Department of Biomedical Engineering
Electric circuit 1 / First stage
Lecturer: Dr. Hussam Jawad Kadhim AL_Janabi
hussam.jawad@uomus.edu.iq

Lecture Four

Methods of analysis

4.1 Nodal Analysis with Voltage Sources

Nodal Analysis with Voltage Sources is a powerful technique used in electrical circuit analysis to determine the voltage at various nodes in a circuit.

There are two cases to determine the node voltage:

CASE 1 If a voltage source is connected between the reference node and a non-reference node, we set the voltage at the nonreference node equal to the voltage of the voltage source as shown in the figure below:

$v1 = 10$ V

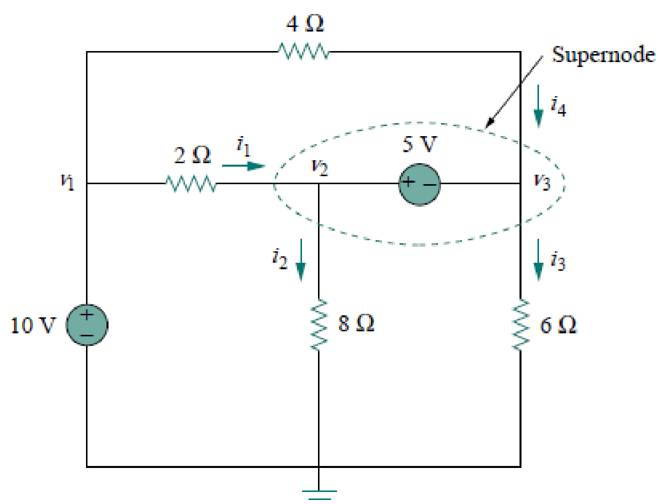


Figure 4.1

CASE 2 If the voltage source is connected between two nonreference nodes, the two nonreference nodes form a *supernode*; we apply both KCL and KVL to determine the node voltages.

Lecturer: Dr. Hussam Jawad Kadhim AL_Janabi
hussam.jawad@uomus.edu.iq

A supernode is formed by enclosing a voltage source connected between two nonreference nodes and any elements connected in parallel with it.

In Fig. 4.1, nodes 2 and 3 form a supernode (two nodes forming a single supernode).

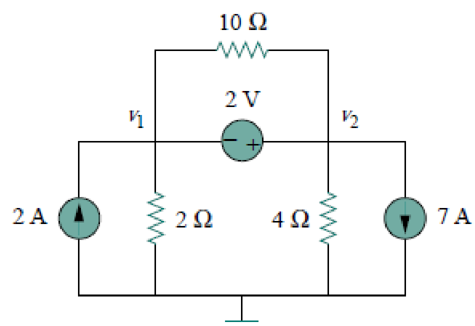
Hence, at the supernode $i_1 + i_4 = i_2 + i_3$ or $\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$

Example: For the circuit shown in the figure below, find the node voltages

Solution

The supernode contains the 2-V source, nodes 1 and 2, and the 10 Ω resistor.

Applying KCL to the supernode.

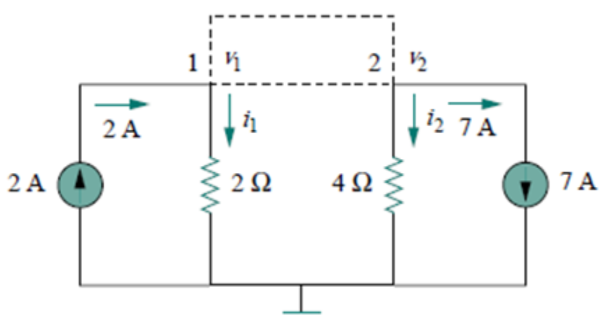


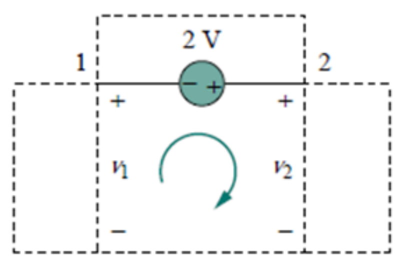
$$2 = i_1 + i_2 + 7$$

Expressing i_1 and i_2 in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Rightarrow 8 = 2v_1 + v_2 + 28$$

$$v_2 = -20 - 2v_1 \qquad \text{Eq(1)}$$





(b)

