Experiment -3-

Mathematical Model Response

Object

1. Learning how to derive the transfer function of a linear electric system.
2. Knowing the state and output equations and driving them & dynamic equations.
3. To represent the system response on the personal computer by using Matlab and Simulink with the system's transfer function.

Transfer function representation

Theory

The transfer function of a single input single output dynamic system is defined as:

$$G(s) = \frac{E_o(s)}{E_i(s)} \qquad (1)$$

where

$$E_o(s) = laplace\ transform\ of\ the\ output\ signal\ e_o(t)$$

$$E_i(s) = laplace\ transform\ of\ the\ input\ signal\ e_i(t)$$

The block diagram, which represents equation (1), is shown in Figure (3-1).

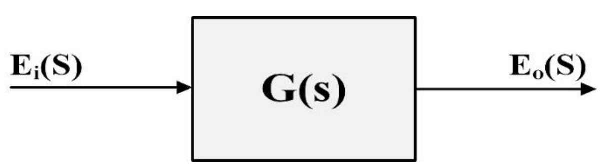


Fig (3-1) Transfer Function block diagram.

Figure (3-2) shows the block diagram for the closed loop control system.

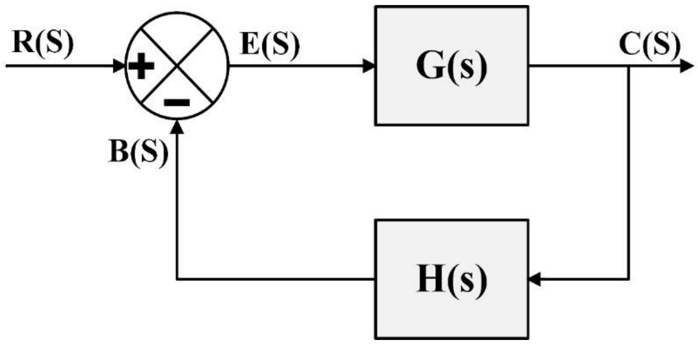


Fig. (3-2) Block diagram of closed-loop control system.

where

G(s) is the process (plant) of the system.

H(s) is the feedback measuring unit (transducer).

If the closed-loop control system output signal $C(s)$ does not follow the changes of input reference $R(s)$, the error signal $E(s)$ is indicated, which may be positive or negative.

$$E(s) = R(s) - B(s) \tag{2}$$

$$B(s) = C(s) * H(s) \tag{3}$$

Substituting for $B(s)$ in (2) from (3) then

$$E(s) = R(s) - C(s) * H(s) \tag{4}$$

here

$$C(s) = E(s)G(s) \tag{5}$$

From (4) and (5), the transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{6}$$

Note: if we have positive feedback, the transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)} \tag{7}$$

We can find the transfer function of any passive electric circuit by applying Kirckoff's laws, for example, to find the transfer function of the (R-L-C) circuit shown in Figure (3-3).

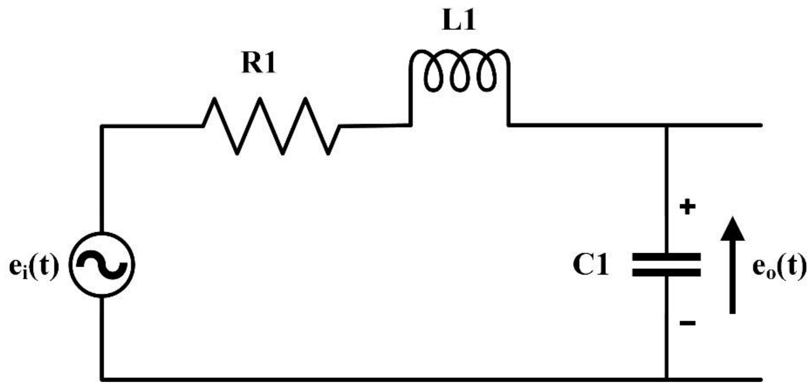


Fig. (3-3): (R-L-C) circuit.

