



Coeff. Of Var. (C. V.)

If S and \bar{y} are the standard deviation and the arithmetic mean of a set of values, respectively, then their coefficient of variation is

$$C.V. = \frac{S}{\bar{y}} * 100$$

Example:

The results of the final exams for two subjects in the second grade were as shown in the following table. Determine in which of the two subjects was the most distracting?

	Statistics	mathematics	
	73	78	arithmetic mean
	7.6	8	Standard deviation

Mathematics

$$C.V. = \frac{S}{\bar{y}} * 100 = \frac{8}{78} * 100 = 10.25\%$$

Statistics

$$C.V. = \frac{S}{\bar{y}} * 100 = \frac{7.6}{73} * 100 = 10.41\%$$



I noticed that the dispersion in the statistics scores was more than mathematics, but if we compare through the standard deviation, we find that the adherence to mathematics scores is more than statistics.

Standardized Scores

When comparing two items from two different groups, they must be converted into standard units in order for the comparison to have a standard reference and real significance. This is done by converting to the standard score, which is calculated according to the relationship:

$$Z_i = \frac{y_i - \bar{y}}{S}$$

Example:

A student got a score of (84) in mathematics, where the average student score is (76) and a standard deviation of (10), and in statistics (90), where the average student score is (82) and a standard deviation of (16). In which of the two subjects was this student's ability higher?

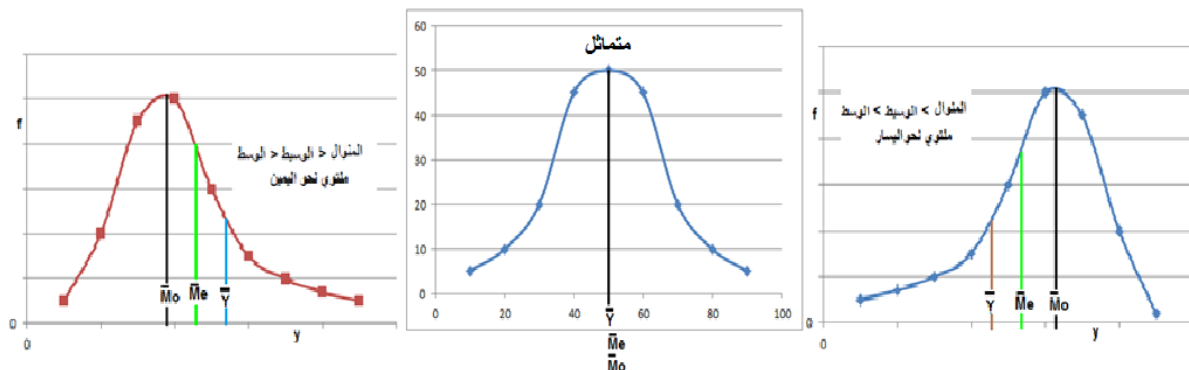
$$Z_1 = \frac{84 - 76}{10} = 0.8$$

$$Z_2 = \frac{90 - 82}{16} = 0.5$$

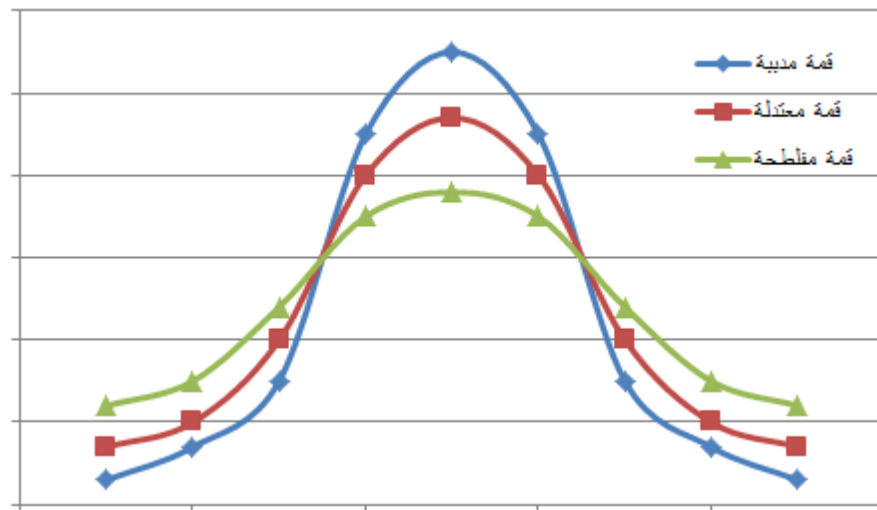
It is clear that the student's ability in mathematics is higher than in statistics.