To get the relationship between v_1 and v_2, we apply KVL to the circuit, we obtain

$-v_1 - 2 + v_2 = 0 \quad \Rightarrow \quad v_2 = v_1 + 2$ Eq(2)

From Eqs. (1) and (2), we write

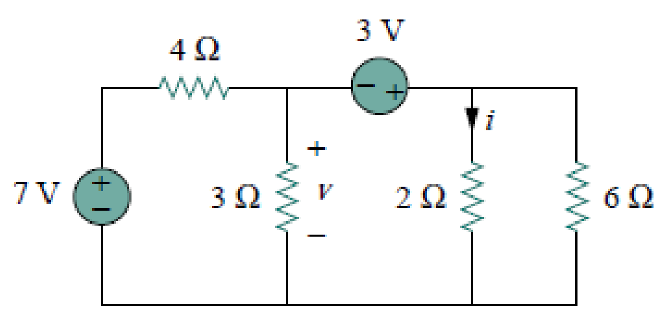
$v_2 = v_1 + 2 = -20 - 2v_1$

$\Rightarrow \quad 3v_1 = -22 \quad \Rightarrow v_1 = -7.333$ V

and $v_2 = v_1 + 2 = -5.333$ V.

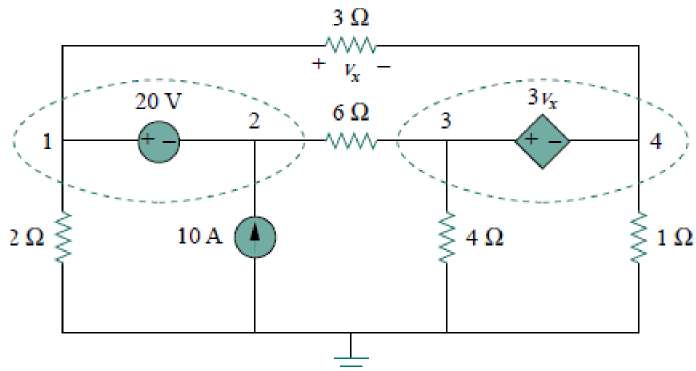
PRACTICE PROBLEM

Find v and i in the circuit in the figure below



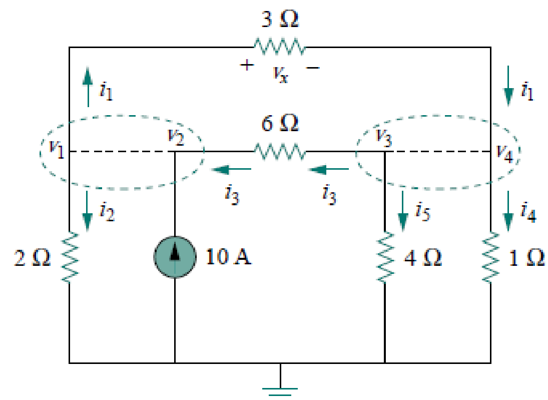
Answer: −0.2 V, 1.4 A.

Example (2): For the circuit shown in the figure below, find *v* and *i*.



Solution:

Nodes 1 and 2 form a supernode; so do nodes 3 and 4. We apply KCL to the two supernodes



