N &= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(5 \Omega \angle 53.13^\circ)(5 \Omega \angle -90^\circ)}{3 \Omega + j4 \Omega - j5 \Omega} = \frac{25 \Omega \angle -36.87^\circ}{3 - j1} \\
 &= \frac{25 \Omega \angle -36.87^\circ}{3.16 \angle -18.43^\circ} = 7.91 \Omega \angle -18.44^\circ = 7.50 \Omega - j2.50 \Omega
 \end{aligned}$$

