Weight–Volume Relationships

- A given volume of soil in natural occurrence consists of solid particles and the void spaces between the particles. The void space may be filled with air and/or water; hence, soil is a three-phase system. If there is no water in the void space, it is a **dry soil**. If the entire void space is filled with water, it is referred to as a **saturated soil**. However, if the void is partially filled with water, it is a **moist soil** (partially saturated soil).

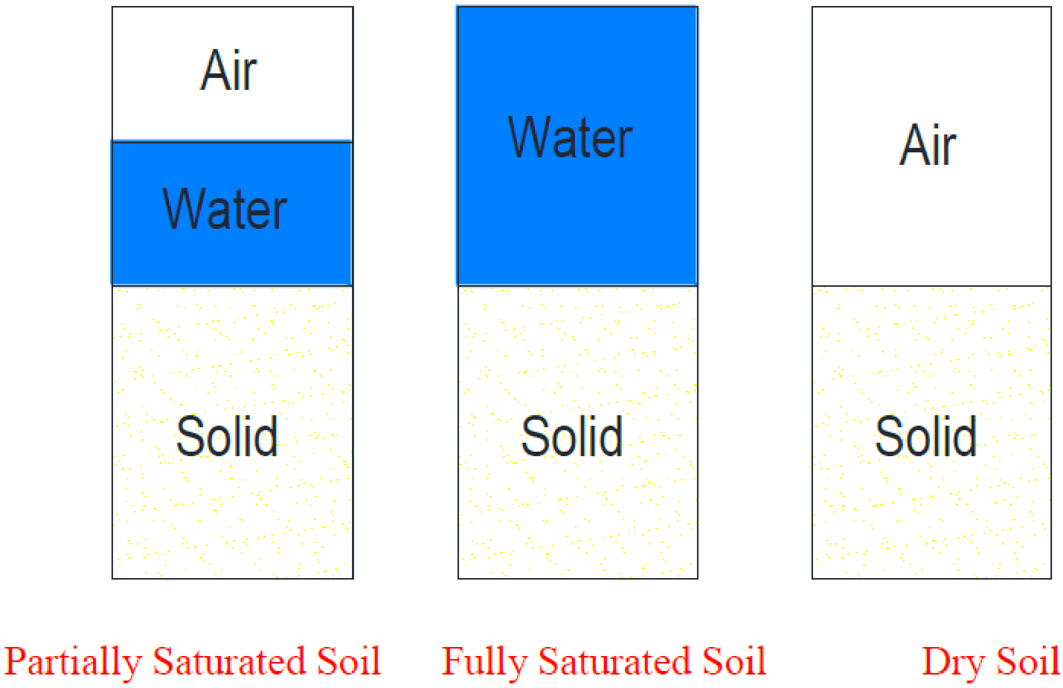
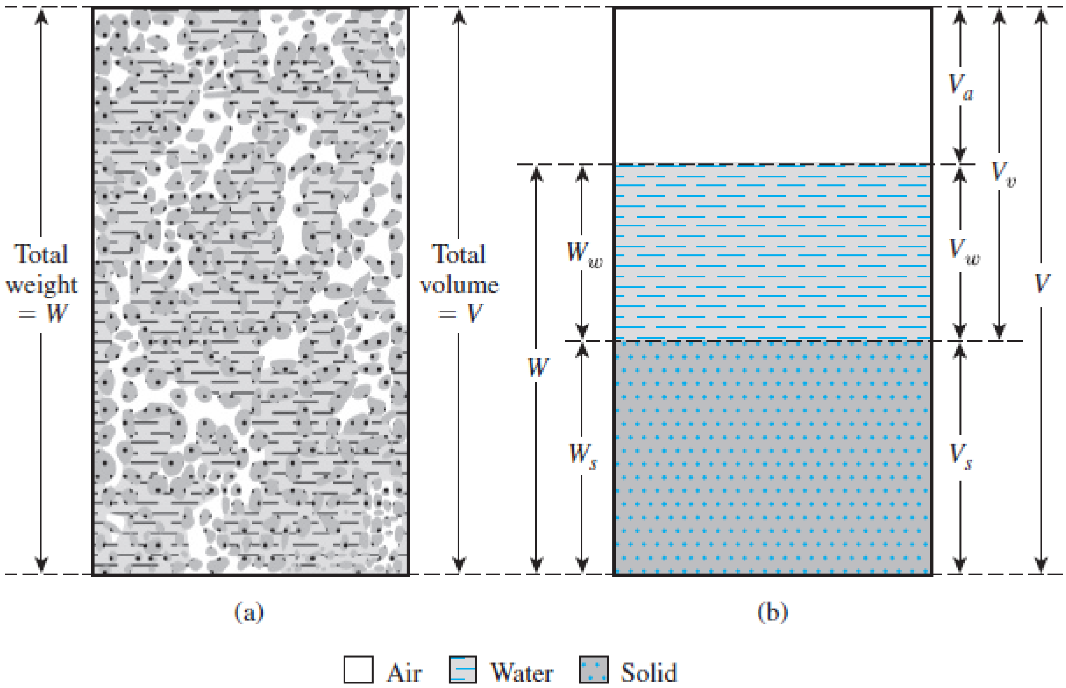


