Bode Plot Method

Objectives:

At the end of this lesson, students will be able to:
1. Define the frequency response of a system.
2. Use the Bode plot method to find a graph of the frequency response of the system
3. Comment on the stability of the system.

Basics of Frequency Domain Analysis

Basic of any frequency response is to plot magnitude M and angle ϕ against input frequency `ω'. When `ω' is varied from 0 to ∞ there is wide range of variations in M and ϕ

Such frequency domain transfer function can be obtained by substituting jω for 's' in the transfer function G(s) of the system.

$$G(j\omega) = G(s)\big|_{s=j\omega} = \text{Frequency domain transfer function}$$

$$G(j\omega) = M\angle\phi$$

$$M = \text{Magnitude} \rightarrow f(\omega)$$

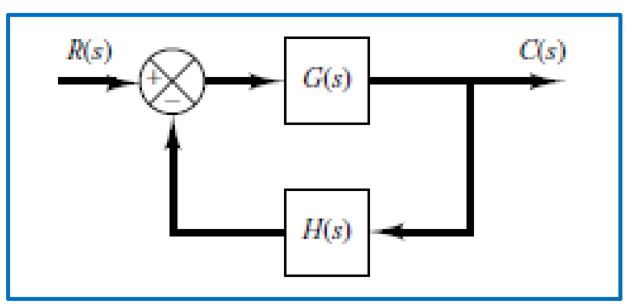
$$\phi = \text{Phase angle} \rightarrow f(\omega)$$

$$\omega = \text{Input frequency}$$

$$|G(j\omega)| = \sqrt{(\text{Real part})^2 + (\text{Imj}\cdot\text{part})^2}$$

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\text{Imaginary part of } G(j\omega)}{\text{Real part of } G(j\omega)}\right]$$

Example 15.1: find M ∟ ϕ for the following:



$$G(s) = \frac{10}{s(s+10)}, \quad H(s) = 1$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\text{i.e.} \quad T(s) = \frac{\frac{10}{s(s+10)}}{1+\frac{10}{s(s+10)}}$$

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$$\therefore \qquad T(s) = \frac{10}{s^2 + 10s + 10}$$

In the frequency domain, replace s by jω

$$\therefore \qquad T(j\omega) = \frac{10}{(j\omega)^2 + 10\,j\omega + 10} = \frac{(10)}{(10-\omega^2) + j\,10\,\omega}$$

$$T(j\omega) = M \angle \phi \quad \text{where}$$

$$M = \frac{|10|}{|(10-\omega^2) + j\,10\,\omega|} = \frac{10}{\sqrt{(10-\omega^2) + 100\,\omega^2}}$$

and

$$\phi = -\tan^{-1}\left(\frac{10\,\omega}{10-\omega^2}\right)$$

Bode Plot

for Bode plot, magnitude in dB and phase angle in degrees are the magnitudes and phase angles of $G(j\omega)H(j\omega)$, plotted against Log ω.

So in general Bode plot consists of two plots which are,

1) *Magnitude expressed in logarithmic values against logarithmic values of frequency called Magnitude Plot.*
2) *Phase angle in degrees against logarithmic values of frequency called Phase Angle Plot.*

Magnitude Plot

For Bode Plot $|G(j\omega)| = 20 \log_{10} |G(j\omega)|$ dB.

Such decibels values are to be plotted against $\log_{10}(\omega)$.

So magnitude plot can be shown as in the Fig. 15.1

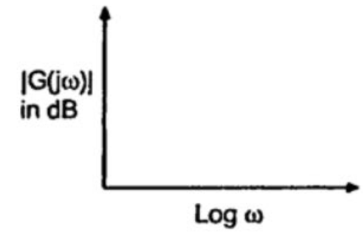

Fig. 15.1 Magnitude plot

The Phase Angle Plot

The phase angle plot can be shown as in the Fig. 15.2

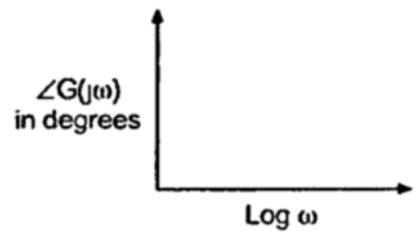

Fig. 15.2 Phase angle plot

Logarithmic Scales (Semilog Papers)

