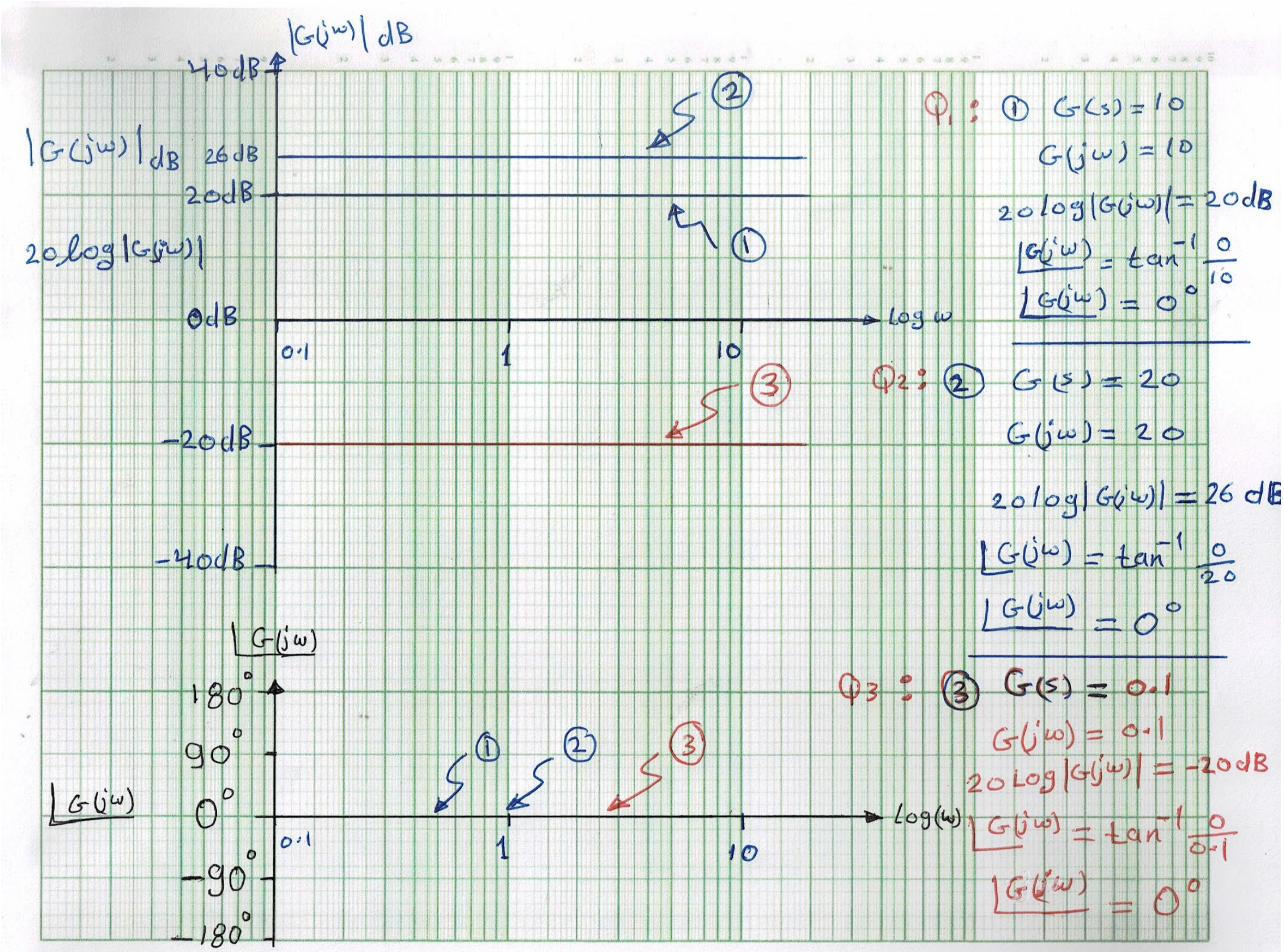
Bode Plot Method

Factor 1 : System Gain 'K'