Figure (1) Soil phases.

- Figure 1.a shows an element of soil of volume *V* and weight W, as it would exist in a natural state. To develop the weight–volume relationships, we must separate the three phases (that is, solid, water, and air) as shown in Figure 1b. Thus, the total volume of a given soil sample can be expressed as

$$V = V_s + V_v = V_s + V_w + V_a$$

where V_s = volume of soil solids
V_v = volume of voids
V_w = volume of water in the voids
V_a = volume of air in the voids



Assuming that the weight of the air is negligible, we can express the total weight of the sample as

$$W = W_s + W_w$$

where W_s = weight of soil solids
W_w = weight of water

volume relationships

The volume relationships commonly used for the three phases in a soil element are void ratio, porosity, and degree of saturation.

Void ratio (e) is defined as the ratio of the volume of voids to the volume of solids.

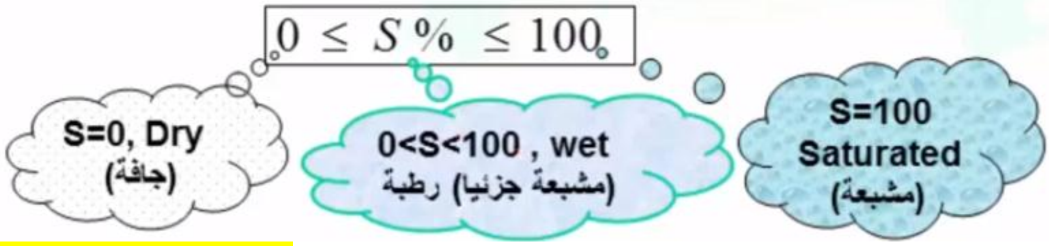
$$e = \frac{Volume \quad of \quad voids}{Volume \quad of \quad solids} = \frac{V_v}{V_s}$$

Porosity (n) is defined as the ratio of the volume of voids to the total volume

$$n = \frac{Volume\ of\ voids}{Total\ volume\ of\ soil\ sample} = \frac{V_v}{V_T} x100$$

Degree of saturation (S) is defined as the ratio of the volume of water to the volume of voids

$$S = \frac{\text{Total volume of voids contains water}}{\text{Total volume of voids}} = \frac{V_w}{V_v} \times 100\%$$



weight relationships

The common terms used for weight relationships are moisture content and unit weight.

Moisture content (w) is also referred to as *water content* and is defined as the ratio of the weight of water to the weight of solids in a given volume of soil:

$$w = \frac{\text{Weight of water}}{\text{Weight of soil solids}} = \frac{W_w}{W_s} \cdot 100\%$$

Unit weight (γ) is the weight of soil per unit volume.

$$\gamma = \frac{W}{V}$$

Soils engineers sometimes refer to the unit weight defined by above Equation as the **moist unit weight**.

Often, to solve earthwork problems, one must know the weight per unit volume of soil, excluding water. This weight is referred to as the dry unit weight, γd. Thus,

$$\gamma_d = \frac{W_s}{V}$$

the relationship of unit weight, dry unit weight, and moisture content can be given as

$$\gamma_d = \frac{\gamma}{1 + w}$$

In SI (Système International), the unit used is kilo Newtons per cubic meter (kN/m^3).

mass densities (ρ)

$$\rho = \frac{M}{V}$$

$$\rho_d = \frac{M_s}{V}$$

where ρ = density of soil (kg/m^3)
ρ_d = dry density of soil (kg/m^3)
M = total mass of the soil sample (kg)
M_s = mass of soil solids in the sample (kg)

The unit of total volume, V, is m^3.
The SI unit of mass density is kilograms per cubic meter (kg/m^3).

