Integration :- التكامل

The process of integration reverses the process of differentiation.

If $f(x) = 2x^2 \longrightarrow f'(x) = 4x$.
The integration of $4x$ is $2x^2$.

Integration is a process of summation or adding parts together & an elongated S, shown as $\int$, is used to replace the words "the integral of"

Types of Integration :- انواع التكامل

① Indefinite Integrals تكامل غير محدد
Integrals containing an arbitrary constant "C" in their results. This constant needs further infos to be found/calculated.

② Definite Integrals تكامل محدد
Integration limits are applied (يتم تطبيق حدود التكامل)

If an expression is written as $[x]_a^b$, b is called the upper limit and a is the lower limit, where,

$$\boxed{[x]_a^b = b - a}$$

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The process of Integration is عكس التفاضل

In integration, the variable of integration is shown by adding d (the derivative) after the function to be integrated.

Thus, $\int 4x\,dx$ means "the integral of $4x$ with respect to x".

and $\int 2t\,dt$ means "the integral of $2t$ with respect to t"

So,

$$\int dx = x + C$$

$$\int dy = y + C$$

$$\int dt = t + C$$

Standard Integrals' تكاملات قياسية

① Integral of constant $\Longrightarrow \int a\,dx = ax + C$, a = cons.

② Power raised variable $\Longrightarrow \int a x^n\,dx = \dfrac{a x^{n+1}}{n+1} + C$

Examples

① $\int 3x^2\,dx \Longrightarrow \int 3x^2\,dx = \dfrac{3x^{2+1}}{2+1} + C = \dfrac{\cancel{3}}{\cancel{3}} x^3 + C = \boxed{x^3 + C}$

② $\int 3x^4\,dx = \dfrac{3x^{4+1}}{4+1} + C = \boxed{\dfrac{3}{5} x^5 + C}$

③ $\int \dfrac{2}{x^2}\,dx = \int 2x^{-2}\,dx = \dfrac{2x^{-2+1}}{-2+1} + C = \dfrac{2x^{-1}}{-1} + C = \boxed{\dfrac{-2}{x} + C}$

④ $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \dfrac{x^{3/2}}{3/2} + C = \boxed{\dfrac{2}{3}\sqrt{x^3} + C}$

(5) $\int (3x + 2x^2 - 5)\,dx = \int 3x\,dx + \int 2x^2\,dx - \int 5\,dx$

$$= 3\frac{x^{1+1}}{1+1} + 2\frac{x^{2+1}}{2+1} - 5x + C$$

$$= \boxed{\frac{3x^2}{2} + \frac{2x^3}{3} - 5x + C}$$

Integrals of the Trigonometric Functions :-

تكاملات الدوال المثلثية :-

(1) $\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$

(2) $\int \cos ax\,dx = \frac{1}{a}\sin ax + C$

(3) $\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$

(4) $\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$

(5) $\int \csc ax \cot ax\,dx = -\frac{1}{a}\csc ax + C$

(6) $\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax + C$

Examples

(1) $\int [8\cos x + 3\sin x]\,dx$

Sol.

$$= 8\sin x + 3(-\cos x) + C$$

$$= \boxed{8\sin x - 3\cos x + C}$$

(2) $\int [4\sec^2 x - \sec x \tan x]\,dx$

Sol.

$$= \boxed{4\tan x - \sec x + C}$$

(3) $\int \csc x(\cot x - \csc x)\,dx = \int (\csc x \cot x - \csc^2 x)\,dx$

$$= -\csc x - (-\cot x) + C = \boxed{\cot x - \csc x + C}$$

④ $\int \cos^3 x \, dx = \int \cos^2 x \cos x \, dx$

$= \int (1 - \sin^2 x) \cos x \, dx$

$= \int \cos x \, dx - \int \sin^2 x \cos x \, dx$

$= \boxed{\sin x - \frac{1}{3}\sin^3 x + C}$

⑤ $\int \cos^5 x \, dx = \int \cos^4 x \cos x \, dx$

$= \int (\cos^2 x)^2 \cos x \, dx$

$= \int (1 - \sin^2 x)^2 \cos x \, dx$

let $u = \sin x$

$du = \cos x \, dx$

$\therefore = \int (1-u^2)^2 \, du = \int (1-u^2)(1-u^2) \, du$

$= \int (1 - 2u^2 + u^4) \, du$

$= u - \frac{2}{3}u^3 + \frac{u^5}{5} + C$

replace u with $\underline{\sin x}$ & du with $\underline{\cos x \, dx}$

$= \boxed{\sin x - \frac{2}{3}\sin^3 x + \frac{\sin^5 x}{5} + C}$

Integration of Inverse Trigonometric Functions

تكامل عكس الدوال المثلثية

① $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$

② $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$

③ $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{u}{a}\right) + C$

④ $\int \frac{-du}{\sqrt{a^2 - u^2}} = \cos^{-1}\left(\frac{u}{a}\right) + C$

⑤ $\int \frac{-du}{a^2 + u^2} = \frac{1}{a} \cot^{-1}\left(\frac{u}{a}\right) + C$

⑥ $\int \frac{-du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \csc^{-1}\left(\frac{u}{a}\right) + C$

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Examples

① $\int \frac{dx}{\sqrt{16-x^2}} \xrightarrow{\text{sol.}} a^2=16 \Rightarrow a=4$

$u=x$

$\therefore \int \frac{dx}{\sqrt{16-x^2}} = \boxed{\sin^{-1}\frac{x}{4}+c}$

② $\int \frac{3}{25+x^2}dx \xrightarrow{\text{sol.}} a^2=25 \Rightarrow a=5$

$u=x$

$\therefore \int \frac{3\,dx}{25+x^2} = 3\int \frac{dx}{25+x^2} = 3 \times \frac{1}{5}\tan^{-1}\frac{x}{5}+c$

$= \boxed{\frac{3}{5}\tan^{-1}\frac{x}{5}+c}$

③ $\int \frac{8}{x\sqrt{4x^2-1}}dx \xrightarrow{\text{sol.}} a^2=1 \Rightarrow \boxed{a=1}$

$u^2=4x^2 \Rightarrow \boxed{u=2x}$

$\therefore \int \frac{8}{x\sqrt{4x^2-1}}dx = 8\int \frac{\cancel{2}dx}{\cancel{2}x\sqrt{4x^2-1}} = \boxed{8\sec^{-1}(2x)+c}$

Another method $\quad du=2dx \Rightarrow dx=\frac{du}{2}, \ \boxed{x=\frac{u}{2}}$

$8\int \frac{dx}{x\sqrt{4x^2-1}} = 8\int \frac{du/\cancel{2}}{\frac{u}{\cancel{2}}\sqrt{u^2-1}} = 8\int \frac{du}{u\sqrt{u^2-1}}$

$= 8\sec^{-1}u + c$

$= \boxed{8\sec^{-1}(2x)+c}$

Standard integration of inverse Tri func so

① $\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2} + C$

② $\int \cos^{-1}x\,dx = x\cos^{-1}x - \sqrt{1-x^2} + C$

③ $\int \tan^{-1}x\,dx = x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + C$

④ $\int \sec^{-1}x\,dx = x\sec^{-1}x - \ln|x+\sqrt{x^2-1}| + C$

⑤ $\int \csc^{-1}x\,dx = x\csc^{-1}x + \ln|x+\sqrt{x^2-1}| + C$

⑥ $\int \cot^{-1}x\,dx = x\cot^{-1}x + \frac{1}{2}\ln|1+x^2| + C$

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