Experiment -5-

First Order Systems Analysis

Object

study the characteristics of time response of first-order control systems.

Theory

The passive filter can represent the simplest first-order control system

(R-C) circuit is shown in Figure. (5-1).

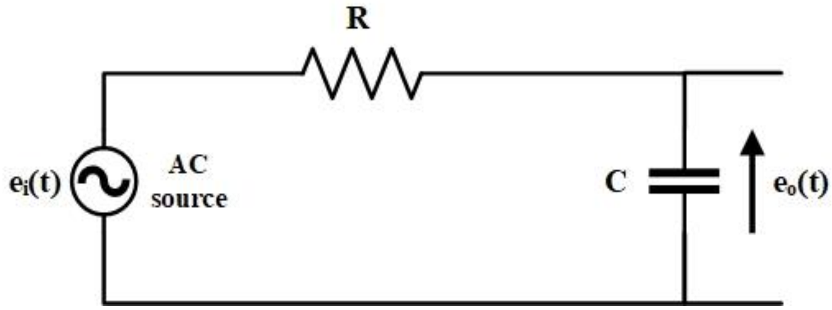

Fig. (5-1): electric (R-C) circuit.

The transfer function can be obtained as follows:

$e_i(t)$ and $e_o(t)$ relation from Kirchhoff's voltage law as

given by

$$e_i(t) = Ri(t) + \frac{1}{C}\int i(t)d(t) \tag{1}$$

$$e_o(t) = \frac{1}{C}\int i(t)d(t) \tag{2}$$

where $i(t)$ is the current passing through R and C.

From the Laplace transform of Eqs. (1) and (2), one can write the transfer function as follows:

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{RCs + 1} \tag{3}$$

For the general system, let $R(s) = E_i(s)$, $C(s) = E_o(s)$, and $RC = T$ where T is the time constant.

The transfer function of the first-order system in terms of T is equal to the following:

$$\frac{C(s)}{R(s)} = \frac{1}{Ts + 1} \tag{4}$$

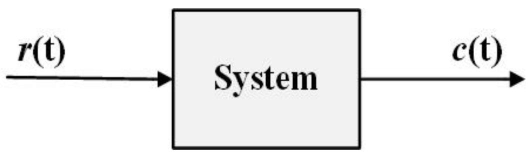

Fig. (5-2): Block diagram of a general control system.

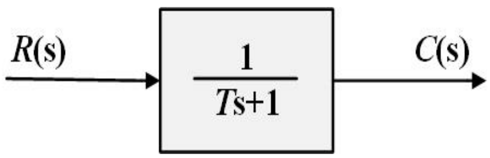

Fig. (5-3): Block diagram of a first-order system.

Unit-step response of the first-order system.

Since the Laplace transform of the unit-step function is $1/s$ and by substituting $R(s) = 1/s$ into Eq. (4), one can obtain

$$C(s) = \frac{1}{s(Ts + 1)} \tag{5}$$

Now, by solving Eq. (5) through the inverse Laplace transform, we get the following:

$$c(t) = 1 - e^{-\frac{t}{T}} \tag{6}$$

Next, $for\ t \geq 0$

Eq. (6) states that initially the output $c(t)$ is zero and finally it becomes to be unity.

One important characteristic of the exponential response curve $c(t)$ is that at $t = T$, the value of $c(t)$ is 0.632, or the response $c(t)$ has reached 63.2% of its total change. This may be easily seen by substituting $t = T$ in $c(t)$, where the following is obtained:

$$c(T) = 1 - e^{-1} = 0.632$$

The smaller the time constant (T), the faster the system response.

The characteristic of the exponential response curve is that the slope of the tangent line at $t = 0$ is $1/T$,

The system response to a unit step input for a first-order control system is represented in Figure. (5-4).

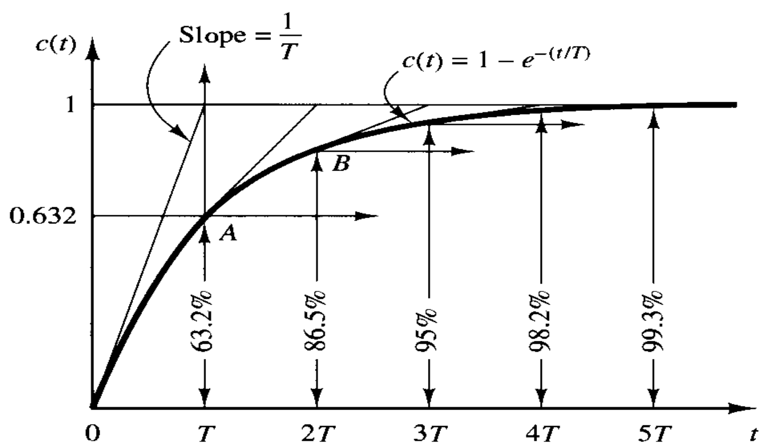

Fig. (5-4) unit step response of the first-order system.