The series connection equation is written as:

$$e_i(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int i\,(t)dt + Ri(t) \tag{8}$$

where $i(t)$ is the current in the loop.

By taking the Laplace transform and assuming all initial conditions are zero, we get:

$$E_i(s) = Ls\,I(s) + \frac{1}{Cs}I(s) + R\,I(s) \tag{9}$$

$$E_o(s) = \frac{1}{Cs}I(s) \tag{10}$$

From (9) and (10), we get

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(\frac{1}{Cs}\right)I(s)}{(LCs^2 + RCs + 1)I(s)} \tag{11}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{(LCs^2 + RCs + 1)} \tag{12}$$

Using Matlab and Simulink, the transfer function can be performed to represent the electric circuit and obtain the system response, as shown in Figure (3-4).

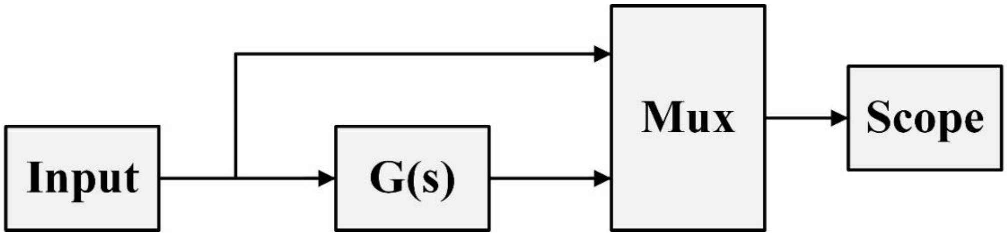
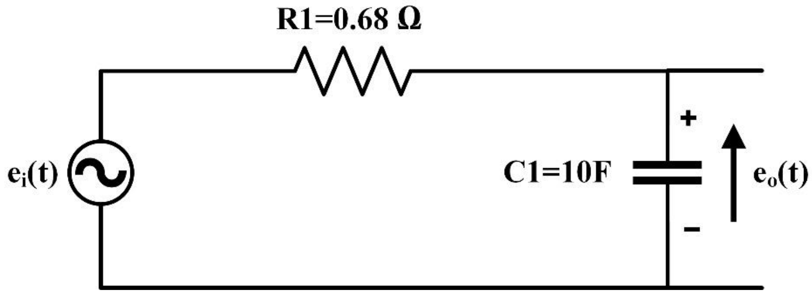


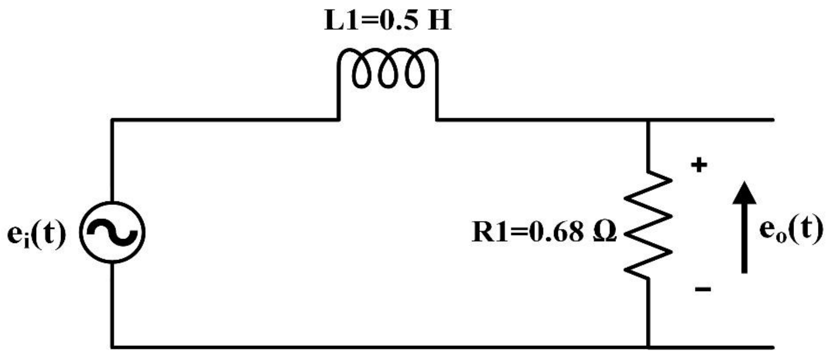
Fig. (3-4): Matlab and Simulink representation.