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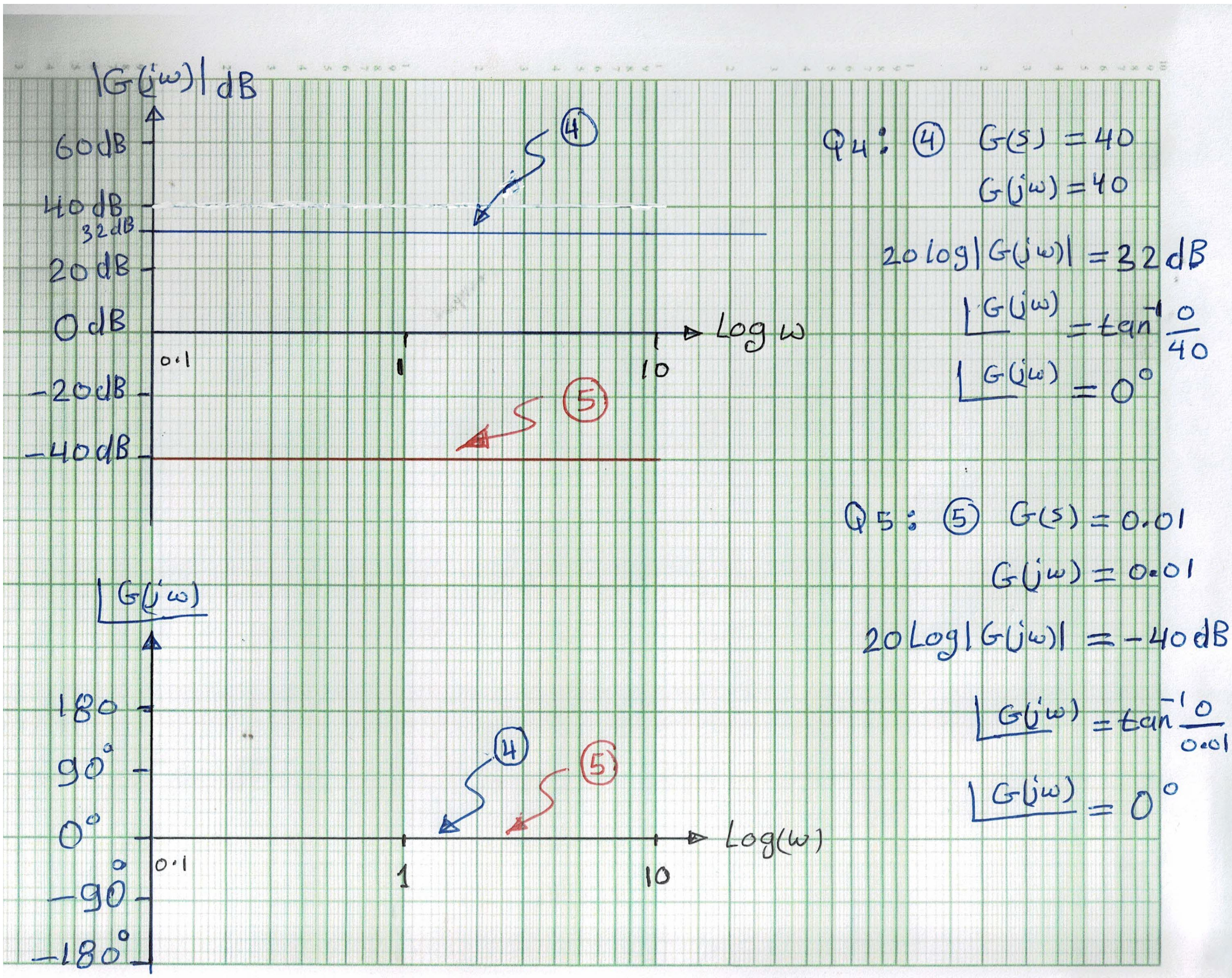

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Factor 2 : Poles or Zeros at the Origin $(j\omega)^{\pm P}$

for `P' number of poles at the origin

$$G(s)H(s) = \frac{1}{s^P}$$

$$G(j\omega)H(j\omega) = \frac{1}{\omega} \cdot \frac{1}{\omega} \ldots\ldots \text{ P times}$$

$$\therefore \quad |G(j\omega)H(j\omega)| = \frac{1}{\omega} \cdot \frac{1}{\omega} \ldots\ldots \text{P times} = \frac{1}{(\omega)^P}$$

$$|G(j\omega)H(j\omega)| \text{ in dB} = 20 \text{ Log } \frac{1}{(\omega)^P} = 20 \text{ Log } (\omega)^{-P} = -20 \times P \text{ Log } \omega$$

So this is a straight line of slope – 20 x P dB/ decade but again intersecting with 0 dB line at ω = 1.

Key Point: *Therefore magnitude plot for `P' poles at the origin gives a family of lines passing through intersection of ω = 1 and 0 dB line having slope [– 20 × P] dB/decade as* shown in figure (16-1)

