Centering or mediation measure

The graphical method in analyzing and studying phenomena to determine characteristics, trends and relationships, depends in its accuracy on the accuracy of the graphic representation itself, and thus the characteristics may differ from one drawing to another for the same phenomenon. Therefore, it is better to resort to quantitative measurement methods, where the researcher uses the mathematical method in measurement. Among the most important measures of central tendency that we will discuss in the study are the arithmetic mean, median and mode for each of .the individual data (ungrouped and grouped data)

First: The arithmetic mean (mean)

The arithmetic mean (mean) is defined as the sum of the values of the observations divided by their number and is symbolized by the symbol ($\bar{Y}$).

A.Calculating the arithmetic mean from ungrouped (single) data

The arithmetic mean is calculated from the ungrouped data from the following relationship:

$$\bar{Y} = \frac{\sum_{i=1}^{n} yi}{n}$$

Example :

yi= 400, 380, 450, 350, 520

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$$\bar{Y} = \frac{\sum_{i=1}^{n} yi}{n} = \frac{400 + 380 + 450 + 350 + 520}{5} = \frac{2100}{5} = 420$$

A.Calculating the arithmetic mean from classified data

It is calculated in the most common way by using the class centers and their frequencies according to the following equation:

$$\bar{Y} = \frac{\sum_{i=1}^{n} fiyi}{\sum_{i=1}^{n} fi}$$

When: fi= frequency of class

yi= class center

Example:

Extract the arithmetic mean from the following table.

Class	Frequency (fi)	Class Center (yi)	$fi * yi$
31−40	1	35.5	35.5
41−50	2	45.5	91
51−60	5	55.5	277.5
61−70	15	65.5	982.5
71−80	25	75.5	1887.5
81−90	20	85.5	1710
91−100	12	95.5	1146
	$\sum fi = 80$		$\sum fi * yi = 6130$

$$\bar{Y} = \frac{\sum_{i=1}^{n} fiyi}{\sum_{i=1}^{n} fi} = \frac{6130}{80} = 76.62$$

Properties of the arithmetic mean:

Sum of deviations of values from their arithmetic mean = zero

- For ungrouped data:

$$\sum (yi - \bar{Y}) = 0$$

$$\sum (yi - \bar{Y}) = \sum yi - \sum \bar{Y} = \sum yi - n\bar{Y} = \sum yi - \sum yi = 0$$

For classified data:

$$\sum fi(yi-\bar{Y})=0$$

$$\sum fi(yi-\bar{Y})=\sum fiyi-\sum fi\bar{Y}=\sum fiyi-\frac{\sum fiyi}{\sum fi}\sum fi$$
$$=\sum fiyi-\sum fiyi=0$$

- When adding a fixed number (k) to each value of the variables, its arithmetic mean = the arithmetic mean of the original values + the fixed number.

$$Ai=yi+k\rightarrow\bar{A}=\bar{Y}+k$$

- If the values of a variable are multiplied by a constant k, the average of the resulting values is equal to the product of the constant k and the arithmetic mean of the variable.

$$Ai=yi*k\rightarrow\bar{A}=\bar{Y}*k$$

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Second: Arithmetic mediator

The median is defined as the value that lies in the middle of a set of values such that the frequencies preceding it are equal to (50%) those following it if the set of values is arranged in ascending or descending order, and its calculation depends on the nature and type of data.

For ungrouped data

If the number of items is odd (n odd): there will be one item representing the median

$$\frac{n+1}{2}$$

and its order is calculated from the relation

Example:

Calculate the median from the following data (20 - 12 - 15 - 10 - 40 - 80 - 61)

Sol/

Arrange the items in ascending order first: (10-12-15-20-40-61-80)

Calculate the order of the median = $\frac{(7+1)}{2} = 4$, the order of the median is **fourth**, so the median = **20**.

