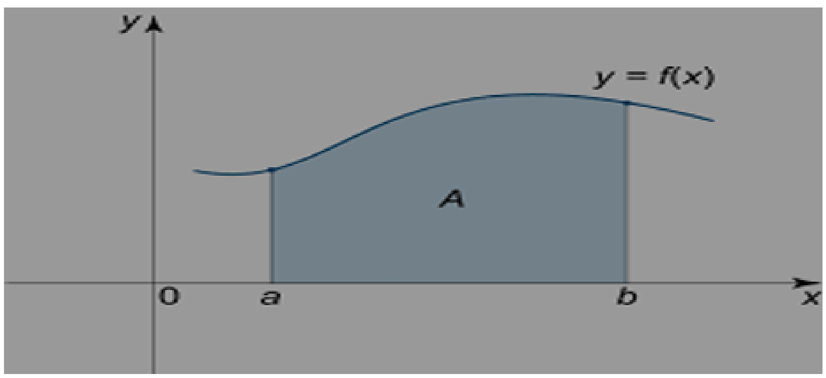
Al-Mustaqbal University / College of Engineering & Technology
Class: first
Subject: integral Mathematics/Code: UOMU024024
Lecturer: M.Sc. Alaa Khalid
Lecture name: introduction of integration
Lecture: 9
2^{nd}term

Integration application/area under the curve

1.The area under the curve

Define: Let F be continuing function over the closed value [a, b], then the area under the curve define:



$$A=\int_a^b f(x)\,dx \quad with\ x-axis$$

Or $$A=\int_a^b f(y)\,dy \quad with\ y-axis$$

Example 1: Find the area under the curve bounded by the curve $y=\sqrt{x}$ $and\ 0 \le x \le 1\ with\ the\ x-axis$

Solution //

$$A=\int_a^b f(x)\,dx \quad \rightarrow A=\int_0^1 \sqrt{x}\,dx = \frac{x^{\frac{3}{2}}}{3/2}\Bigg|_0^1 = \frac{2}{3}unit^2$$

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Example 1: Find the area bounded by the curve $y = x - x^2$. with x − axis .

Solution /

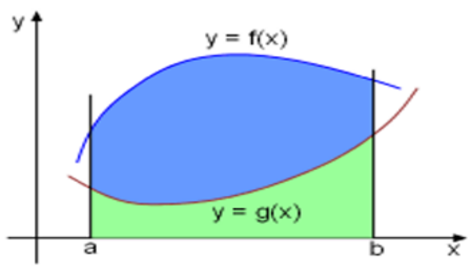
$$y = x - x^2 .\text{ with } x - \text{axis} \rightarrow 0 = x - x^2 \rightarrow x(1 - x^2) = 0$$

x=0 & x=1

$$A = \int_0^1 (x - x^2)\, dx = \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 = \frac{1}{6} unit^2$$

2. The area between two curve

Define: Let F1&F2 are two functions over the closed value [a,b],then between two curves define as follows:-



$$A=\int_a^b |f1(x) - f2(x)|\, dx \quad with\ x - axis$$

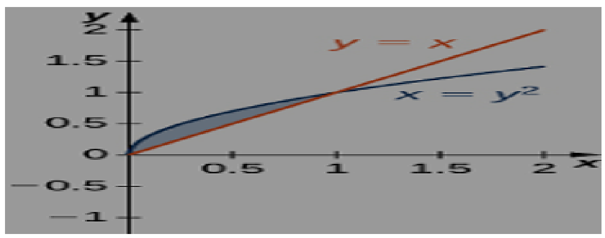
Or $$A=\int_a^b |f1(y) - f2(y)| dx \quad with\ y - axis$$

$$Area = \int_a^b upper\ curve - lower\ curve\ dx$$

Example 1: Find the area of region bonded by the curves

$$y=\sqrt{x} \quad \& \quad y = x$$

Solution //



$$\sqrt{x} = x \rightarrow x - x^2 = 0 \rightarrow x(1-x) = 0$$

$$x=0 \ \& \ x=1$$

$$A = \int_0^1 (\sqrt{x} - x)\, dx = \frac{x^{\frac{3}{2}}}{3/2} - \frac{x^2}{2}\Bigg|_0^1 = \frac{1}{6}\, unit^2$$

Find the area enclosed between $y = x^2 - 3x + 2$ and $y = 2x - 2$

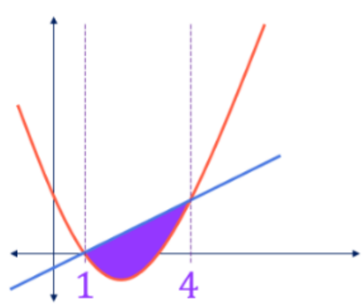
1. Find coordinates of intersection

$$x^2 - 3x + 2 = 2x - 2$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$x = 4 \quad x = 1$$



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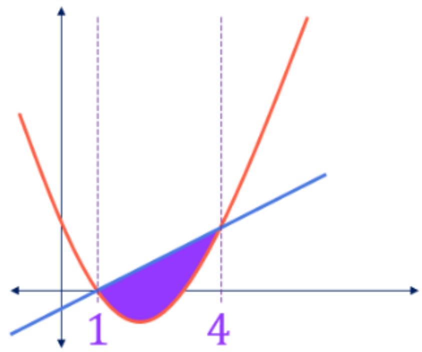
2. Integrate using these points as limits

$$\int_a^b \text{upper curve} - \text{lower curve}\; dx$$

$$\int_1^4 (2x-2)-(x^2-3x+2)\,dx$$

$$\int_1^4 2x-2-x^2+3x-2\;dx$$

$$\int_1^4 -x^2+5x-4\;dx$$



$$\int_1^4 -x^2+5x-4\;dx$$

$$\left[-\frac{x^3}{3}+\frac{5x^2}{2}-4x\right]_1^4$$

$$\left[-\frac{(4)^3}{3}+\frac{5(4)^2}{2}-4(4)\right]-\left[-\frac{(1)^3}{3}+\frac{5(1)^2}{2}-4(1)\right]$$

$$2\frac{2}{3} - -1\frac{5}{6}$$

$$2\frac{2}{3} + 1\frac{5}{6} = 4.5 \text{ units}^2$$

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