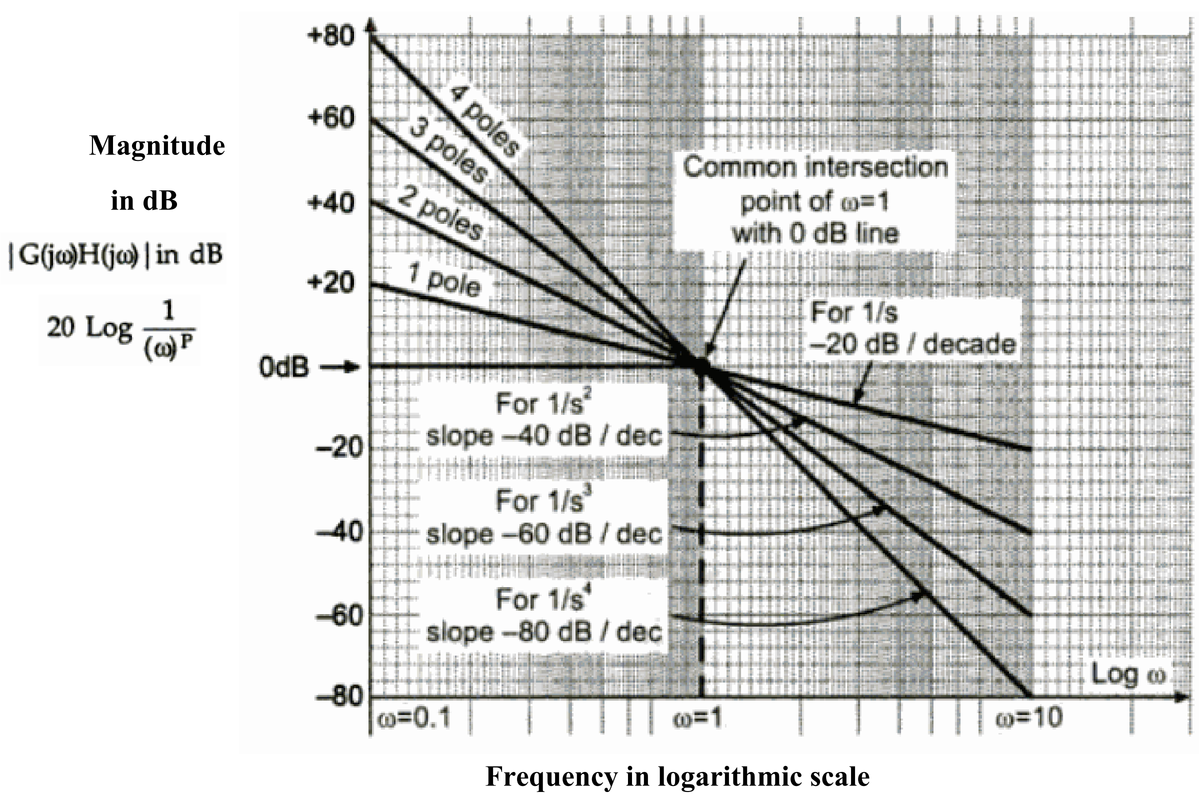

Fig. (16-1): Magnitude plot of poles at the origin

for P number of zeros at the origin

$$G(s)H(s) = s^P$$

$$\therefore \quad G(j\omega)H(j\omega) = j\omega \cdot j\omega \cdot j\omega \ldots\ldots P \text{ time}$$

$$\therefore \quad |G(j\omega)H(j\omega)| = \omega^P$$

$$\therefore \quad \text{Magnitude in dB} = 20 \times P \operatorname{Log} \omega$$

$$\text{i.e.} \quad \text{Slope} = +20 \times P \text{ dB/decade}$$

Key Point : *Each zero at the origin increases the magnitude at a rate of + 20 dB/decade.*