<u>Electric circuits</u>



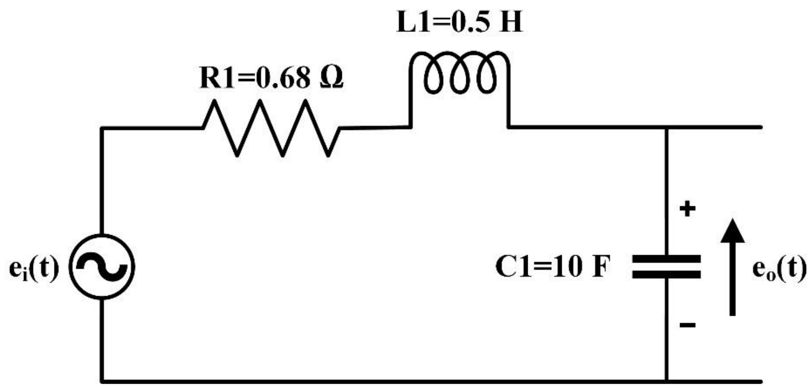
$$G(s) = \frac{1}{R1C1s + 1}$$

Fig. (3-5): (R-C) circuit.



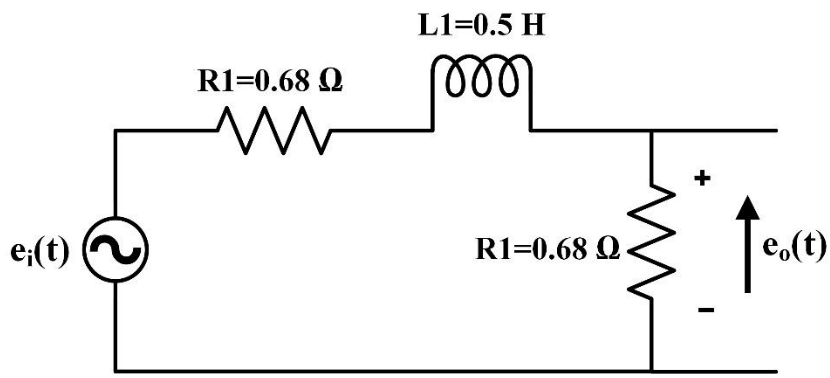
$$G(s) = \frac{1}{(L1/R1)s + 1}$$

Fig. (3-6): (L-R) circuit.



$$G(s) = \frac{1}{L1C1s^2 + R1C1s + 1}$$

Fig. (3-7): (R-L-C) circuit.



$$G(s) = \frac{1}{2((L1/2R1)s + 1)}$$

Fig. (3-8): (R-L-R) circuit.

Procedure

1- From the transfer functions of figures (3-5), (3-6), (3-7), and (3-8), show the output response $e_o(t)$ on a personal computer using Matlab and Simulink.

2- Plot the output response $e_o(t)$.

Discussion

1- Derive the transfer function $G(s)$ for Figure (3-9) below using the same concept used in Figure (3-3).

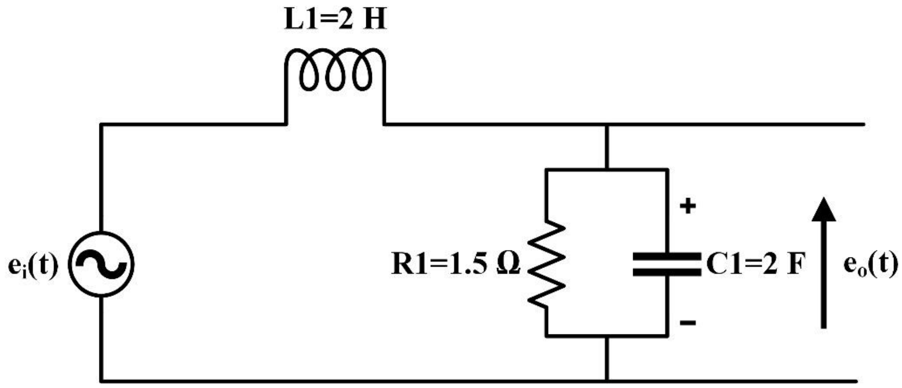


Fig. (3-9): (L-R-C) circuit.

2- For the parallel (L-R-C) circuit shown in Figure (3-9), plot the output response $e_o(t)$ for a unit step input $e_i(t)$.