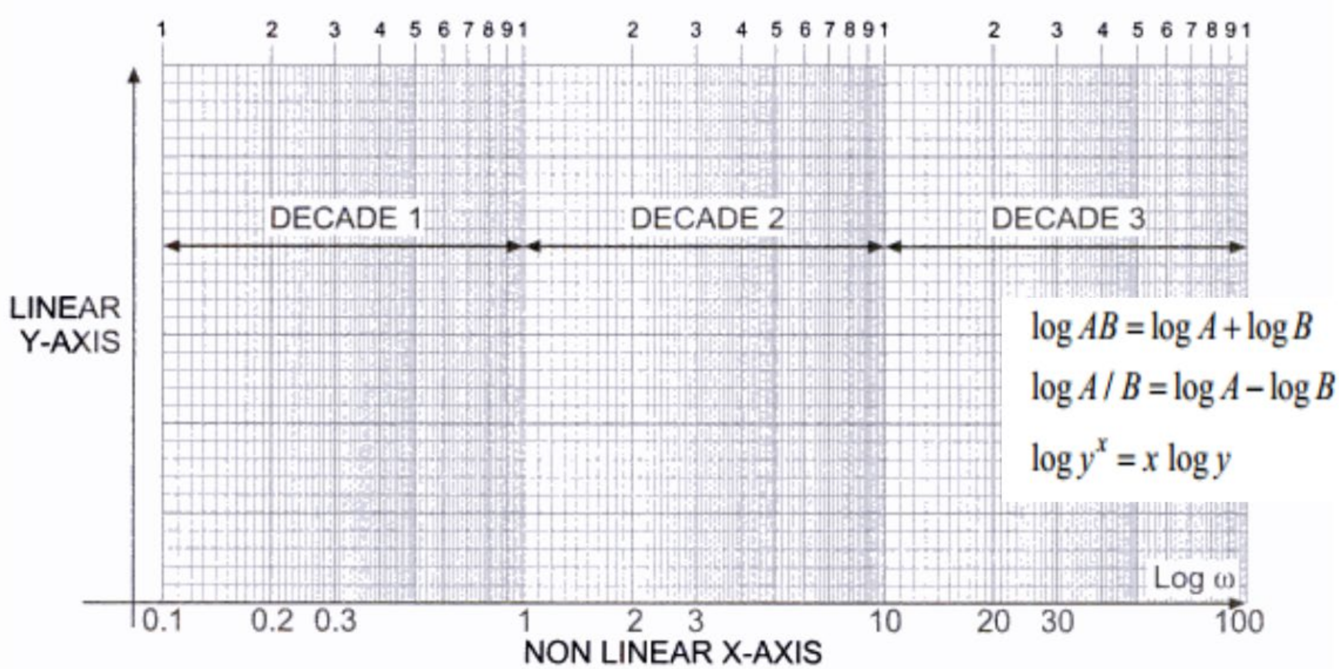

Fig. 15.3 Semilog paper

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Standard Form of Open Loop T.F. G(jw)H(jw)

Consider $$G(s)H(s) = \frac{K' s^Z (s+Z_1)(s+Z_2) \ldots\ldots}{s^P (s+P_1)(s+P_2) \ldots\ldots\ldots}$$

The standard time constant form can be denoted as,

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s) \ldots\ldots}{s^P (1+T_a s)(1+T_b s)}$$

K = Resultant system gain P = Type of the system

$T_1, T_2, T_a, T_b, \ldots\ldots$ = Time constants of different poles and zeros.

Frequency domain O.L.T.F.

$$G(j\omega)H(j\omega) = \frac{K(1+T_1 j\omega)(1+T_2 j\omega) \ldots\ldots}{(j\omega)^P (1+T_a j\omega)(1+T_b j\omega) \ldots\ldots}$$

List of such basic factors is ,

1) Resultant system gain K, constant factor.

2) Poles or zeros at the origin. (Integral and Derivative factors) i.e. $(j\omega)^{\pm P}$

3) Simple poles and zeros also called first order factors of the form $(1+j\omega T)^{\pm 1}$

4) Quadratic factors which cannot be factorised into real factors, of the form

$$\left(1+\frac{2\xi}{\omega_n}s+\frac{s^2}{\omega_n^2}\right) \approx 1+2\xi j\left(\frac{\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2$$

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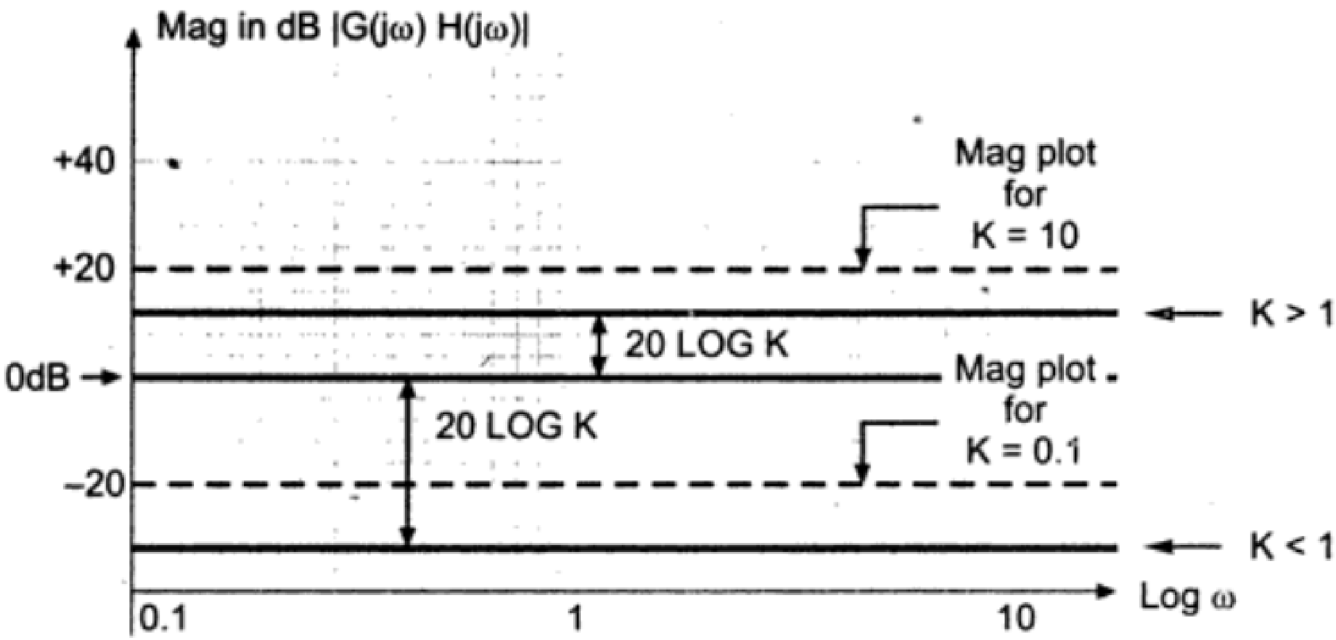
Bode Plots of Standard Factors of G(jω)H(jω)