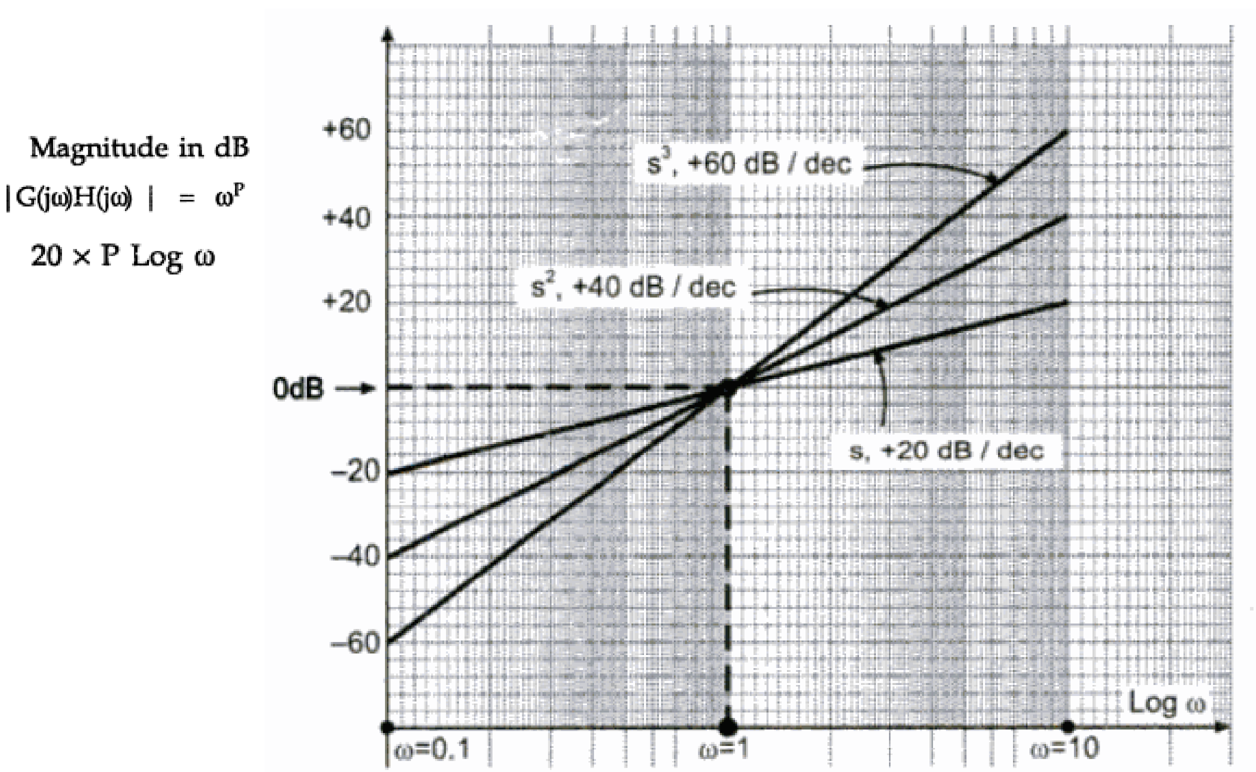

Figure (16-2): Magnitude plot of zeros at the origin

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Phase Angle Plot : Consider 1 pole at the origin

$$G(s)H(s) = \frac{1}{s} \qquad G(j\omega)H(j\omega) = \frac{1}{j\omega}$$

$$\therefore \quad \angle G(j\omega)H(j\omega) = \angle\frac{1}{j\omega} = \frac{0^\circ}{90^\circ} = -90^\circ$$

For 2 poles at origin,

$$G(s)H(s) = \frac{1}{s^2}$$

$$\therefore \quad G(j\omega)H(j\omega) = \frac{1}{j\omega}\cdot\frac{1}{j\omega}$$

$$\therefore \quad \angle G(j\omega)H(j\omega) = \angle\frac{1}{j\omega}\angle\frac{1}{j\omega} = \frac{0^o}{90^o + 90^o} = -180^o$$

Key Point: *In general P number of poles at the origin contribute $-90^\circ \times P$ angle to overall phase angle plot. This contribution is constant irrespective of 'ω'.*

Similarly for a zero at the origin,

$$G(s)H(s) = s \qquad \text{i.e.} \qquad G(j\omega)H(j\omega) = j\omega$$

$$\therefore \quad \angle G(j\omega)H(j\omega) = \angle 0 + j\omega = +\tan^{1}\frac{\omega}{0} = +90^\circ$$

Key Point: *In general 'P' number of zeros at the origin, the total angle contribution is $+90^\circ \times P$, irrespective of value of ω.*

Key Point: *In general `P' number of zeros at the origin, the total angle contribution is* $+90° \times P$*, irrespective of value of* ω.

