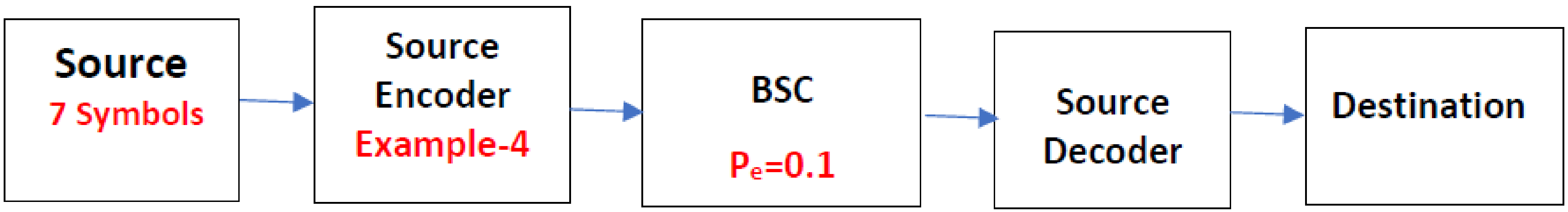
Example-5:

If the code of Example-4 is connected to BSC with P_e=0.1 as shown below, find probabilities of binary “0” and binary “1” at both the input and output of the channel.



Solution:

From the results of Example-4 we have the code table below. Now to find P(0) and P(1) at the encoder output, we must average the lengths of zeros ($l_{i,0}$) and the lengths of ones ($l_{i,1}$) and then find their probabilities using the following formulas:

$L = \sum_x P(x_i) \, . \, l_i$ $\quad L_0 = \sum_x P(x_i) \, . \, l_{i,0}$ $\quad L_1 = \sum_x P(x_i) \, . \, l_{i,1}$

$P(0) = \frac{L_0}{L}$ $\quad P(1) = \frac{L_1}{L}$ or P(1)=1-P(0)

It is also clear that: $L = L_0 + L_1$ since $l_i = l_{i,0} + l_{i,1}$

Symbol x_i	Probability $P(x_i)$	Assigned Codeword	l_i	No. of 0's $l_{i,0}$	No. of 1's $l_{i,1}$
x_4	0.3	00	2	2	0
x_2	0.2	11	2	0	2
x_5	0.15	010	3	2	1
x_7	0.12	100	3	2	1
x_3	0.1	101	3	1	2
x_6	0.08	0110	4	2	2
x_1	0.05	0111	4	1	3

Using the above formulas one can find that:

We already have L=2.63 Bits/Symbol,

$L_0 = \sum_x P(x_i) \, . \, l_{i,0} = 1.45$ $\quad L_1 = \sum_x P(x_i) \, . \, l_{i,1} = 1.18$

Then $P(0) = \frac{L_0}{L} = 0.55$ $\quad P(1) = \frac{L_1}{L} = 0.45$ or P(1)=1-P(0)=1-0.55=0.45