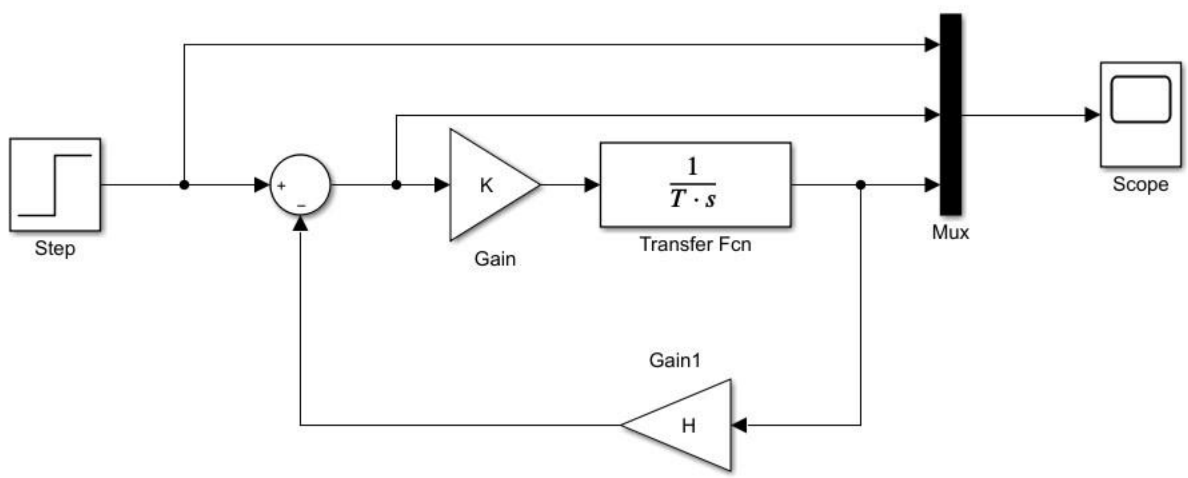

Fig. (5-5): Simulink program for first-order system.

The output response $\boldsymbol{c(t)}$ for step input is depicted in Figure. (5-6) when Figure. (5-5) connected with K=1 and H=1.