Phase angle

In degree

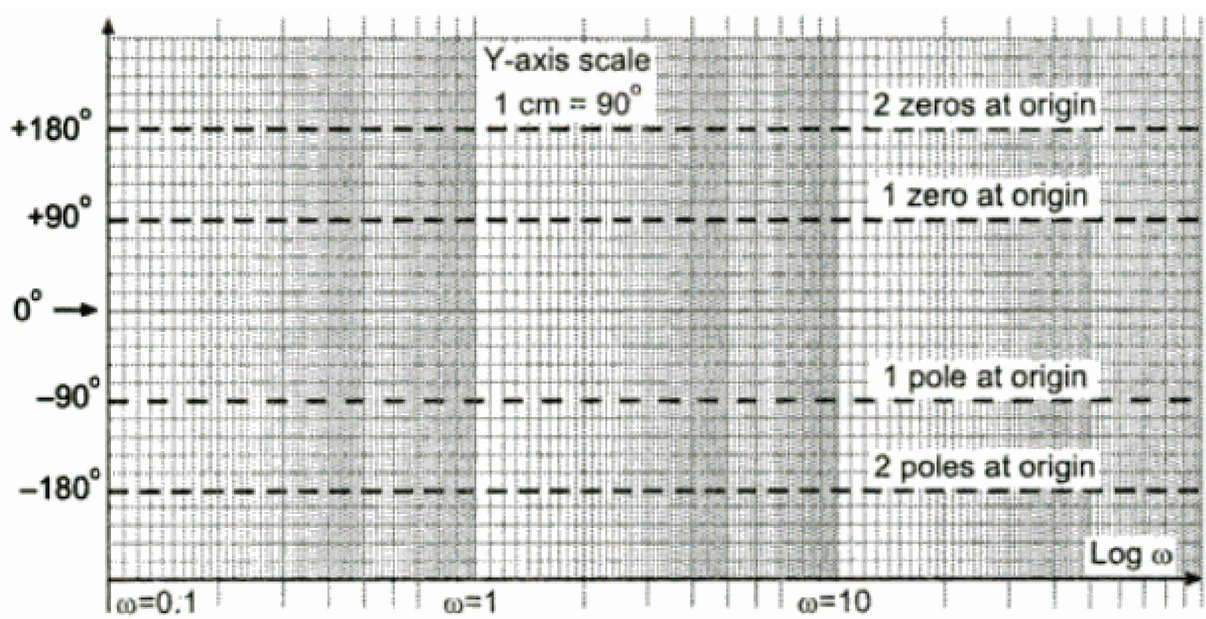

Fig. (16-3) phase angle plot

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Example: Consider $G(s)H(s) = \frac{10}{s}$ so $G(j\omega)H(j\omega) = \frac{10}{j\omega}$

Factor	w_c	Slope	Effect on magnitude plot
K = 10	-	0 dB / dec	Shift up by 20 log 10 = 20 dB/dec
1/ s	-	-20 dB / dec	Slope = 0 – 20 = -20 dB/dec

ω	Contribution by K	By 1 pole at origin	Resultant ϕ_R
0	0°	– 90°	– 90°
10	0°	– 90°	– 90°
40	0°	– 90°	– 90°
1000	0°	– 90°	– 90°
∞	0°	– 90°	– 90°

So phase angle plot is straight line parallel to X-axis as shown with phase angle – 90°.

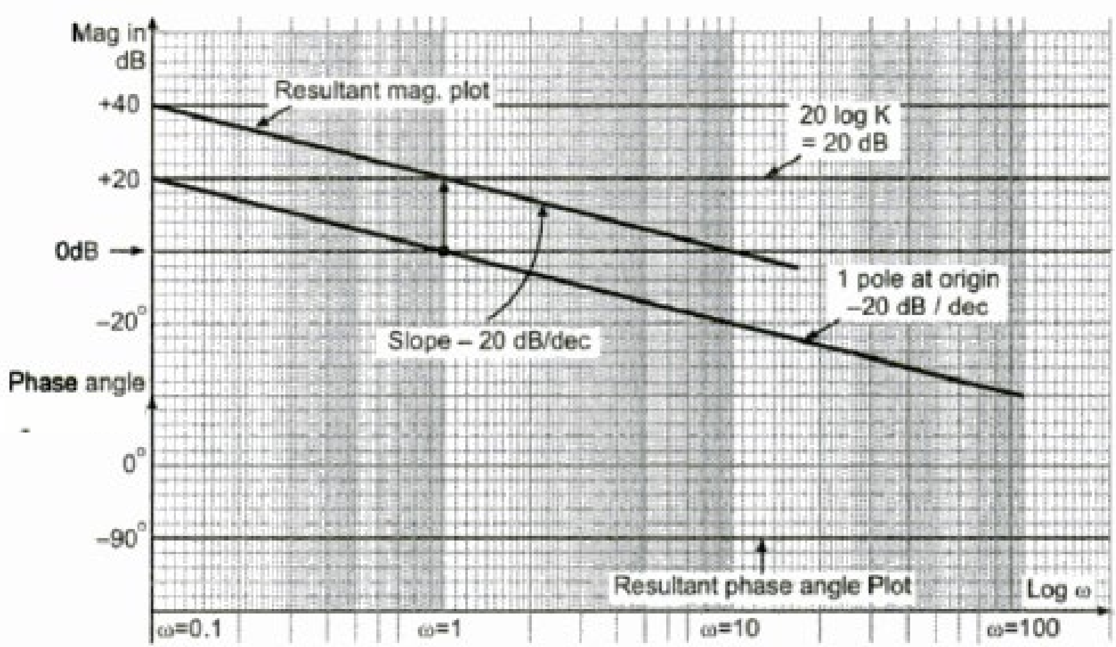


Fig. (16-4) phase angle plot

Factor 3 : Simple Poles or Zeros (First Order Factors)

The factor is represented as $(1+Ts)^{\pm 1}$ i.e. $(1+j\omega T)^{\pm 1}$

Let us start with a simple pole

$$G(s)\,H(s) = (1+Ts)^{-1} = \frac{1}{(1+Ts)}$$

$$G(j\omega)\,H(j\omega) = \frac{1}{1+T\,j\omega}$$

$$\therefore \quad |G(j\omega)\,H(j\omega)| = \frac{1}{\sqrt{1+(\omega T)^2}} = \left[\sqrt{1+(\omega T)^2}\right]^{-1}$$

$$\therefore \quad \text{In dB magnitude} = 20\,\text{Log}\left[\sqrt{1+(\omega T)^2}\right]^{-1}$$

$$= -\,20\,\text{Log}\sqrt{1+\omega^2 T^2}\ \text{dB}$$

The approximation is,

i) For low frequency range $\omega << \frac{1}{T}$ i.e. $\omega^2 T^2 << 1$ hence can be neglected.