At supernode 1-2

$i_3 + 10 = i_1 + i_2$

Expressing this in terms of the node voltages,

$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$ Multiply by 6,

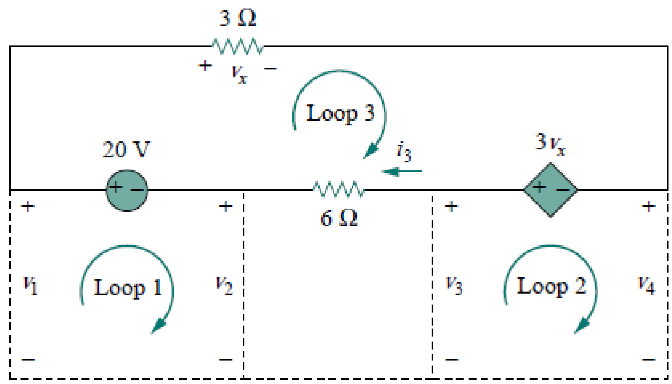
$\Rightarrow \quad 5v_1 + v2 - v_3 - 2v_4 = 60$ Eq(1)

At supernode 3-4

$i_1 = i_3 + i_4 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$ Multiply by 12

$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$ Eq(2)

We now apply KVL to the branches involving the voltage sources as shown in the figure below.



For loop 1,

$-v_1 + 20 + v2 = 0 \quad \Rightarrow v_1 - v2 = 20$ Eq(3)

For loop 2,

$-v_3 + 3v_x + v_4 = 0$

But $v_x = v1 - v4$ so that $-v_3 + 3(v1 - v4) + v_4 = 0 \;\Rightarrow\; 3v_1 - v_3 - 2v_4 = 0$ Eq(4)

For loop 3,

$v_x - 3v_x + 6i3 - 20 = 0$

But $6i_3 = v_3 - v_2$ and $v_x = v_1 - v_4$. Hence $-2v_1 - v_2 + v_3 + 2v_4 = 20$ Eq(5)

We need four node voltages, v_1, v_2, v_3, and v_4, and it requires only four out of the five Eqs. (1) to (5) to find them. Although the fifth equation is redundant, it can be used to check results. We can eliminate one node voltage so that we solve three simultaneous equations instead of four. From Eq. (3), $v2 = v1 - 20$. Substituting this into Eqs. (1) and (2), respectively, gives

$6v_1 - v_3 - 2v_4 = 80$ Eq(6)

And $6v_1 - 5v_3 - 16v_4 = 40$ Eq(7)

Equations (4), (6), and (7) can be cast in matrix form as

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

Using Cramer's rule,

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18, \qquad \Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{vmatrix} = -480$$

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{vmatrix} = -3120, \qquad \Delta_4 = \begin{vmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{vmatrix} = 840$$

Thus, we arrive at the node voltages as

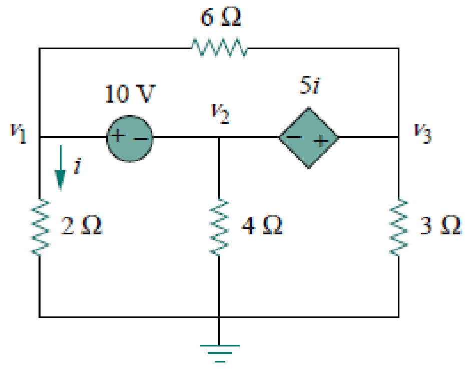
$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.667 \text{ V}, \qquad v_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.333 \text{ V}$$

$$v_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.667 \text{ V}$$

and $v_2 = v_1 - 20 = 6.667$ V

P R A C T I C E P R O B L E M

Find v_1, v_2, and v_3 in the following circuit using nodal analysis



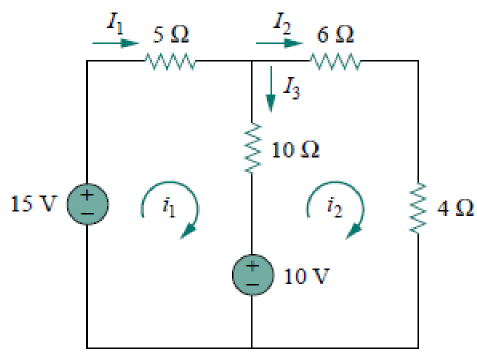
Answer: $v1 = 3.043$ V, $v2 = -6.956$ V, $v3 = 0.6522$ V.

