**Relative Density**

is the ratio of the actual density to the maximum possible density of the soil. The term relative density is commonly used to indicate the in situ denseness or looseness of granular soil. It is expressed in terms of void ratio

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

where $D_r$ = relative density, usually given as a percentage
$e$ = *in situ* void ratio of the soil
$e_{max}$ = void ratio of the soil in the loosest state
$e_{min}$ = void ratio of the soil in the densest state

The relationships for relative density can also be defined in terms of porosity

$$e = \frac{n}{1 - n}$$

$$e_{max} = \frac{n_{max}}{1 - n_{max}}$$

$$e_{min} = \frac{n_{min}}{1 - n_{min}}$$

where $n_{max}$ and $n_{min}$ = porosity of the soil in the loosest and densest conditions, respectively.

$$D_r = \frac{(1 - n_{min})(n_{max} - n)}{(n_{max} - n_{min})(1 - n)}$$

By using the definition of dry unit weight, we can express relative density in terms of maximum and minimum dry unit weights. Thus,

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w}{1 + e}$$

$$D_r = \frac{\left[\dfrac{1}{\gamma_{d(min)}}\right] - \left[\dfrac{1}{\gamma_d}\right]}{\left[\dfrac{1}{\gamma_{d(min)}}\right] - \left[\dfrac{1}{\gamma_{d(max)}}\right]} = \left[\frac{\gamma_d - \gamma_{d(min)}}{\gamma_{d(max)} - \gamma_{d(min)}}\right]\left[\frac{\gamma_{d(max)}}{\gamma_d}\right]$$

where $\gamma_{d(min)}$ = dry unit weight in the loosest condition (at a void ratio of $e_{max}$)
$\gamma_d$ = *in situ* dry unit weight (at a void ratio of $e$)
$\gamma_{d(max)}$ = dry unit weight in the densest condition (at a void ratio of $e_{min}$)

By using the definition of density, we can express relative density in terms of density. Thus,

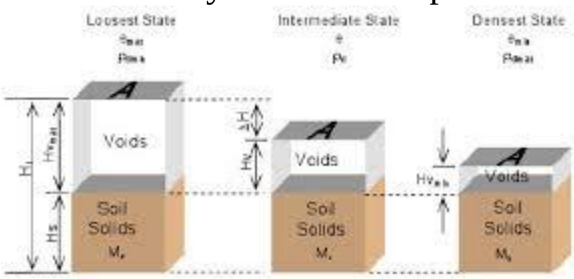
$$D_r = \left[\frac{\rho_d - \rho_{d(\min)}}{\rho_{d(\max)} - \rho_{d(\min)}}\right]\frac{\rho_{d(\max)}}{\rho_d}$$

The values of $D_r$ may vary from a minimum of 0% for very loose soil to a maximum of 100% for very dense soils. Soils engineers qualitatively describe the granular soil deposits according to their relative densities, as shown in Table 3.3. In-place soils seldom have relative densities less than 20 to 30%. Compacting a granular soil to a relative density greater than about 85% is difficult.

**Table 3.3** Qualitative Description of Granular Soil Deposits

| Relative density (%) | Description of soil deposit |
|---|---|
| 0–15 | Very loose |
| 15–50 | Loose |
| 50–70 | Medium |
| 70–85 | Dense |
| 85–100 | Very dense |

The relative density of a soil in its densest possible state ($e = e_{min}$) is 1 (or 100%) and the Relative Density in its loosest possible state ($e = e_{max}$) is 0.




$$\text{volumetric strain, } \frac{\Delta V}{V_o} = \frac{\Delta H}{H_o} = \frac{\Delta e}{1+e_o}$$

Example 1: for a granular soil, given, $\gamma_{dry} = 17.3 \frac{kN}{m^3}$, relative density = 82%, $\omega = 8\%$ and $G_s = 2.65$. If $e_{min} = 0.44$ . what would be $e_{max}$ ? what would be the dry unit weight in the loosest state?