$\therefore$ Magnitude in dB $= -\,20\,\text{Log}\,1 = 0$ dB.

So for low frequencies it is straight line of 0 dB only. Thus the contribution by such factor can be completely neglected for low frequency range, as it is very small.

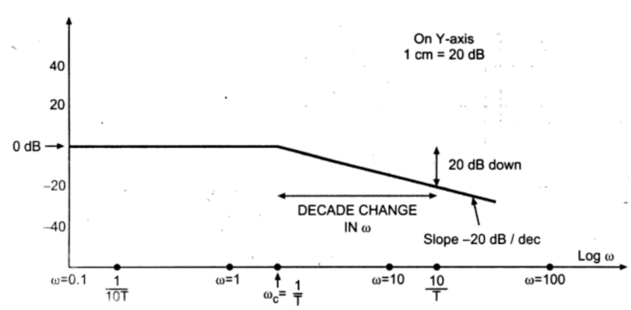
ii) For high frequency range $\omega >> \frac{1}{T}$ $\therefore\ 1 << \omega^2 T^2$

Magnitude in dB $= -\,20\,\text{Log}\,\omega T$ dB

i.e. it is straight line of slope – 20 dB/ decade.

This frequency at which change of slope from 0 dB to – 20 dB/decade occurs is called **Corner Frequency**, denoted by ω_c.

$$\therefore \quad \omega_c = 1/T$$



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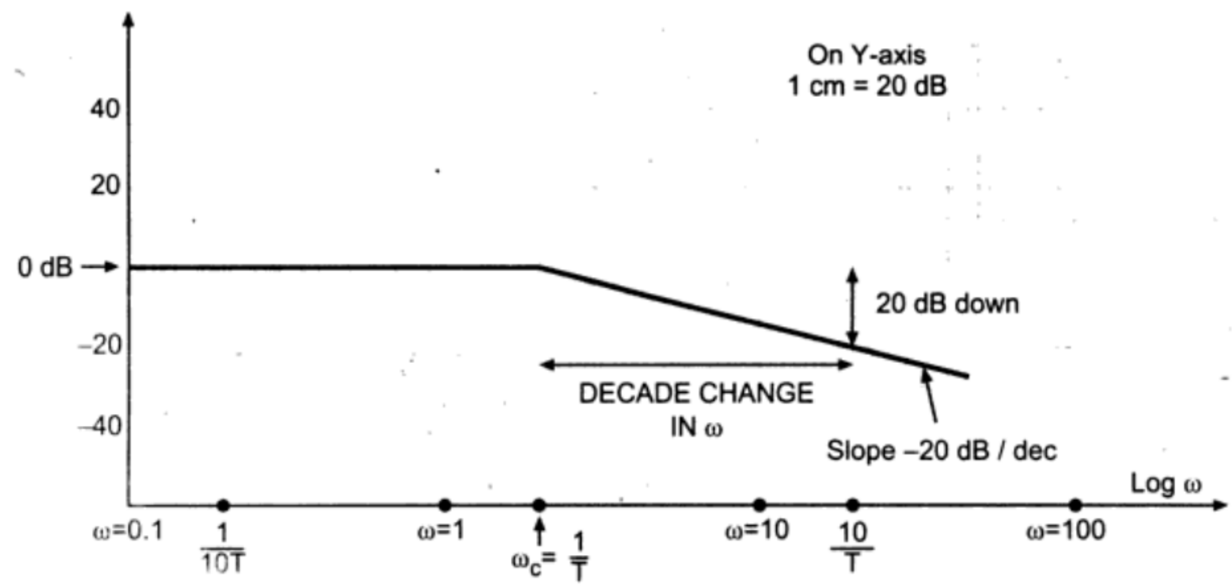


