Communication Technical Engineering Department

1st Stage

Digital Logic- UOMU028021

Lecture 2 – Number Systems: Conversion

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Introduction

- For **digital information** to be processed by a circuit, it must be represented in a suitable format for that circuit.
- To achieve this, a base B number system (B a natural number ≥ 2) needs to be chosen.
- Several number systems are used in digital technology, with the most commonly used systems are
  - **Decimal** (base **10**)
  - **Binary** (base **2**)
  - **Octal** (base **8**)
  - **Hexadecimal** (base **16**).

Number Systems

- A number system of base (**radix**), **r** is a system that uses distinct symbols of **r** digits.
- Normally numbers are represented by a string of Digits/Symbols, (i.e. in decimal system these digits are (0,1,2,3,4,5,6,7,8, and 9).
- To determine the quantity that the number represents in any number system, it is necessary to multiply each digit by an integer power of (r) and then form the sum of all weighted digits.

Decimal Numbers System

- Decimal number system is a system which has a radix (r = 10) and ten digits (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9). The value of the number is the sum of digits after each has been multiplied by its weight.
- **Example 1 : Decimal Number System (Weighted Positional Notation)**

Decimal Number: $(83)_{10}$

Expanded Form:

$= 8 \times 10^1 + 3 \times 10^0$

$= 8 \times 10 + 3 \times 1$

$= 80 + 3$

Digit Weights:

- Digit "8" → Weight = 10 (10^1)
- Digit "3" → Weight = 1 (10^0)

Try Yourself!

$(724)_{10}$

Binary Numbers System

- Binary numbers system is another way to count which have a radix (r = 2) and two digits or bits (0 and 1)
  - Counting in Binary System
    - **In decimal numbers system** we count by starting at 0 and count up to 9. Then start another digit position to the left and continue counting 10 through 99. Then start a third digit position to the left and continue counting from 100 to 999, and so on.
    - **In binary system**, the same situation occurs when we count, except that in binary system we have only ***two bits 0 and 1***. Including another bit position and continue counting, 10, 11. With three bits, we can continue to count, 100, 101, 110, and 111. To continue counting, we need a fourth bit, and so on.

Binary Numbers System

Binary Numbers Table

Number	Binary Number	Number	Binary Number	Number	Binary Number
1	1	11	1011	21	10101
2	10	12	1100	22	10110
3	11	13	1101	23	10111
4	100	14	1110	24	11000
5	101	15	1111	25	11001
6	110	16	10000	26	11010
7	111	17	10001	27	11011
8	1000	18	10010	28	11100
9	1001	19	10011	29	11101
10	1010	20	10100	30	1111

Conversion from Binary to Decimal

- The value of the binary number in decimal system can be computed as the sums of the bits after each have been multiplied by its weight as illustrated in the following examples:
  - **Binary Number: $(110)_2$**
    - **Step 1: Expand Using Positional Weights**
      - $= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
    - **Step 2: Calculate Powers of 2**
      - $= 1 \times 4 + 1 \times 2 + 0 \times 1$
    - **Step 3: Sum the Values**
      - $= 4 + 2 + 0$
        $= (6)_{10}$
- Starting from the left:
  - the first bit (1) has a weight of 4 (2^2),
  - the second bit (1) has a weight of 2 (2^1), and
  - the third bit (0) has a weight of 1 (2^0).

Try Yourself!

$(110110)_2$

Conversion from Binary to Decimal

- **Binary Number: $(10110.1)_2$**
- **Step-by-Step Conversion:**
  - **Integer Part (Left of Binary Point):**
    $= 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0$
    $= 1\times16 + 0\times8 + 1\times4 + 1\times2 + 0\times1$
  - **Fractional Part (Right of Binary Point):**
    $= 1\times2^{-1}$
    $= 1\times0.5$
  - **Summation:**
    $= 16 + 0 + 4 + 2 + 0 + 0.5$
    $= (22.5)_{10}$

- Starting from the left:
  - the first bit (1) has a weight of 16 (2^4),
  - the second bit (0) has a weight of 8 (2^3),
  - the third bit (1) has a weight of 4 (2^2),
  - the fourth bit (1) has a weight of 2 (2^1), and
  - the fifth bit (0) has a weight of 1 (2^0).
  - After the binary point, the following bit (1) has a weight of 0.5 (2^{-1}).

Conversion from Binary to Decimal

- Binary Number: $(101.1011)_2$
- Step-by-Step Conversion:
  - Integer Part (Left of Binary Point):
    a) $1\times2^2 = 1\times4 = 4$
    b) $0\times2^1 = 0\times2 = 0$
    c) $1\times2^0 = 1\times1 = 1$
  - Fractional Part (Right of Binary Point):
    a) $1\times2^{-1} = 1\times0.5 = 0.5$
    b) $0\times2^{-2} = 0\times0.25 = 0$
    c) $1\times2^{-3} = 1\times0.125 = 0.125$
    d) $1\times2^{-4} = 1\times0.0625 = 0.0625$
  - Summation:
    = 4 (integer) + 0 + 1 + 0.5 (fraction) + 0 + 0.125 + 0.0625
    = $(5.6875)_{10}$

Try Yourself! : convert the following binary number into decimal number.
$(11010.101)_2$

Conversion from Decimal to Binary

There are two methods for converting decimal numbers to binary numbers:

1. **Sum of Weights Method**
   - This method determines the binary equivalent by identifying which combination of binary weights sum to the decimal value.
   - The weight positions in a binary number are represented as:

$$2^{n-1} \ldots 2^5 \;\; 2^4 \;\; 2^3 \;\; 2^2 \;\; 2^1 \;\; 2^0 \bullet 2^{-1} \;\; 2^{-2} \ldots 2^{-n}$$

↑

Binary Point

Where **n** is the number of bits from the binary point.