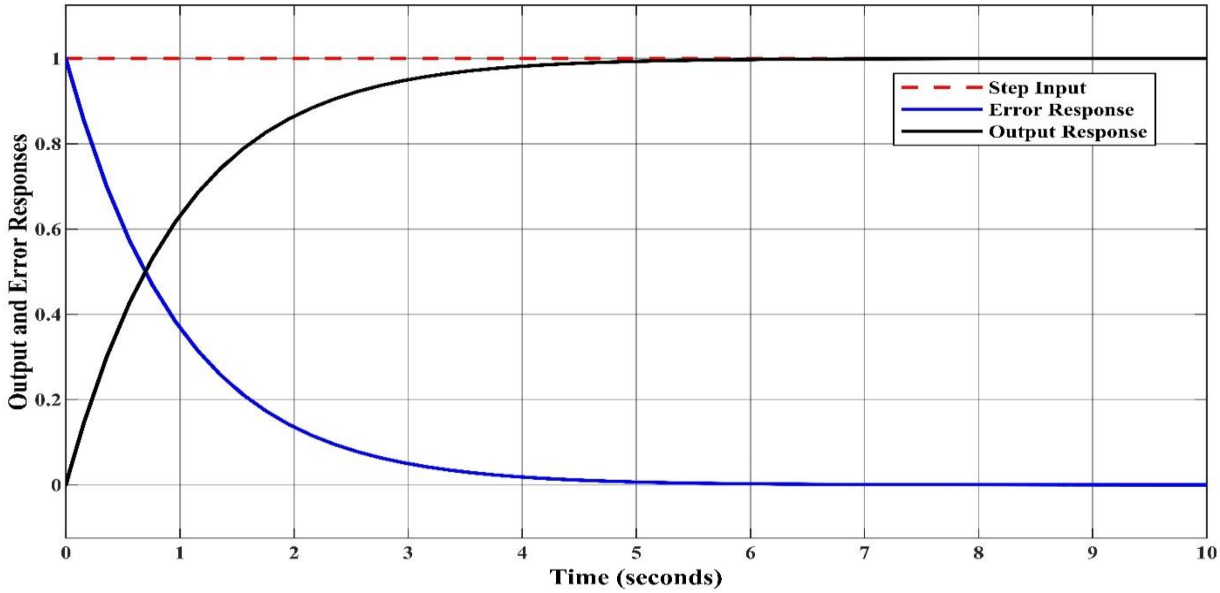

Fig. (5-6): Step and Error responses of Figure. (5-5).

The Error response $\boldsymbol{e(t) = r(t) - c(t)}$ for step input is illustrated in Figure. (5-6).

Procedure

1- Connect the circuit as shown in Figure. (5-5) apply unit step function assuming that the time constant T=1 and the values of k = 0.1, 0.5, 1, 2 then sketch the system response c (t) & error for k=0.1, 0.5, 1, 2.

2-Recored the values of ess & tr for each case as in table (3-1).

Table (5-1) The parameter values for point (1).

K / T	0.1		0.5		1		2	
parameters	ess	tr	ess	tr	ess	tr	ess	tr
1								
2								
3								

Discussion

1- Drive the T.F for the circuit shown in Fig. (5-1).

2- Repeat step 1 in the procedure when we have an impulse, ramp, and sinusoidal inputs.

3- Connect Figure. (5-7) Estimate the value of T to get the output response equal to (1) as shown in the display block, knowing that k=1 and H=1. Draw the output and error responses on the same graph when k=0.5, 1, 1.5, and record the values of ess and tr at each k as shown in Table (5-2).

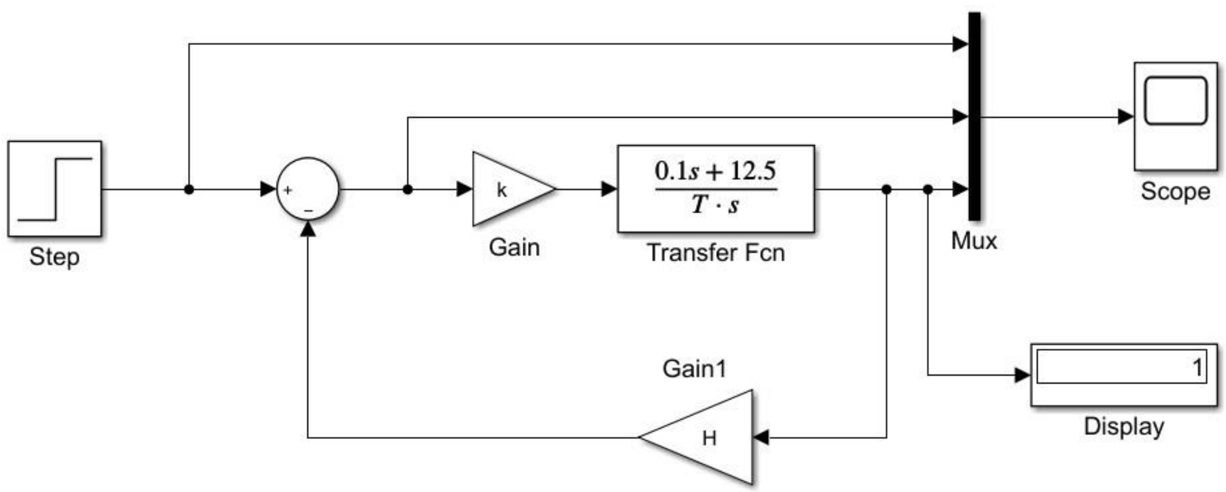

Fig. (5-7): Simulink first-order system.

Table (5-2) The parameter values for point (3) in the discussion.

k / T	0.5		1		1.5	
parameters	ess	tr	ess	tr	ess	tr
?						

Recommended Textbooks:

- K. Ogata “Modern Control Engineering “Prentice-Hall Pub.1997
- Norman S. Nise “CONTROL SYSTEMS ENGINEERING“John Wiley & *Sons*, Inc. Sixth Edition