Factor 1 : System Gain 'K'

Key Point: *This means `K' shifts the magnitude plot of* $|G(j\omega)H(j\omega)|$ *by a distance of 20 Log K dB upwards if K > 1 and downwards if K < 1.*



Phase Angle Plot :

As $G(j\omega)H(j\omega) = K + j0$

Corresponding $\phi = \tan^{-1}\dfrac{\text{imj part}}{\text{real part}} = \tan^{-1}\dfrac{0}{K} = 0°$

But if `K' is negative, it always contributes – 180° to the phase angle plot independent of frequency.

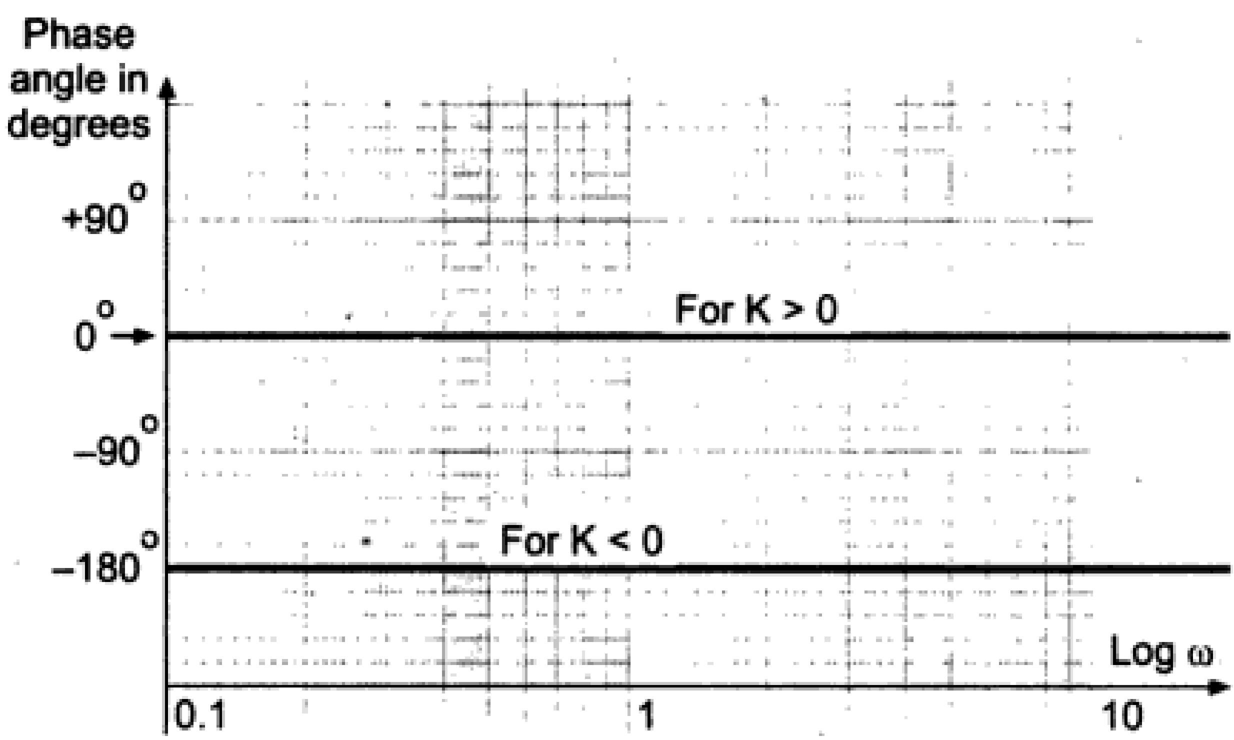


Fig (15.5): phase plot of G(jω) in Bode plot diagram for basics factor K