① Derivatives of Exponential Functions

مشتقات الدوال الاسية

1- $\frac{d}{dx}(a^u) = a^u \ln a \times u'$, a = constant

2- $\frac{d}{dx}(e^u) = e^u \times u'$

Examples :-

① Show that $\frac{d}{dx} a^x = a^x \ln a$

Solution

$\because a^x = e^{\ln a^x} = e^{x \ln a}$

$\therefore \frac{d}{dx} a^x = \frac{d}{dx} e^{x \ln a}$

$= e^{x \ln a} \times \ln a$

$= e^{\ln a^x} \times \ln a$

$\therefore \frac{d}{dx} a^x = \boxed{a^x \times \ln a}$ Ans

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② Show that $\frac{d}{dx} e^x = e^x$?

Solution

By using the derivative by definition

let $f(x) = e^x$ —— ①

$f(x+\Delta x) = e^{x+\Delta x} = e^x \cdot e^{\Delta x}$ —— ②

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \text{—— ③}$$

substitute ① & ② into ③, yields,

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{e^x \cdot e^{\Delta x} - e^x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{e^x (e^{\Delta x} - 1)}{\Delta x}$$

$$= e^x \lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x}$$

one of the definition of "e" is

$$\lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x} = 1$$

$$\therefore \frac{df}{dx} = \frac{d}{dx} e^x = \boxed{e^x} \quad \text{Ans}$$

③ Derive $R(\omega) = 4^{\omega} - 5\log_9 \omega$

Solution

$$\frac{dR}{d\omega} = \boxed{4^{\omega}\ln 4 - \frac{5}{\omega \ln 9}} \quad \text{Ans}$$

④ Derive $f(x) = 3e^{x} + 10x^{3}\ln x$

Solution

$$\frac{df}{dx} = f' = 3e^{x} + 10x^{3} * \frac{1}{x} + \ln x * 30x^{2}$$

$$= 3e^{x} + \frac{10x^{\cancel{3}\,2}}{\cancel{x}} + 30x^{2}\ln x$$

$$= \boxed{3e^{x} + 10x^{2} + 30x^{2}\ln x} \quad \text{Ans}$$

⑤ Derive $y = \dfrac{5e^{x}}{3e^{x}+1}$

Solution

$$\frac{dy}{dx} = y' = \frac{(3e^{x}+1)\cdot 5e^{x} - 5e^{x}\cdot 3e^{x}}{(3e^{x}+1)^{2}}$$

$$= \frac{\cancel{15e^{2x}} + 5e^{x} \cancel{-15e^{2x}}}{(3e^{x}+1)^{2}} = \boxed{\frac{5e^{x}}{(3e^{x}+1)^{2}}} \quad \text{Ans}$$

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⑥ $\frac{d}{dx}\left(e^{(5x+3)}\right) = e^{5x+3} * 5 = \boxed{5e^{5x+3}}$

⑦ $\frac{d}{dx}\left(e^{x^3}\right) = e^{x^3} * 3x^2 = \boxed{3x^2 e^{x^3}}$

⑧ $\frac{d}{dx}\left(3^{x^2}\right) = 3^{x^2} * 2x * \ln 3 = \boxed{2x\, 3^{x^2} \ln 3}$

⑨ $\frac{d}{dx}\left(7^{(2x+5)}\right) = 7^{2x+5} * 2 * \ln 7 = \boxed{2 * 7^{2x+5} \ln 7}$

⑩ $\frac{d}{dx}\left(9^{\tan x}\right) = \boxed{9^{\tan x} * \sec^2 x * \ln 9}$

⑪ $\frac{d}{dx}\left(x^4 e^{4x}\right) = x^4 * e^{4x} * 4 + e^{4x} * 4x^3$

$= \boxed{4x^3 e^{4x}(x+1)}$

⑫ $\frac{d}{dx}\left[\frac{e^x + e^{-x}}{e^x - e^{-x}}\right]$

$$= \frac{(e^x - e^{-x})(e^x - e^{-x}) - (e^x + e^{-x})(e^x + e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2}$$

$$= \boxed{1 - \frac{(e^x + e^{-x})^2}{(e^x - e^{-x})^2}}$$ Ans

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(5) Derivatives of Hyperbolic Functions

مشتقات الدوال الزائدية

1- $\frac{d}{dx}\sinh u = \cosh u * u'$

2- $\frac{d}{dx}\cosh u = \sinh u * u'$

3- $\frac{d}{dx}\tanh u = \operatorname{sech}^2 u * u'$

4- $\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \tanh u * u'$

5- $\frac{d}{dx}\operatorname{cosech} u = -\operatorname{cosech} u \coth u * u'$

6 $\frac{d}{dx}\coth u = -\operatorname{cosech}^2 u * u'$

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Examples :-

① Find the derivative of

$y = 4 \sinh 2x - \frac{3}{7} \cosh 3x$

Solution

$$\frac{dy}{dx} = y' = 4(\cosh 2x * 2) - \frac{3}{7}(\sinh 3x * 3)$$

$$= \boxed{8 \cosh 2x - \frac{9}{7} \sinh 3x} \quad \text{Ans}$$

② Derive $y = 5 \tanh \frac{x}{2} - 2 \coth 4x$

Solution

$$\frac{dy}{dx} = y' = 5\left(\operatorname{sech}^2 \frac{x}{2} * \frac{1}{2}\right) - 2\left(-\operatorname{cosech}^2 4x * 4\right)$$

$$= \boxed{\frac{5}{2} \operatorname{sech}^2 \frac{x}{2} + 8 \operatorname{cosech}^2 4x} \quad \text{Ans}$$