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

$$17.3 = \frac{2.65 * 9.81}{1+e}$$

e=0.53

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} * 100$$

$$0.82 = \frac{e_{max} - 0.53}{e_{max} - 0.44} * 100$$

$$e_{max} = 0.94$$

$$\gamma_{dry}\,(at\ loosest) = \frac{G_s}{1+e_{max}}\gamma_w = \frac{2.65}{1+0.94} * 10$$

$$= 13.65\ kN/m^3$$

Example 2: a granular soil is compacted to moist unit weight of 20.45 $kN/m^3$ at moisture content of 18% . What is relative density of the compacted soil? Given, $e_{max} = 0.85$ , $e_{min} = 0.42$ $and$ $G_s = 2.65$ ?

$$\gamma = \frac{G_s(1+\omega)\gamma_w}{1+e}$$

$$20.45 = \frac{2.65(1+0.18)}{1+e} * 10$$

e=0.52

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} * 100$$

$$= \frac{0.85 - 0.52}{0.85 - 0.42} * 100 = 76.74\%$$

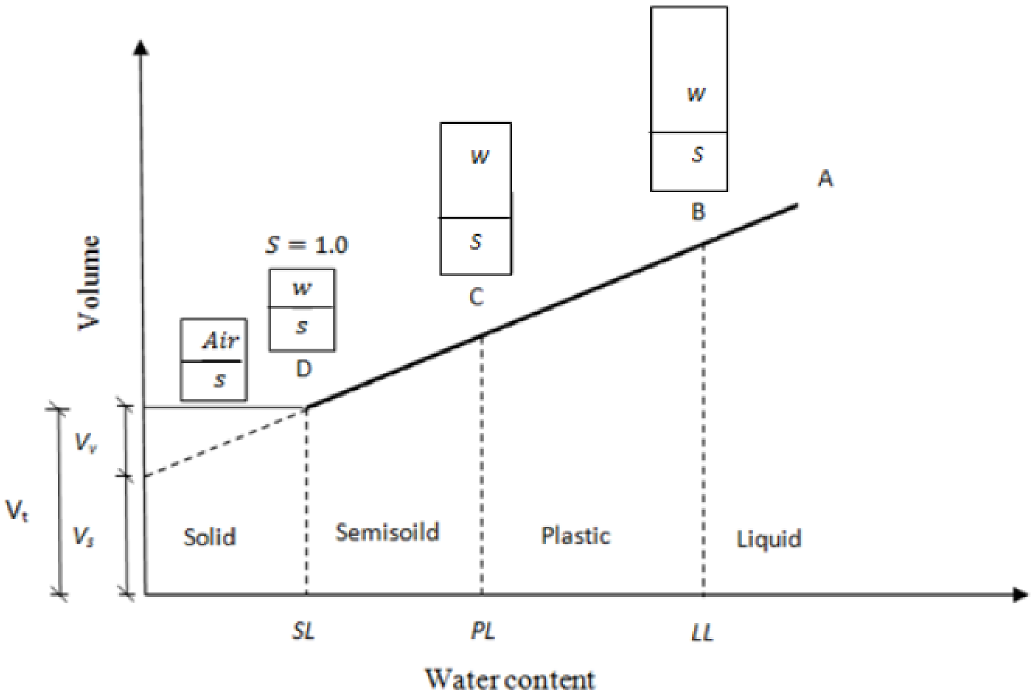
Example 3: A dry sample of soil having the following properties, L.L. = 52%, P.L. = 30%, $G_s$= 2.7, e= 0.53. Find: Shrinkage limit, dry density, dry unit weight.