If the number of items is even (n is even): there will be two items adjacent to the median, which is calculated by finding their arithmetic mean, and their order is calculated from the relationship

$$(\frac{n}{2}, \frac{n}{2}+1)$$

Example:

Calculate the median from the following data

40-33-20-18- 14 - 15 - 12 - 15

Sol/

Arrange the items in ascending order first:

(12-14-15-15-18-20-33-40)

$$(\frac{n}{2}, \frac{n}{2}+1)$$

$$\left(\frac{8}{2}, \frac{8}{2}+1\right) \rightarrow (4, 5)$$

The median is the average of the fourth and fifth values

(15+18)/2=16.5 Me

For classified data

using the ascending cumulative frequency table according to the following relationship:

$$Me = Lm + \left(\frac{\sum f/2 - Fi}{fm}\right) * C$$

When:

Lm: The true lower bound of the median class (i.e. the class that contains the median item of the distribution).

$\sum f/2$: Sum of frequencies divided by 2

Fi: Sum of frequencies of classes preceding the median class

Fm: Frequency of the median class

C: Length of the class

Example:

Calculate the median for the table

Class	Frequency	Ascending cumulative repetition
≤31	0	0
31−40	1	1
41−50	2	3
51−60	5	8
61−70	15	23
71−80	25	48
81−90	20	68
91−100	12	80
	$\sum fi = 80$	

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$$Me = Lm + \left(\frac{\sum f/2 - Fi}{fm}\right) * C$$

$$\sum \frac{f}{2} = \frac{80}{2} = 40$$

Class is (71-80)

$$Me = 71 + \left(\frac{40 - 23}{25}\right) * 10 = 77.8$$

Example:

Extract the arithmetic median from the following table.

class	Frequency
34-40	10
41-47	8
48-54	12
55-61	12
62-68	10

Sol/

Class Interval	Frequency (fm)	Cumulative Frequency
≤ 34	0	0
34 - 40	10	10
41 - 47	8	18
48 - 54	12	30
55 - 61	12	42
62 - 68	10	52

$$Me = Lm + \left(\frac{\sum f/2 - Fi}{fm}\right) * C$$

$Lm = 48,\ f = 52,\ Fi = 18,\ fm = 12,\ C = 7$

$$\text{Median} = 48 + \left(\frac{26 - 18}{12}\right) \times 7$$

$$= 48 + \left(\frac{8}{12}\right) \times 7$$

$$= 48 + (0.6667 \times 7)$$

$$= 48 + 4.67$$

$$\approx 52.67$$

H.W:

Extract the arithmetic median from the following table.

class	Frequency
50-55	15
56-61	10
62-67	14
68-73	8
74-79	3

Third: Mode

Mode is the most frequently occurring value in the data set.

For ungrouped data

Example:

Find the Mode (12-8-10-8-9-8-7)

Sol/

Mode = 8

For classified data

$$Mo = Lm + \left(\frac{D1}{D2 + D1}\right) * C$$

Lm: The true lower bound of the median class (i.e., the class that contains the median item of the distribution).

D1: The difference between the repetition of the modal class and the class before it

D2: The difference between the frequency of the modal class and the class that follows it

C: Length of the class

Example:

Find the Mode

Class	Frequency (fi)
31−40	1
41−50	2
51−60	5
61−70	15
71−80	25
81−90	20
91−100	12

Sol/

$$Mo = Lm + \left(\frac{D1}{D2 + D1}\right) * C$$

$$Mo = 71 + \left(\frac{25 - 15}{(25 - 20) + (25 - 15)}\right) * 10 = 77.666$$

Example:

Extract the mode from the following table.

Class	Frequency
0.51-0.59	5
0.60-0.68	4
0.69-0.77	15
0.78-0.86	10
0.87-0.95	14

Sol/

$$Mo = Lm + \left(\frac{D1}{D1 + D2}\right) * C$$

$$Mo = 0.69 + \left(\frac{11}{11+5}\right) \times 0.09$$

$$= 0.69 + \left(\frac{11}{16}\right) \times 0.09$$

$$= 0.69 + (0.6875 \times 0.09)$$

$$= 0.69 + 0.0619$$

$$\approx 0.75$$

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H.W/

Extract the mode from the following table.

Class	Frequency
0.51-0.57	10
0.58-0.64	8
0.65-0.71	10
0.72-0.78	12
0.79-0.85	28
0.86-0.92	16
0.93-0.99	9