- The unit weight in kN/m^3 can be obtained from densities in kg/m^3 as

$$Unitweight,\ \gamma = \frac{Weight}{Volume} = \frac{Mass \cdot g}{Volume} = \frac{W}{V_T} = \frac{M.g}{V_T}$$

$$So \rightarrow \quad \gamma = \rho \cdot g$$

$$\gamma\ (\text{kN/m}^3) = \frac{g\rho(\text{kg/m}^3)}{1000}$$

where g = acceleration due to gravity = 9.81 m/sec^2.
Note that unit weight of water (γ_w) is equal to 9.81 kN/m^3 or 1000 kgf/m^3.

Specific gravity of soil (Gs)

Specific gravity is defined as the ratio of the unit weight of soil to the unit weight of water.

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{g * \rho_s}{g * \rho_w} = \frac{\rho_s}{\rho_w}$$

$$G_s = \frac{w_s}{V_s\ \gamma_w}\ or\ \frac{M_s}{V_s\ \rho_w}$$

Type of Soil	Gs
Sand	2.65 - 2.67
Silty sand	2.67 – 2.70
Inorganic clay	2.70 – 2.80
Soils with mica or iron	2.75 – 3.00
Organic soils	< 2.00

- The relationship between void ratio and porosity can be derived as follows:

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{\left(\frac{V_v}{V}\right)}{1 - \left(\frac{V_v}{V}\right)} = \frac{n}{1 - n}$$

$$n = \frac{e}{1 + e}$$

- The relationship between void ratio, Degree of saturation, Specific gravity and water content can be derived as follows:

$$w = \frac{w_w}{w_s} = \frac{\gamma_w V_w}{\gamma_s V_s} = \frac{\gamma_w V_w}{\gamma_w G_s V_s} = \frac{V_w}{G_s V_s} = \frac{S * Vv}{G_s V_s} = \frac{S * e}{G_s}$$

$$Se = wG_s$$

- The relationship between void ratio, Degree of saturation, Specific gravity and unit weight can be derived as follows:

$$\gamma = \frac{W_T}{V_T} = \frac{W_w + W_s}{V_s + V_v} = \frac{\gamma_w V_w + \gamma_s V_s}{V_s + V_v} = \frac{\gamma_w S V_v + \gamma_w G_s V_s}{V_s + V_v}$$

$$\gamma = \frac{(Se + G_s)}{1 + e} \gamma_w$$

At dry state (S=0)

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w}{1 + e}$$

At saturated state (S=1)

$$\gamma_{sat} = \frac{W_T(\text{void filled with wate r})}{V_T} = \frac{(G_s + e)\gamma_w}{1+e}$$

- The relationship between porosity, water content, Specific gravity and unit weight can be derived as follows:

$$\gamma = \frac{(Se + G_s)}{1+e}\gamma_w$$

$$\gamma = \frac{G_s(1+w)}{1+e}\gamma_w = \gamma_d(1+w)$$

$$\gamma = \frac{W_T}{V_T} = \text{Gs}\,\gamma_w\,(1\text{-n})(1+\text{w})$$

$$\gamma_d = \frac{W_s}{V_T} = \text{Gs}\,\gamma_w\,(1\text{-n})$$

Table 3.1 Various Forms of Relationships for γ, γ_d, and γ_{sat}

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	**Relationship**	**Given**	**Relationship**	**Given**	**Relationship**
w, G_s, e	$\frac{(1+w)G_s\gamma_w}{1+e}$	γ, w	$\frac{\gamma}{1+w}$	G_s, e	$\frac{(G_s+e)\gamma_w}{1+e}$
S, G_s, e	$\frac{(G_s+Se)\gamma_w}{1+e}$	G_s, e	$\frac{G_s\gamma_w}{1+e}$	G_s, n	$[(1-n)G_s+n]\gamma_w$
w, G_s, S	$\frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1-n)$	G_s, w_{sat}	$\left(\frac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$	G_s, w, S	$\frac{G_s\gamma_w}{1+\left(\frac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1+w_{sat}}{1+e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1-n)+nS\gamma_w$	e, w, S	$\frac{eS\gamma_w}{(1+e)w}$	n, w_{sat}	$n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat} - \frac{e\gamma_w}{1+e}$	γ_d, e	$\gamma_d + \left(\frac{e}{1+e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat} - n\gamma_w$	γ_d, n	$\gamma_d + n\gamma_w$
		γ_{sat}, G_s	$\frac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$	γ_d, G_s	$\left(1-\frac{1}{G_s}\right)\gamma_d + \gamma_w$
				γ_d, w_{sat}	$\gamma_d(1+w_{sat})$