جامـــعـة المـــستـقـبـل
AL MUSTAQBAL UNIVERSITY

كلية العلـــوم
قــســـــــم علوم الذكاء الاصطناعي

المحاضرة الثانية

Discrete Structures : المادة

المرحلة : الاولى/ الكورس الثاني

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Set of numbers:

Several sets are used so often, they are given special symbols.

N = the set of natural numbers or positive integers

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

Z = the set of all integers: . . . ,−2,−1, 0, 1, 2, . . .

$$\mathbb{Z} = \mathbb{N} \cup \{\ldots, -2, -1\}$$

Q = the set of rational numbers

$$\mathbb{Q} = \mathbb{Z} \cup \{\ldots, -1/3, -1/2, 1/2, 1/3, \ldots, 2/3, 2/5, \ldots\}$$

Where $Q = \{ a/b : a , b \in Z, b \neq 0\}$

R = the set of real numbers

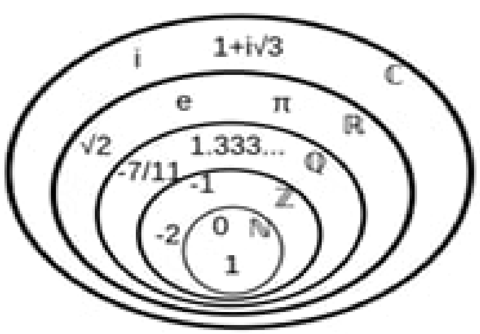
$$\mathbb{R} = \mathbb{Q} \cup \{\ldots, -\pi, -\sqrt{2}, \sqrt{2}, \pi, \ldots\}$$

C = the set of complex numbers

$$\mathbb{C} = \mathbb{R} \cup \{i, 1+i, 1-i, \sqrt{2}+\pi i, \ldots\}$$

Where $C=\{ x + iy ; x , y \in R; i = \sqrt{-1}\}$

Observe that $N \subset Z \subset Q \subset R \subset C$.



Theorem 1:

For any set A, B, C:

1. $\varnothing \subset A \subset U$.
2. $A \subset A$.
3. If $A \subset B$ and $B \subset C$, then $A \subset C$.
4. $A = B$ if and only if $A \subset B$ and $B \subset A$.

Set operations:

1) UNION:

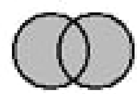
The *union* of two sets A and B, denoted by $A \cup B$, is the set of all elements which belong to A or to B;

$$A \cup B = \{ x : x \in A \text{ or } x \in B\}$$

Example

A={1,2,3,4,5} B={5,7,9,11,13}

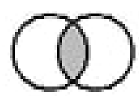
$A \cup B = \{1,2,3,4,5,7,9,11,13\}$



A ∪ B

2) INTERSECTION

The intersection of two sets A and B, denoted by $A \cap B$, is the set of elements which belong to both A and B;



A ∩ B

$$A \cap B = \{ x : x \in A \text{ and } x \in B\}.$$

Example 1

A={1,3,5,7,9} B={2,3,4,5,6}

The elements they have in common are 3 and 5

$A \cap B = \{3,5\}$

Example 2

A={The English alphabet} B={vowels}

So $A \cap B$ = {vowels}

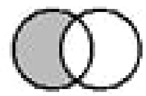
Example 3

A={1,2,3,4,5}   B={6,7,8,9,10}

In this case A and B have nothing in common. $A \cap B = \varnothing$

3) THE DIFFERENCE:

The difference of two sets A\B or A-B is those elements which belong to A but which do not belong to B.

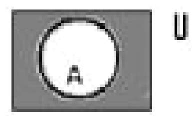


A – B

$A \setminus B = \{x : x \in A, x \notin B\}$

4) COMPLEMENT OF SET:

Complement of set A^c or A' , is the set of elements which belong to U but which do not belong to A .



$A^c = \{x : x \in U, x \notin A\}$

Example 1:

let A={1,2,3}

B = {3,4}

U={1,2,3,4,5,6}

Find:

$A \cup B = \{1, 2, 3, 4\}$

$A \cap B = \{3\}$

$A - B = \{1, 2\}$

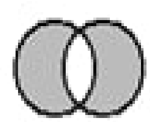
$A^c = \{4, 5, 6\}$

5) Symmetric difference of sets

The symmetric difference of sets A and B, denoted by $A \oplus B$, consists of those elements which belong to A or B but not to both. That is,

$A \oplus B = (A \cup B) \setminus (A \cap B)$ or

$A \oplus B = (A \setminus B) \cup (B \setminus A)$



A ⊕ B

Example:

Suppose $U = N = \{1, 2, 3, \ldots\}$ is the universal set.

Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7\}$,

$C = \{2, 3, 8, 9\}$, $E = \{2, 4, 6, 8, \ldots\}$

Then:

$A^c = \{5, 6, 7, \ldots\}$,

$B^c = \{1, 2, 8, 9, 10, \ldots\}$,

$C^c = \{1,4,5,6,7,10,\ldots\}$

$E^c = \{1, 3, 5, 7, \ldots\}$

$A \backslash B = \{1, 2\}$,

$A \backslash C = \{1, 4\}$,

$B \backslash C = \{4, 5, 6, 7\}$,

$A \backslash E = \{1, 3\}$,

$B \backslash A = \{5, 6, 7\}$,

$C \backslash A = \{8, 9\}$,

$C \backslash B = \{2, 8, 9\}$,

$E \backslash A = \{6, 8, 10, 12, \ldots\}$.

Furthermore:

$A \oplus B = (A \backslash B) \cup (B \backslash A) = \{1, 2, 5, 6, 7\}$,

$B \oplus C = \{2, 4, 5, 6, 7, 8, 9\}$,

$A \oplus C = (A \backslash C) \cup (A \backslash C) = \{1, 4, 8, 9\}$,

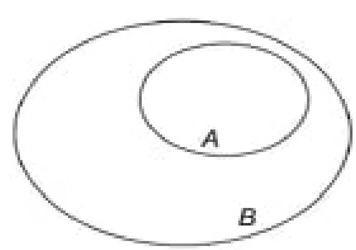
$A \oplus E = \{1, 3, 6, 8, 10, \ldots\}$.

Theorem 2 :

$A \subset B$,

$A \cap B = A$,

$A \cup B = B$ are equivalent



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Theorem 3: (Algebra of sets)
Sets under the above operations satisfy various laws or identities which are listed below:

1- $A \cup A = A$
$A \cap A = A$

2- $(A \cup B) \cup C = A \cup (B \cup C)$ — Associative laws
$(A \cap B) \cap C = A \cap (B \cap C)$

3- $A \cup B = B \cup A$ — Commutativity
$A \cap B = B \cap A$

4- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ — Distributive laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5- $A \cup \varnothing = A$ — Identity laws
$A \cap U = A$

6- $A \cup U = U$ — Identity laws
$A \cap \varnothing = \varnothing$

7- $(A^c)^c = A$ — Double complements

8- $A \cup A^c = U$ — Complement intersections and unions

$A \cap A^c = \varnothing$

9- $U^c = \varnothing$
$\varnothing^c = U$

10- $(A \cup B)^c = A^c \cap B^c$ — De Morgan's laws
$(A \cap B)^c = A^c \cup B^c$

Power set

The power set of some set S, denoted P(S), is the set of all subsets of S (including S itself and the empty set)

$P(S) = \{e : e \subseteq S\}$

Example 1:

Let A = { 1,2 3}

Power set of set A = P(A)

=\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{\},A]

Example 2:

P({0,1})={{},{0},{1},{0,1}}