Al-Mustaqbal University / College of Engineering & Technology
Class: first
Subject: integral Mathematics/Code: UOMU024024
Lecturer: M.Sc. Alaa Khalid

Lecture name: method of integration

Lecture: 8

2^{nd}term

Evaluate $I = \int \frac{\sin^2(2x)}{1+\cos(2x)}\,dx$

Solution :- $I = \int \frac{\sin^2(2x)}{1+\cos(2x)}\,dx = \int \frac{1-\cos^2(2x)}{1+\cos(2x)}\,dx = \int \frac{(1-\cos 2x)(1+\cos 2x)}{1+\cos(2x)}$

$$\Rightarrow \int [1-\cos(2x)]\,dx = x - \frac{1}{2}\sin(2x) + c$$

Evaluate $\int (sinx + cosx)^2$

$$\int (\sin x + cox\ x)^2 = \int \left(\sin^2 x + 2 sinxcos\ x + cos^2 x\right)dx$$

$$= \int \left(\sin^2 x + cox^2 x\right) + 2 \sin x\ cosx\ )dx$$

$$= \int (1 + 2sinx\ cosx\ )dx$$

$$= \int 1 + 2 \int sinx\ \cos x\ dx$$

$$= x + 2 \int \sin x\ cosx\ dx$$

Email : alaa.khalid.abdalreda@uomus.edu.iq

$$u = sinx \qquad du = \cos x dx \qquad dx = \frac{du}{cox}$$

$$= \; x + 2\int u \;\; cosx \;.\frac{du}{cosx}$$

$$= x + 2\int u \; du$$

$$= x + 2\,.\frac{u^2}{2} + c$$

$$= x + u^2 + c \qquad = x \; + sin^2 x + c$$

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}} dx$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x)^{\frac{-1}{2}} \cos x \; dx$$

$$u = sin\, x \quad , \; du = cosx\, dx \;\; , \;\; dx = \frac{du}{cos\, x}$$

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$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (u)^{\frac{-1}{2}} \cos x \,.\, \frac{du}{\cos x} = \left[\frac{(u)^{\frac{1}{2}}}{\frac{1}{2}}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left[2\sqrt{\sin x}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 2\sqrt{\sin\frac{\pi}{2}} - 2\sqrt{\sin\frac{\pi}{4}} = 2\sqrt{1} - 2\sqrt{\frac{1}{2}}$$

$$= 2 - \sqrt{2}\cdot\sqrt{2}\cdot\frac{1}{\sqrt{2}} = 2 - \sqrt{2}$$

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Evaluate $\int \sqrt[3]{27e^{9x} + e^{12x}}\ \mathbf{dx}$

$$\int (27e^{9x} + e^{12x})^{\frac{1}{3}}\mathbf{dx} = \int (27e^{9x} + e^{3x+9x})^{\frac{1}{3}}\mathbf{dx}$$

$$= \int (27e^{9x} + e^{3x}\, e^{9x})^{\frac{1}{3}}\ \mathbf{dx}$$

$$= \int((e^{9x})\,(27 + e^{3x}\,))^{\frac{1}{3}}\ dx$$

$$= \int(e^{9x})^{\frac{1}{3}}\,(\ 27 + e^{3x}\,)^{\frac{1}{3}}\ dx$$

$$= \int e^{3x}\,(\ 27 + e^{3x}\,)^{\frac{1}{3}}\, dx$$

$$u = 27 + e^{3x} \qquad , \quad du = 3e^{3x}dx \qquad , \quad dx = \frac{du}{3e^{3x}}$$

$$\int e^{3x}\,(\ 27 + e^{3x}\,)^{\frac{1}{3}}\, dx \quad = e^{3x}\ u^{\frac{1}{3}}\ \cdot \frac{du}{3e^{3x}}$$

$$= \frac{1}{3} \int u^{\frac{1}{3}}\ du$$

$$= \frac{1}{3}\ \frac{u^{\frac{4}{3}}}{4/3} \ + \ C$$

$$= \frac{1}{3} \cdot \frac{3}{4}\ \ 27 + e^{3x}\,)^{\frac{4}{3}}$$

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Evaluate $\int_0^{\sqrt{3}} x\, e^{\left(\frac{x^2}{3}\right)}$

$$u = \frac{x^2}{3} \quad , \quad \frac{du}{dx} = \frac{1}{3} \cdot 2x$$

$$du = \frac{2}{3}\, x\, dx \quad , \quad dx = \frac{3}{2}\, x\, du$$

$$\int_0^{\sqrt{3}} x\, e^u \cdot \frac{3}{2}\, x\, du = \frac{3}{2} \int_0^{\sqrt{3}} e^u\, du$$

$$= \frac{3}{2}\, [e^u]_0^{\sqrt{3}} = \frac{3}{2} \left[e^{\frac{x^2}{3}}\right]_0^{\sqrt{3}}$$

$$= \frac{3}{2} \left[e^{\frac{(\sqrt{3})^2}{3}} - e^{\frac{0^2}{3}}\right] = \frac{3}{2}\, [e^1 - e^0]$$

$$= \frac{3}{2}\, [e - 1] = \approx 2.577 \ldots$$

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