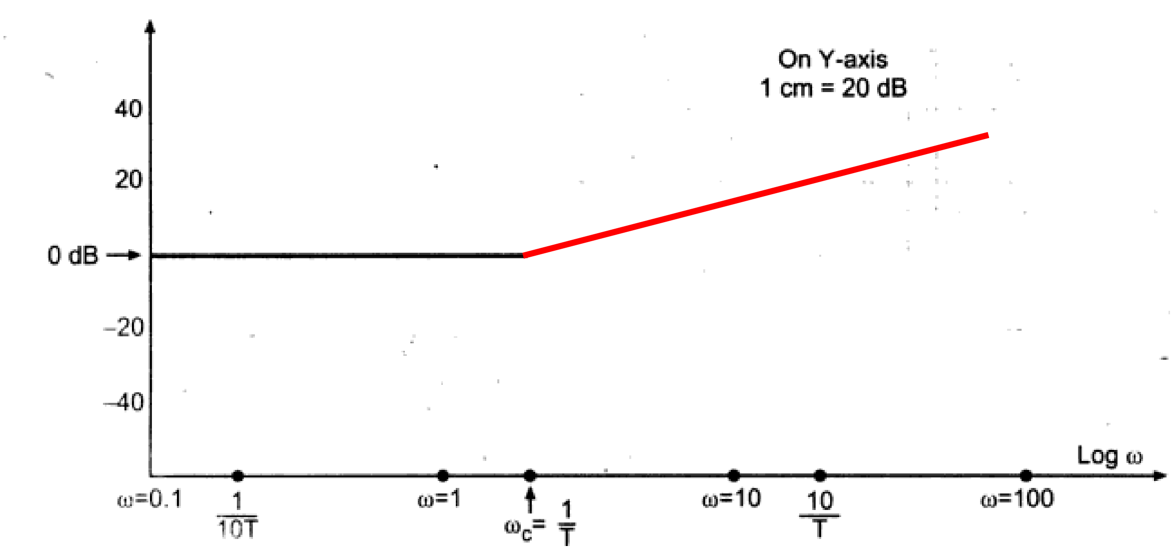
For a simple zero, i.e. first order zero,

$$G(s)H(s) = (1 + Ts) \qquad \text{i.e.} \quad G(j\omega)H(j\omega) = (1 + j\omega T)$$

$$\therefore \qquad |G(j\omega)H(j\omega)| = \sqrt{1+(\omega T)^2}$$

$$\therefore \qquad \text{Magnitude in dB} = 20 \text{ Log } \sqrt{1+\omega^2 T^2} \cdot \text{dB}$$

The magnitude plot for simple zero is a straight line of 0 dB upto $\omega_C = 1/T$ and then straight line of slope + 20 dB/decade for all frequencies more than corner frequency.



Phase Angle Plot : Consider a simple pole

$$G(s)H(s) = \frac{1}{1+Ts}$$

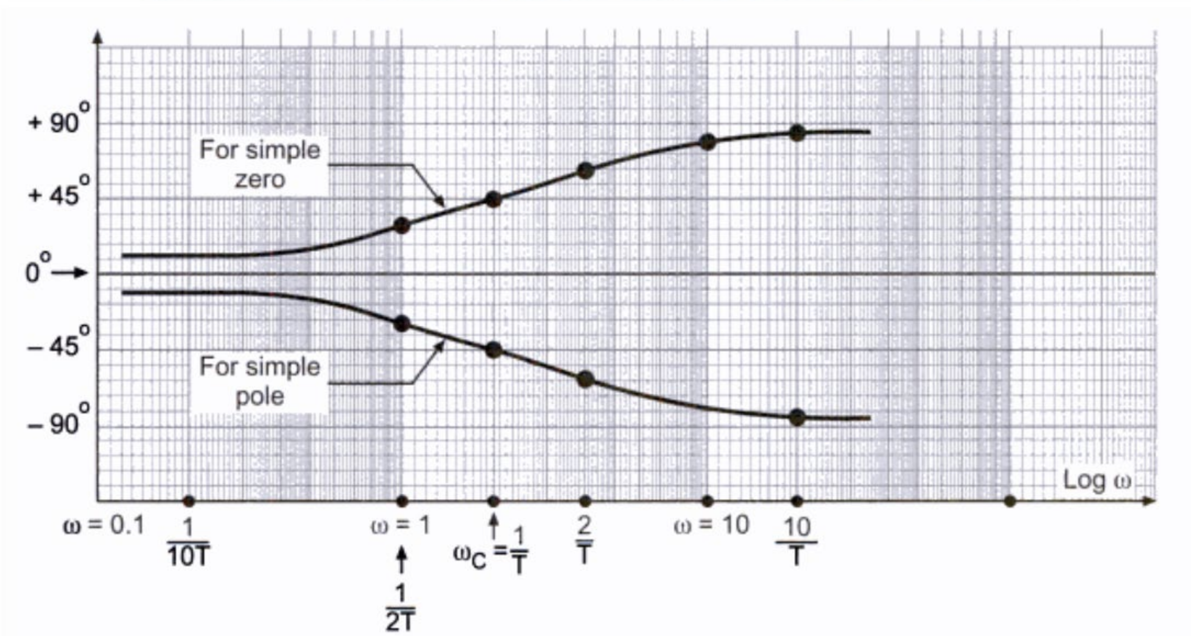
$$G(j\omega)H(j\omega) = \frac{1}{1+j\omega T} \quad \therefore \angle G(j\omega)H(j\omega) = \frac{0^\circ}{\tan^{-1}\frac{\omega T}{1}} = -\tan^{-1}\omega T$$