Lecturer: Dr. Hussam Jawad Kadhim AL_Janabi
hussam.jawad@uomus.edu.iq

4.2 MESH ANALYSIS

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Recall that a loop is a closed path with no node passed more than once. A mesh is a loop that does not contain any other loop within it. Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents

Example (1): For the circuit in the figure below, find the branch currents I_1, I_2, and I_3 using mesh analysis



Solution

For mesh 1

$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0 \quad \Rightarrow \quad 3i_1 - 2i_2 = 1$ Eq(1)

For mesh 2,

$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0 \quad \Rightarrow \quad i_1 = 2i_2 - 1$ Eq(2)

METHOD 1 Using the substitution method

$6i_2 - 3 - 2i_2 = 1 \Rightarrow i_2 = 1$ A

$i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$I_1 = i_1 = 1$ A, $I_2 = i_2 = 1$ A, $I_3 = i_1 - i_2 = 0$

***METHOD 2** Use Cramer's rule*

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

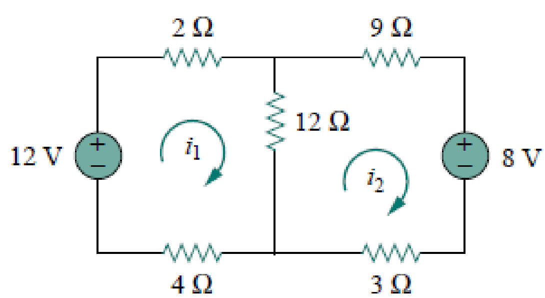
Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

as before.

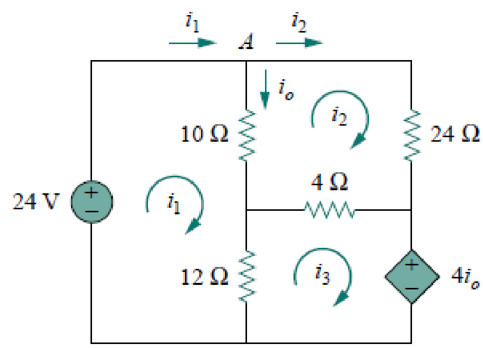
PRACTICE PROBLEM

Calculate the mesh currents i_1 and i_2 in the following circuit.



Answer: $i_1 = 23$ A, $i_2 = 0$ A.

Example (2): *Use mesh analysis to find the current i_o in the following circuit*



Solution:

We apply KVL to the three meshes in turn

For mesh 1

$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$

$\Rightarrow \quad 11i_1 - 5i_2 - 6i_3 = 12$ Eq(1)

For mesh 2,

$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$

$\Rightarrow -5i_1 + 19i_2 - 2i_3 = 0$ Eq(2)

For mesh 3,

$4i_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$

But at node A, $i_o = i_1 - i_2$, so that $4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$

$-i_1 - i_2 + 2i_3 = 0$ Eq(3)

In matrix form, Eqs. (1) to (3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \\ 11 & -5 & -6 \\ -5 & 19 & -2 \end{vmatrix}$$

= 418 - 30 - 10 - 114 - 22 - 50 = 192

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{vmatrix} \qquad \Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{vmatrix} \qquad \Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{vmatrix}$$

= 456 − 24 = 432 = 24 + 120 = 144 = 60 + 228 = 288

We calculate the mesh currents using Cramer's rule as

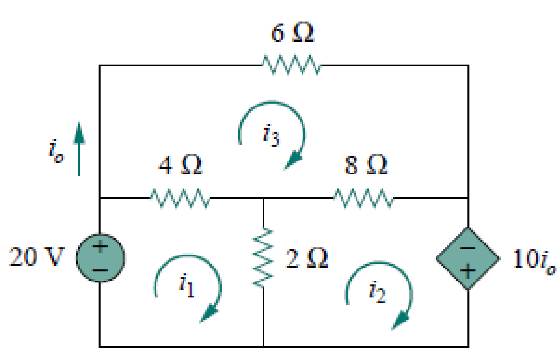
$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $i_o = i_1 - i_2 = 1.5$ A.

P R A C T I C E P R O B L E M

Using mesh analysis, find i_o in the following circuit.



Answer: -5 A.