128 64 32 16 8 4 2 1

- $(7)_{10} = 4 + 2 + 1$
  $= 1\times4 + 1\times2 + 1\times1$
  $= 1\times2^2 + 1\times2^1 + 1\times2^0$
  $= (111)_2$
- $(13)_{10} = 8 + 4 + 0 + 1$
  $= 1\times8 + 1\times4 + 0\times2 + 1\times1$
  $= 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$
  $= (1101)_2$

Conversion from Decimal to Binary

2. Repeated Division by 2 Method

- To convert a decimal number N to binary:

i. Divide N by 2

ii. Divide each resulting quotient by 2 until the quotient becomes 0

iii. The remainders form the binary number:

1. First remainder = Least Significant Bit (LSB)
2. Last remainder = Most Significant Bit (MSB)

- **$(11)_{10}$ to Binary:**

Division	Quotient	Remainder	Bit Position
11 ÷ 2	5	1	LSB (2^0)
5 ÷ 2	2	1	2^1
2 ÷ 2	1	0	2^2
1 ÷ 2	0	1	MSB (2^3)

- **Result**: write remainders from bottom to top → **$(1011)_2$**

Conversion from Decimal to Binary

- $(34)_{10}$ to Binary

Division	Quotient	Remainder	Bit Position
34 ÷ 2	17	0	LSB (2^0)
17 ÷ 2	8	1	2^1
8 ÷ 2	4	0	2^2
4 ÷ 2	2	0	2^3
2 ÷ 2	1	0	2^4
1 ÷ 2	0	1	MSB (2^5)

- **Result**: write remainders from bottom to top → $(100010)_2$

- $(45)_{10}$ to Binary

Division	Quotient	Remainder	Bit Position
45 ÷ 2	22	1	LSB (2^0)
22 ÷ 2	11	0	2^1
11 ÷ 2	5	1	2^2
5 ÷ 2	2	1	2^3
2 ÷ 2	1	0	2^4
1 ÷ 2	0	1	MSB (2^5)

- **Result:** write remainders from bottom to top → $(101101)_2$

Binary Weights Table

Integer Part (Positive Powers)	Fractional Part (Negative Powers)
$2^7 = 128$	$2^{-1} = 0.5$
$2^6 = 64$	$2^{-2} = 0.25$
$2^5 = 32$	$2^{-3} = 0.125$
$2^4 = 16$	$2^{-4} = 0.0625$
$2^3 = 8$	$2^{-5} = 0.03125$
$2^2 = 4$	$2^{-6} = 0.015625$
$2^1 = 2$	$2^{-7} = 0.0078125$
$2^0 = 1$	$2^{-8} = 0.00390625$

Conversion of decimal fractions to binary fractions

There are two methods for converting decimal numbers to binary numbers:

1. **Sum of Weights Method**
   - The sum of weights method can be applied to fractional decimal numbers as shown before. The following example illustrates the method:

- **Fraction $(0.625)_{10}$ → Binary**
  - **Weights: 0.5, 0.25, 0.125**
  - **Steps:**

1. **0.5 ≤ 0.625? Yes → 1 (Remainder: 0.625 - 0.5 = 0.125)**
2. **0.25 ≤ 0.125? No → 0**
3. **0.125 ≤ 0.125? Yes → 1 (Remainder: 0.125 - 0.125 = 0)**

- **Result: 0.101_2 → (0.5 + 0 + 0.125).**
- **Table Format:**

Weight	0.5	0.25	0.125
Bit	1	0	1

Conversion of decimal fractions to binary fractions

2. Repeated Multiplication by 2 Method

– To convert a decimal fraction to binary fraction by this method, we need to apply the following steps:

1. **Multiply** the decimal fraction by 2.
2. **Separate** the result into its integer part (carry) and fractional part.
3. **Repeat** the multiplication with the fractional part until it becomes 0 (or reaches the desired precision).
4. **Collect** the integer parts (carries) in order—the **first carry** is the **Most Significant Bit (MSB)**, and the **last carry** is the **Least Significant Bit (LSB)**.

Conversion of decimal fractions to binary fractions

- Convert $(0.625)_{10} \rightarrow$ Binary

Step	Multiply by 2	Integer Part (Carry)	Fractional Part	Bit Position
1	0.625 × 2 = 1.25	1 (MSB)	0.25	2^{-1}
2	0.25 × 2 = 0.5	0	0.5	2^{-2}
3	0.5 × 2 = 1.0	1 (LSB)	0.0	2^{-3}

- Result:
  - write carries top to bottom → 0.101_2

$= 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3} = 0.5 + 0 + 0.125$

$= 0.625_{10}$

- Convert $(0.3125)_{10} \rightarrow$ Binary

Step	Multiply by 2	Integer Part (Carry)	Fractional Part	Bit Position
1	0.3125 × 2 = 0.625	0 (MSB)	0.625	2^{-1}
2	0.625 × 2 = 1.25	1	0.25	2^{-2}
3	0.25 × 2 = 0.5	0	0.5	2^{-3}
4	0.5 × 2 = 1.0	1 (LSB)	0.0	2^{-4}

- Result:
  - write carries top to bottom → 0.0101_2

$= 0\times2^{-1} + 1\times2^{-2} + 0\times2^{-3} + 1\times2^{-4} = 0 + 0.25 + 0 + 0.0625$

$= 0.3125_{10}$

Homework!

- Convert the decimal number $(37.375)_{10}$ to binary number using:
  1. Sum-of-weights method.
  2. Repeated division-by-2 method.

Answer $(100101.011)_2$

THANK YOU ☺