While for a simple zero,

$$G(s)H(s) = 1 + Ts$$

$$G(j\omega)H(j\omega) = 1 + j\,\omega T \quad \therefore \angle G(j\omega)H(j\omega) = \tan^{-1}\frac{\omega T}{1} = +\tan^{-1}\omega T$$

ω	$\pm\tan^{-1}\omega T$ (+ for zero, – for pole)
$0.1\omega_C = \frac{1}{10T}$	± 5.71°
$0.5\omega_C = \frac{1}{2T}$	± 26.6°
$\omega_C = \frac{1}{T}$	± 45°
$2\,\omega_C = \frac{2}{T}$	± 63.4°
$10\,\omega_C = \frac{10}{T}$	± 84.3°



Example 10.2 : *Sketch the Bode Plot for the system having*

$$G(s)H(s) = \frac{20}{s(1+0.1\,s)}$$

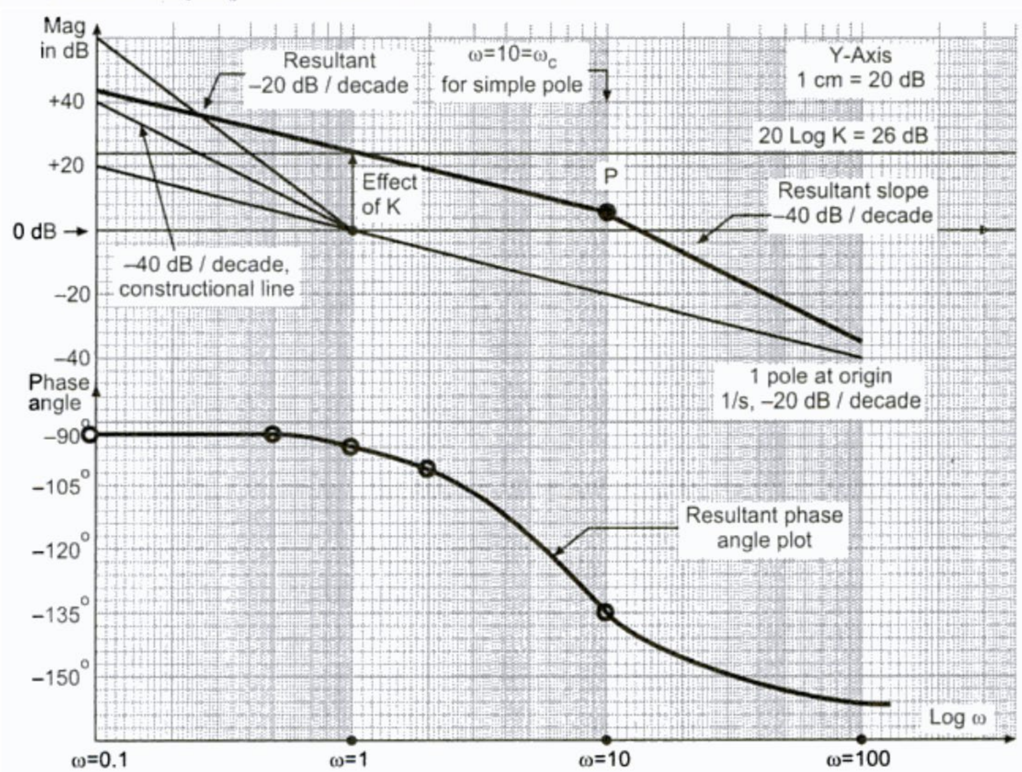
K = 20 ∴ Its magnitude = 20 Log 20 = + 26 dB

factor	w_c	slope	Effect on magnitude plot
K=20	-	0 dB/dec	Shift up by 20 log 20 = 26 dB
$\frac{1}{s}$	-	-20 dB/dec	Slope = 0 – 20 = -20 dB/dec
$\frac{1}{1+0.1s}$	10	-20 dB/dec	Slope = -20 – 20 = -40 dB/dec

For the phase angle plot prepare the table of angles as below :

ω in rad/sec	φ due to 1 pole at origin	φ due to simple pole = – $\tan^{-1} 0.1\omega$	ϕ_R Resultant
0.1	– 90°	– 0.57°	– 90.57°
0.5	– 90°	– 2.86°	– 92.86°
1	– 90°	– 5.7°	– 95.7°
2	– 90°	– 11.3°	– 101.3°
10	– 90°	– 45°	– 135°
50	– 90°	– 78.79°	– 168°

φ due to simple pole = $-\tan^{-1}\omega T = -\tan^{-1} 0.1\,\omega$



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