Al-Mustaqbal University / College of Engineering & Technology
Class: first
Subject: integral Mathematics/Code: UOMU024024
Lecturer: M.Sc. Alaa Khalid
Lecture name: Method of integration
Lecture: 7
2nd term

Integration By Substitution

The goal of this method is to transform the integral into a standard from

To evaluate the integral $I = \int f[g(x)]\, g'(x)dx$ **carry out the following steps**

1- substitute $u = g(x)$ **the** $du = g'(x)dx$ **to obtain** $I = \int f(u)du$

2- Evaluate $I = \int f(u)du$ **by integrating w.r.t** u

3- Replace u **by** $g(x)$ **in the final result**

Evaluate $I = \int \frac{dx}{\sqrt[3]{1-2x}}$

Solution :- $I = \int (1-2x)^{-\frac{1}{3}} dx$ **Let** $u = 1-2x \Rightarrow du = -2dx \Rightarrow dx = \frac{du}{-2}$

$$I = \int (1-2x)^{-\frac{1}{3}} dx \Rightarrow I = \int u^{-\frac{1}{3}} \frac{du}{-2} = \frac{-1}{2}\int u^{-\frac{1}{3}} du = \frac{-1}{2}\frac{u^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{-3}{4}(1-2x)^{\frac{2}{3}} + c$$

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Evaluate $\int \frac{-\,e^{x^{-1}}}{x^2}\,dx$

$$= -\int x^{-2}\, e^{x^{-1}}\,dx$$

$$u = x^{-1} \quad , \quad du = -1.\,x^{-2}\,dx$$

$$= -\int x^{-2}\, e^{u} \,.\, \frac{du}{-\,x^{-2}}$$

$$= \int e^{u}\,du = e^{u} + c = e^{x^{-1}} + C$$

Evaluate $\int \frac{(ln\,x)^2}{x}\,dx$

$$u = \ln x \quad , \quad du = \frac{1}{x}dx \quad , \quad dx = x\,du$$

$$\int \frac{u^2}{x}\,.\,x\;du = \int u^2 du$$

$$= \frac{u^3}{3} + C$$

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Evaluate $\int \sin(4x)dx$

let u = 4x $\rightarrow \frac{du}{dx} = 4 \qquad du = 4\,dx \qquad dx = \frac{du}{4}$

$$\int \sin(u)\,.\,\frac{du}{4} = \frac{1}{4}\int \sin(u)du$$

$$= \frac{1}{4}(-\cos u) + c$$

$$= -\frac{1}{4}\cos(4x) + c$$

Evaluate $I = \int \sin^2(5x)\cos(5x)\,dx$

Solution :- **Let** $u = \sin(5x) \Rightarrow du = 5\cos(5x)\,dx \Rightarrow dx = \frac{du}{5\cos(5x)}$

$$I = \int \sin^2(5x)\cos(5x)\,dx \Rightarrow I = \int u^2 \cos(5x)\frac{du}{5\cos(5x)} = \frac{1}{5}\int u^2 du = \frac{1}{5}\frac{u^3}{3} + c = \frac{1}{15}[\sin(5x)]^3 + c$$

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Evaluate $\int_0^1 (1+e^x)^2 e^x \, dx$

$$u = 1 + e^x \;,\quad du = e^x dx \,,\quad dx = \frac{du}{e^x}$$

$$\int_0^1 (1+e^x)^2 e^x \, dx = \int_0^1 (u)^2 e^x \cdot \frac{du}{e^x}$$

$$\int_0^1 (u)^2 .\; du = \left[\frac{u^3}{3}\right]_0^1 = \frac{1}{3}[u^3]_0^1$$

$$= \frac{1}{3}[1+e^x]_0^1 = \frac{1}{3}\;[1+e^1] - [1+e^0]$$

$$= \ldots$$

Good Luck ..

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