Dry sample $\Longrightarrow e_{dry} = e_{shrinkage} = 0.53$

$$Se = wG_s$$

$\therefore S.e_{s.l} = G_s\,.\omega_{c_{s.l}} \Longrightarrow$ 1*0.53 =2.7 *S.L

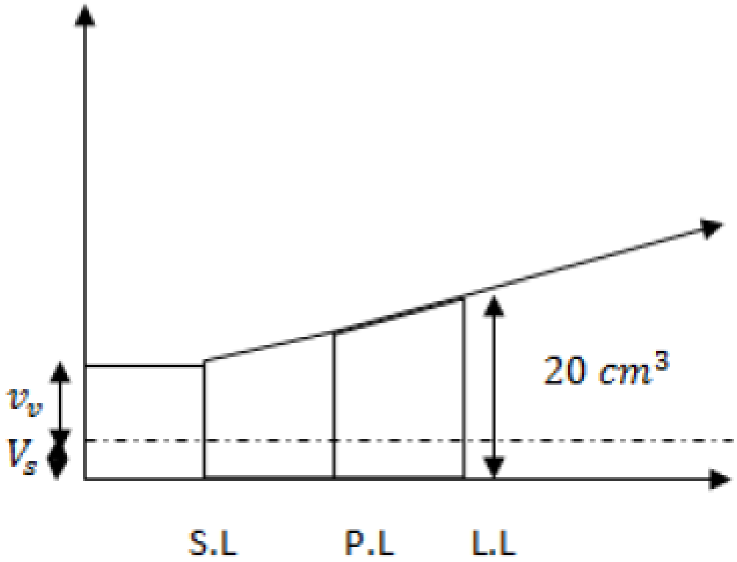
S.L.= 19.6%




$\rho_{dry} = \frac{G_s}{1+e}\,\rho_w \Longrightarrow \rho_{dry} = \frac{2.7}{1+0.53}\,1 = 1.764\ \frac{gm}{cm^3}$

$\therefore\ \gamma_{dry} = \rho_{dry} * g = 1.764 * 10 = 17.64\ kN/m^3$

Example 4: A saturated soil sample has a volume of 20 $cm^3$ at its L.L Given L.L= 42% , P.L.= 30% , S.L.= 17% , $G_s = 2.74$. Find the min. volume the soil can attain.




The minimum volume occurs at S.L. or at dry state.

$$v_t = v_v + v_s$$

$v_s$ is constant along all state.

At L.L.

$$S.e = G_s.\omega_c$$

$$1*e= 2.74 *0.42$$

$$e_{L.L.}= 1.1508$$

$$e = \frac{v_v}{v_s} = \frac{20 - v_s}{v_s} = 1.1508$$

$$\therefore v_s = 9.3\ cm^3$$

At S.L

$$S.e = G_s . \omega_{S.L.}$$

$$1 * e_{S.L.} = 2.74 * 0.17$$

$$e_{S.L.} = 0.4658$$

$$e = \frac{v_v}{v_s} \Longrightarrow$$

$$v_{v\,S.L.} = 0.465 * 9.3 = 4.33\ cm^3$$

$$v_t = v_v + v_s$$

$$= 4.33+9.3=13.63\ cm^3$$

**Example 5**

A sample of Soil was Placed in a Container of diameter 10 cm and height 15 cm. The Soil filled the Container, and the volume of the Soil solids was found to be equal to 520.3 Cm³, Determine the voids ratio and the Porosity of the Soil

Sol: $V_s = 520.3\ Cm^3$ , $D = 10\ cm$ , $H = 15\ cm$

$$V_T = \frac{\pi D^2}{4} H = \frac{\pi (10)^2}{4} * 15 \Rightarrow V_T = 1178.1\ cm^3$$

$$V_v = V_T - V_s = 1178.1 - 520.3 \Rightarrow V_v = 657.8\ cm^3$$

$$e = \frac{V_v}{V_s} = \frac{657.8}{520.3} \Rightarrow e = 1.26$$

$$n = \frac{e}{1+e} = \frac{1.26}{1+1.26} \Rightarrow n = 0.558$$