Skewness & Kurtosis Measures

The measure of skewness: is the deviation of the distribution curve from symmetry (positive, negative, moderate skewness), and this measure is specific to unimodal distributions. As shown in the figure below.



The measure of Kurtosis: It is a degree at the top of the distribution. The higher the peak, the more flat the distribution is. The lower the height, the more flat the distribution is, as shown in the following figure.



To calculate these measures, the theory of moments can be used in its various degrees, including:

1- The moment around zero

a) For ungrouped data

$$\bar{y}^r = \frac{\sum yi^r}{n}$$

The first moment around zero is the arithmetic mean

$$\bar{y}^1 = \frac{\sum yi^1}{n}$$

As for the second moment around zero, it is



$$\bar{y}^2 = \frac{\sum yi^2}{n}$$

b) For grouped data

$$\bar{y}^r = \frac{\sum f i y_i^r}{\sum f i}$$

2- Momentum about the arithmetic mean

a) For ungrouped data

$$M_r = \frac{\sum (y_i - \bar{y})^r}{n}$$

If (r=1 so $M_r=0$)

If (r=2)

$$M_r = \frac{\sum (y_i - \bar{y})^2}{n} = \frac{SS}{n} = \sigma^2$$

b) For classified data

$$M_r = \frac{\sum f i (y_i - \bar{y})^r}{\sum f i}$$



Example : Find the first, second and fourth moments of the data

A- About zero

B-B- About the arithmetic mean

$y_i = 4, 7, 5, 9, 8, 3, 6$

sol/

a- About zero $\bar{y}^r = \frac{\sum y_i^r}{n}$

First moments:

$$\bar{y} = \frac{\sum y_i}{n} = \frac{4 + 7 + 5 + 9 + 8 + 3 + 6}{7} = \frac{42}{7} = 6$$

Second moments:

$$\bar{y}^2 = \frac{\sum y_i^2}{n} = \frac{16 + 49 + 25 + 81 + 64 + 9 + 36}{7} = \frac{280}{7} = 40$$

Fourth moments:

$$\begin{aligned}\bar{y}^4 &= \frac{\sum y_i^4}{n} = \frac{256 + 2401 + 625 + 6661 + 4096 + 81 + 1296}{7} = \frac{15316}{7} \\ &= 2188\end{aligned}$$

b- About the arithmetic mean ($\bar{y} = 6$)

First moments:



$$M_1 = \frac{\Sigma(yi - \bar{y})^1}{n} = 0$$

Second moments:

$$M_2 = \frac{\Sigma(yi - \bar{y})^2}{n}$$

$$\begin{aligned} M_2 &= \frac{(4 - 6)^2 + (7 - 6)^2 + (5 - 6)^2 + (9 - 6)^2 + (8 - 6)^2 + (3 - 6)^2 + (6 - 6)^2}{7} \\ &= \frac{28}{7} = 4 \end{aligned}$$

Fourth moments:

$$M_4 = \frac{\Sigma(yi - \bar{y})^4}{n}$$

$$\begin{aligned} M_4 &= \frac{(4 - 6)^4 + (7 - 6)^4 + (5 - 6)^4 + (9 - 6)^4 + (8 - 6)^4 + (3 - 6)^4 + (6 - 6)^4}{7} \\ &= \frac{196}{7} = 28 \end{aligned}$$