



③ Derivative of logarithm Functions: مشتقة الدوال اللوغاريتمية

$$1 - \frac{d}{dx} \log_a u = \frac{u'}{u \ln a}$$

$$2 - \frac{d}{dx} \ln u = \frac{u'}{u}$$

Examples:

① Show that $\frac{d}{dx} \ln x = \frac{1}{x}$?

Solution:

By using the fact

$$g'(x) = \frac{1}{f'(g(x))}$$

$$\text{Let } f(x) = e^x \Rightarrow f'(x) = e^x$$

$$\& \quad g(x) = \ln x$$

$$\text{Then, } \frac{d}{dx} \ln x = \frac{1}{e^{\ln x}} = \boxed{\frac{1}{x}} \quad \underline{\text{Ans}}$$

Note e^x & $\ln x$ are inverses of each other.
الـ e^x و $\ln x$ هما معكوسات لبعضهما البعض



② show that $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$?

Solution

From the previous course,

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\therefore \frac{d}{dx} \log_a x = \frac{d}{dx} \left(\frac{\ln x}{\ln a} \right)$$

$$= \frac{1}{\ln a} \times \frac{d}{dx} \ln x$$

$$= \frac{1}{\ln a} \times \frac{1}{x} = \boxed{\frac{1}{x \ln a}} \quad \underline{\text{Ans}}$$

$$\textcircled{3} \frac{d}{dx} (\log_2 x) = \boxed{\frac{1}{x \ln 2}} \quad \underline{\text{Ans}}$$

$$\textcircled{4} \frac{d}{dx} (\log_2 x^2) = \frac{1}{x^2 \ln 2} \times 2x = \boxed{\frac{2}{x \ln 2}} \quad \underline{\text{Ans}}$$

$$\textcircled{5} \frac{d}{dx} (\log_7 (5-2x)) = \frac{1}{(5-2x) \ln 7} \times (-2) = \boxed{\frac{-2}{(5-2x) \ln 7}} \quad \underline{\text{Ans}}$$

$$\textcircled{6} \frac{d}{dx} (\ln x^2) = \frac{1}{x^2} \times 2x = \boxed{\frac{2}{x}} \quad \underline{\text{Ans}}$$

$$\textcircled{7} \frac{d}{dx} (\ln (x^2+5)) = \frac{1}{x^2+5} \times 2x = \boxed{\frac{2x}{x^2+5}} \quad \underline{\text{Ans}}$$



$$\textcircled{B} \frac{d}{dx} (\ln(\sin x)) = \frac{1}{\sin x} \times \cos x = \frac{\cos x}{\sin x} \\ = \boxed{\cot x}$$

$$\textcircled{A} \frac{d}{dx} (\ln(\ln x)) = \frac{1}{\ln x} \times \frac{1}{x} = \boxed{\frac{1}{x \ln x}}$$

Questions For Discussing ٢٠٢٠/١١/٢١

1- $\frac{d}{dx} (\log_5 (\tan x))$

2- $\frac{d}{dx} (\sqrt[3]{\ln x})$

3- $\frac{d}{dx} (\log_3 (\ln(\sin^2 x)))$

4- $\frac{d}{dx} (\ln(\sin(\log_5(\sec x))))$