

Class: first

Subject: integral Mathematics/Code: UOMU024024

Lecturer: M.Sc. Alaa Khalid Lecture name: Method of integration

Lecture: 7 2ndterm

Integration By Substitution

The goal of this method is to transform the integral into a standard from

To evaluate the integral $I = \int f[g(x)] g'(x) dx$ carry out the following steps

- **1- substitute** u = g(x) the du = g'(x)dx to obtain $I = \int f(u)du$
- **2- Evaluate** $I = \int f(u)du$ by integrating w.r.t u
- **3- Replace** u by g(x) in the final result

Evaluate
$$I = \int \frac{dx}{\sqrt[3]{1-2x}}$$

Solution:
$$I = \int (1-2x)^{-\frac{1}{3}} dx$$
 Let $u = 1-2x \Rightarrow du = -2dx \Rightarrow dx = \frac{du}{-2}$

$$I = \int (1 - 2x)^{-\frac{1}{3}} dx \implies I = \int u^{-\frac{1}{3}} \frac{du}{-2} = \frac{-1}{2} \int u^{-\frac{1}{3}} du = \frac{-1}{2} \frac{u^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{-3}{4} (1 - 2x)^{\frac{2}{3}} + c$$

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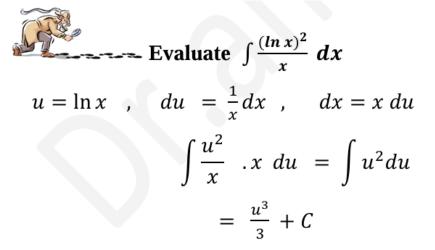
Evaluate
$$\int \frac{-e^{x^{-1}}}{x^2} dx$$

$$=-\int x^{-2} e^{x^{-1}} dx$$

$$u = x^{-1}$$
 , $du = -1.x^{-2} dx$

$$= -\int x^{-2} e^{u} \cdot \frac{du}{-x^{-2}}$$

$$= \int e^{u} du = e^{u} + c = e^{x^{-1}} + C$$



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Evaluate
$$\int \sin(4x) dx$$

let
$$u = 4x \rightarrow \frac{du}{dx} = 4$$
 $du = 4 dx$ $dx = \frac{du}{4}$

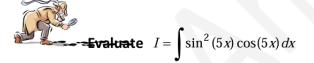
$$du = 4 dx$$

$$dx = \frac{du}{4}$$

$$\int \sin(u) \cdot \frac{du}{4} = \frac{1}{4} \int \sin(u) du$$

$$= \frac{1}{4} \left(-\cos u \right) + c$$

$$= -\frac{1}{4} \cos(4x) + c$$
3



Let $u = \sin(5x) \implies du = 5\cos(5x) dx \implies dx = \frac{du}{5\cos(5x)}$

$$I = \int \sin^2(5x)\cos(5x) dx \Rightarrow I = \int u^2\cos(5x) \frac{du}{5\cos(5x)} = \frac{1}{5} \int u^2 du = \frac{1}{5} \frac{u^3}{3} + c = \frac{1}{15} [\sin(5x)]^3 + c$$



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$$u = \mathbf{1} + \mathbf{e}^{\mathbf{x}} , \quad du = e^{x} dx , \quad dx = \frac{du}{e^{x}}$$

$$\int_{0}^{1} (\mathbf{1} + \mathbf{e}^{x})^{2} \mathbf{e}^{x} dx = \int_{0}^{1} (\mathbf{u})^{2} \mathbf{e}^{x} . \quad \frac{du}{e^{x}}$$

$$\int_0^1 (\mathbf{u})^2 \cdot du = \left[\frac{u^3}{3} \right]_0^1 = \frac{1}{3} [\mathbf{u}^3]_0^1$$
$$= \frac{1}{3} [\mathbf{1} + \mathbf{e}^x]_0^1 = \frac{1}{3} [\mathbf{1} + \mathbf{e}^1] - [\mathbf{1} + \mathbf{e}^0]$$
$$= \dots$$

Good Luck ..

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