جامعة المستقبل
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Department of Cyber Security

Subject:

Types Of Relations

Class:

First

Lecturer:

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Lecture: (5)

Properties of binary relations (Types of relations)

Let R be a relation on the set A

1)Reflexive :

R is said to be *reflexive* if ordered couple $(x, x) \in R$ for $\forall x \in X$.

$\forall$ a $\in$A $\rightarrow$ aRa or (a,a) $\in$ R ; $\forall$ a, b $\in$A. .

Thus R is not reflexive if there exists a $\in$ A such that

(a, a) $\notin$ R.

Example i:

Consider the following five relations on the set $A = \{1, 2, 3, 4\}$:

$R1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$

$R2 = \{(1, 1)(1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

$R3 = \{(1, 3), (2, 1)\}$

$R4 = \emptyset$, the empty relation

$R5 = A \times A$, the universal relation

Determine which of the relations are reflexive.

Since A contains the four elements 1, 2, 3, and 4,

a relation R on A is reflexive if it contains the four pairs

$(1, 1), (2, 2), (3, 3)$, and $(4, 4)$.

Thus only $R2$ and the universal relation $R5 = A \times A$ are reflexive.

Note that $R1$,$R3$, and $R4$ are not reflexive since, for example, $(2, 2)$ does not belong to any of them.

Example ii

Consider the following five relations:

(1) Relation $\leq$ (less than or equal) on the set **Z** of integers.

(2) Set inclusion $\subseteq$ on a collection C of sets.

(3) Relation $\perp$ (perpendicular) on the set L of lines in the plane.

(4) Relation $\parallel$ (parallel) on the set L of lines in the plane.

Determine which of the relations are reflexive.

The relation (3) is not reflexive since no line is perpendicular to itself.

Also (4) is not reflexive since no line is parallel to itself.

The other relations are reflexive; that is,

$x \leq x$ for every $x \in \mathbf{Z}$,

$A \subseteq A$ for any set $A \in C$

2) **Symmetric** :

R is said to be *symmetric* if, ordered couple $(x, y) \in$ R and also ordered couple $(y, x) \in$ R for $\forall x$, $\forall y \in$ X.

aRb $\rightarrow$ bRa $\forall$ a,b $\in$A. [if whenever (a, b) $\in$ R then (b, a) $\in$ R.]

Thus R is not symmetric if there exists a, b $\in$ A such that (a, b) $\in$ R but (b, a) $\notin$R.

Example

(a) Determine which of the relations in Example i are symmetric

$R1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$

$R2 = \{(1, 1)(1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

$R3 = \{(1, 3), (2, 1)\}$

R4 = ∅, *the empty relation*

R5 = *A* × *A*, *the universal relation*

R1 is not symmetric since $(1, 2) \in R1$ but $(2, 1) \notin R1$.

R3 is not symmetric since $(1, 3) \in R3$ but $(3, 1) \notin R3$.

The other relations are symmetric.

(b) Determine which of the relations in Example ii are symmetric.

(1) Relation ≤ (less than or equal) on the set **Z** of integers.

(2) Set inclusion ⊆ on a collection *C* of sets.

(3) Relation ⊥ (perpendicular) on the set *L* of lines in the plane.

(4) Relation ∥ (parallel) on the set *L* of lines in the plane.

The relation ⊥ is symmetric since if line *a* is perpendicular to line *b* then *b* is perpendicular to *a*.

Also, ∥ is symmetric since if line *a* is parallel to line *b* then *b* is parallel to line *a*.

The other relations are not symmetric. For example:

$3 \leq 4$ but 4 not≤ 3; $\{1, 2\} \subseteq \{1, 2, 3\}$ but $\{1, 2, 3\}$ not $\subseteq \{1, 2\}$.

3) **Transitive** :

R is said to be *transitive* if ordered couple $(x, z) \in R$ whenever

both ordered couples $(x, y) \in R$ and $(y, z) \in R$.

$aRb \wedge bRc \rightarrow aRc$. that is, if whenever $(a, b), (b, c) \in R$

then $(a, c) \in R$.

Thus R is not transitive if there exist $a, b, c \in R$ such that

$(a, b), (b, c) \in R$ but $(a, c) \notin R$.

Example

(a) Determine which of the relations in example i are transitive.

$R1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$

$R2 = \{(1, 1)(1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

$R3 = \{(1, 3), (2, 1)\}$

$R4 = \emptyset$, *the empty relation*

$R5 = A \times A$, *the universal relation*

The relation R3 is not transitive since $(2, 1), (1, 3) \in R3$ but $(2,3) \notin R3$. All the other relations are transitive.

(b) Determine which of the relations in example ii are transitive.

(1) Relation $\leq$ (less than or equal) on the set Z of integers.

(2) Set inclusion $\subseteq$ on a collection C of sets.

(3) Relation $\perp$ (perpendicular) on the set L of lines in the plane.

(4) Relation $\parallel$ (parallel) on the set L of lines in the plane.

The relations ≤, ⊆, and || are transitive, but certainly not ⊥.

Also, since no line is parallel to itself, we can have a ∥ b and b ∥ a, but a ∥ a. Thus ∥ is not transitive.

Relation	Reflexive	Symmetric	Transitive
≤	✅ Yes	❌ No	✅ Yes
⊆	✅ Yes	❌ No	✅ Yes
||	✅ (in most definitions)	✅ Yes	✅ Yes
⊥	❌ No	✅ Yes	❌ No