



جامعة المستقبل
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المحاضرة الثالثة



المادة : Discrete Structures
المرحلة : الاولى / الكورس الثاني
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Cardinality

The cardinality of a set S , denoted $|S|$, is simply the number of elements a set has, so

$$|\{a,b,c,d\}| = 4,$$

The cardinality of the power set

Theorem:

If $|A| = n$ then $|P(A)| = 2^n$

(Every set with n elements has 2^n subsets)

Problem set

Write the answers to the following questions.

1. $|\{1,2,3,4,5,6,7,8,9,0\}|$
2. $|P(\{1,2,3\})|$
3. $P(\{0,1,2\})$
4. $P(\{1\})$

Answers

1. 10
2. $2^3=8$
3. $\{\{\},\{0\},\{1\},\{2\},\{0,1\},\{0,2\},\{1,2\},\{0,1,2\}\}$
4. $\{\{\},\{1\}\}$

The Cartesian product

The Cartesian Product of two sets is the set of all tuples made from elements of two sets.

We write the Cartesian Product of two sets A and B as $A \times B$. It is defined as:

$$A \times B = \{(a,b)|a \in A \text{ and } b \in B\}$$

It may be clearer to understand from examples;

$$\{0,1\} \times \{2,3\} = \{(0,2), (0,3), (1,2), (1,3)\}$$

$$\{a,b\} \times \{c,d\} = \{(a,c), (a,d), (b,c), (b,d)\}$$

$$\{0,1,2\} \times \{4,6\} = \{(0,4), (0,6), (1,4), (1,6), (2,4), (2,6)\}$$



Example:

If $A = \{1, 2, 3\}$ and $B = \{x, y\}$ then

$$A \times B = \{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}$$

$$B \times A = \{(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)\}$$

It is clear that, the cardinality of the Cartesian product of two sets A and B is:

$$|A \times B| = |A| |B|$$

A Cartesian Product of two sets A and B can be produced by making tuples of each element of A with each element of B; this can be visualized as a grid (which *Cartesian* implies) or table: if, *e.g.*,

$A = \{0, 1\}$ and $B = \{2, 3\}$, the grid is

		A	
		0	1
B	2	(0,2)	(1,2)
	3	(0,3)	(1,3)

Problem set

Answer the following questions:

1. $\{2,3,4\} \times \{1,3,4\}$
2. $\{0,1\} \times \{0,1\}$
3. $|\{1,2,3\} \times \{0\}|$
4. $|\{1,1\} \times \{2,3,4\}|$

Answers

1. $\{(2,1), (2,3), (2,4), (3,1), (3,3), (3,4), (4,1), (4,3), (4,4)\}$
2. $\{(0,0), (0,1), (1,0), (1,1)\}$
3. 3
4. 6



Computer Representation of Sets

There are various ways to represent sets using a computer. One method is to store the elements of the set in an unordered fashion. However, if this is done, the operations of computing the union, intersection, or difference of two sets would be time-consuming, because each of these operations would require a large amount of searching for elements. We will present a method for storing elements using an arbitrary ordering of the elements of the universal set. This method of representing sets makes computing combinations of sets easy.

Assume that the universal set U is finite (and of reasonable size so that the number of elements of U is not larger than the memory size of the computer being used). First, specify an arbitrary ordering of the elements of U , for instance:

a_1, a_2, \dots, a_n . Represent a subset A of U with the bit string of length n , where the i th bit in this string is 1 if a_i belongs to A and is 0 if a_i does not belong to A .

Example

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and the ordering of elements of U has the elements in increasing order; that is,

$a_i = i$. What bit strings represent

- 1- the subset of all odd integers in U ,
- 2- The subset of all even integers in U , and
- 3- the subset of integers not exceeding 5 in U ?

Solution:

- 1- The bit string that represents the set of odd integers in U , namely, $\{1, 3, 5, 7, 9\}$, has a one bit in the first, third, fifth, seventh, and ninth positions, and a zero elsewhere. It is:
10 1010 1010.
- 2- we represent the subset of all even integers in U , namely, $\{2, 4, 6, 8, 10\}$, by the string 01 0101 0101.
- 3- The set of all integers in U that do not exceed 5, namely, $\{1, 2, 3, 4, 5\}$, is represented by the String 11 1110 0000.



Example

We have seen that the bit string for the set $\{1, 3, 5, 7, 9\}$ (with universal set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$) is 10 1010 1010. What is the bit string for the complement of this set?

Solution:

The bit string for the complement of this set is obtained by replacing 0s with 1s and vice versa. This yields the string 01 0101 0101, which corresponds to the set $\{2, 4, 6, 8, 10\}$.

EXAMPLE

The bit strings for the sets $\{1, 2, 3, 4, 5\}$ and $\{1, 3, 5, 7, 9\}$ are 11 1110 0000 and 10 1010 1010, respectively. Use bit strings to find the union and intersection of these sets.

Solution:

The bit string for the **union** of these sets is:

$11\ 1110\ 0000 \vee 10\ 1010\ 1010 = 11\ 1110\ 1010$, which corresponds to the set $\{1, 2, 3, 4, 5, 7, 9\}$.

The bit string for the **intersection** of these sets is

$11\ 1110\ 0000 \wedge 10\ 1010\ 1010 = 10\ 1010\ 0000$, which corresponds to the set $\{1, 3, 5\}$.