③ Show that $\frac{d}{dx} \sinh x = \cosh x$

Solution

From past course $\sinh x = \frac{e^{x} - e^{-x}}{2}$

$$\therefore \frac{d}{dx} \sinh x = \frac{1}{2}\left(e^{x} - e^{-x} * (-1)\right) = \boxed{\frac{e^{x} + e^{-x}}{2}}$$

$$= \boxed{\cosh x} \quad \text{Ans}$$

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4) Show that $\frac{d}{dx}\cosh x = \sinh x$

Solution

From the previous course

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$\therefore \frac{d}{dx}\cosh x = \frac{1}{2}\left(e^{x} + e^{-x} * (-1)\right) = \boxed{\frac{e^{x} - e^{-x}}{2}}$$

$$= \boxed{\sinh x} \quad \underline{Ans}$$

5) Show that $\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$

Solution

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\frac{d}{dx}\tanh x = \frac{\cosh x * \cosh x - \sinh x * \sinh x}{\cosh^2 x}$$

$$= \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \boxed{\frac{1}{\cosh^2 x}}$$

$$= \boxed{\operatorname{sech}^2 x} \quad \underline{Ans}$$

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(6) Differentiate $f(x) = 2x^5 \cosh x$

Solution

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$$\frac{df}{dx} = f'(x) = 2x^5 \sinh x + \cosh x * 10x^4$$

$$= \boxed{2x^5 \sinh x + 10x^4 \cosh x} \quad \text{Ans}$$

(7) Derive $h(t) = \dfrac{\sinh t}{t+1}$

Solution

مشتق قسمة دالتين

$$\frac{dh}{dt} = h'(t) = \boxed{\frac{(t+1)\cosh t - \sinh t}{(t+1)^2}} \quad \text{Ans}$$

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⑥ Derivatives Inverse Hyperbolic Fns

مشتقات معكوس الدوال الزائدية

1- $$\frac{d}{dx}\sinh^{-1}u = \frac{u'}{\sqrt{u^2+1}}$$

2- $$\frac{d}{dx}\cosh^{-1}u = \frac{u'}{\sqrt{u^2-1}}$$

3- $$\frac{d}{dx}\tanh^{-1}u = \frac{u'}{1-u^2}$$

4- $$\frac{d}{dx}\operatorname{cosech}^{-1}u = \frac{-u'}{|u|\sqrt{1+u^2}}$$

5- $$\frac{d}{dx}\operatorname{sech}^{-1}u = \frac{-u'}{u\sqrt{1-u^2}}$$

6- $$\frac{d}{dx}\coth^{-1}u = \frac{u'}{1-u^2}$$

Examples:

① Find the derivative of $y = \sinh^{-1}(4x)$

solution

$$\frac{dy}{dx} = y' = \frac{4}{\sqrt{(4x)^2+1}} = \boxed{\frac{4}{\sqrt{16x^2+1}}} \quad \underline{Ans}$$

② Derive $y = \cosh^{-1}(x^3)$

solution

$$\frac{dy}{dx} = y' = \frac{3x^2}{\sqrt{(x^3)^2-1}} = \boxed{\frac{3x^2}{\sqrt{x^6-1}}} \quad \underline{Ans}$$

③ Derive $y = \tanh^{-1}(\sqrt{x})$

Solution

$$\frac{dy}{dx} = y' = \frac{\frac{1}{2\sqrt{x}}}{1-(\sqrt{x})^2} = \frac{\frac{1}{2\sqrt{x}}}{1-x} = \boxed{\frac{1}{2\sqrt{x}(1-x)}} \quad \underline{Ans}$$

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④ Derive $y = (\operatorname{cosech}^{-1}(x))^3$

Solution

$$\frac{dy}{dx} = y' = 3(\operatorname{cosech}^{-1}(x))^2 * \frac{-1}{|x|\sqrt{1+x^2}}$$

$$= \boxed{\frac{-3(\operatorname{cosech}^{-1}(x))^2}{|x|\sqrt{1+x^2}}} \quad \text{Ans}$$

⑤ Derive $y = 6x \sinh^{-1}(3x) - 2\sqrt{1+9x^2}$

Solution

$$\frac{dy}{dx} = y' = 6x * \frac{3}{\sqrt{(3x)^2+1}} + \sinh^{-1}(3x) * 6 - \left[\cancel{2} * \frac{1}{\cancel{2}} * \frac{9 * 2x}{\sqrt{1+9x^2}}\right]$$

$$= \frac{18x}{\sqrt{9x^2+1}} + 6\sinh^{-1}(3x) - \frac{18x}{\sqrt{1+9x^2}}$$

$$= \boxed{6 \sinh^{-1}(3x)} \quad \text{Ans}$$