## 4.4.1 Relation between thickness of cake and volume of filtrate:

In equation 2, the variables ***l*** and ***V*** are connected, and the relation between them may be obtained by making a material balance between the solids in both the slurry and the cake as follows.

Volume of solids in filter cake = (volume of cake) – (volume of pores)

$= Al - Al\,e = (1-e)\,Al$ …………..…4a

Mass of solids in filter cake $= (1-e)\,Al\rho_s$ …………………………...4b

where $\rho_s$ is the density of the solids

Mass of liquid retained in the filter cake $= eAl\rho$……………………….5

Where $\rho$ is the density of the filtrate.

If $J$ is the mass fraction of solids in the original suspension then:

$$(1-e)lA\rho_s = \frac{(V+eAl)\rho J}{1-J} \quad \text{……………….……….6}$$

$$(1-J)(1-e)Al\rho_s = JV\rho + AeJl\rho \quad \text{……………..…..7}$$

So that: $$l = \frac{JV\rho}{A\{(1-J)(1-e)\rho_s - Je\rho\}} \quad \text{…………………..……..8}$$

and: $$V = \frac{\{\rho_s(1-e)(1-J) - e\rho J\}Al}{\rho J} \quad \text{………….............9}$$

If $v$ is the volume of cake deposited by unit volume of filtrate then:

$$v = \frac{lA}{V} \quad \text{or} \quad l = \frac{vV}{A} \quad \text{………………………….10}$$

and from equation(9)

$$v = \frac{J\rho}{(1-J)(1-e)\rho_s - Je\rho} \quad \text{…………………..…..….11}$$

Substituting for $l$ in equation(2)

$$\frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)}{r\mu}\frac{A}{vV}$$

or $$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu vV} \quad \text{……………………..…….12}$$

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Equation (12) may be regarded as the basic relation between **-ΔP**, ***V***, and ***t***. Two important types of operation are: (i) where the pressure difference is maintained constant and (ii) where the rate of filtration is maintained constant.

*For a filtration at constant rate:*

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{\mathbf{r}\mu v V}$$

$$\frac{dV}{dt} = \frac{V}{t} = \text{constant}$$

So that: $$\frac{V}{t} = \frac{A^2(-\Delta P)}{\mathbf{r}\mu V v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 13$$

or $$\frac{t}{V} = \frac{\mathbf{r}\mu v}{A^2(-\Delta P)} V \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 14$$

and $-\Delta P$ is directly proportional to $V$.

*For a filtration at constant pressure difference*

$$\frac{V^2}{2} = \frac{A^2(-\Delta P)t}{\mathbf{r}\mu v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 15$$

or: $$\frac{t}{V} = \frac{\mathbf{r}\mu v}{2A^2(-\Delta P)} V \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 16$$

Thus, for a constant pressure filtration, there is a linear relation between $V^2$ and ***t*** or between t/***V*** and ***V***.

Filtration at constant pressure is more frequently adopted in practice, although the pressure difference is normally gradually built up to its ultimate value.

If this takes a time $t_1$ during which a volume $V_1$ of filtrate passes, then integration of equation (16) gives:

$$\frac{1}{2}(V^2 - V_1^2) = \frac{A^2(-\Delta P)}{\mathbf{r}\mu v}(t - t_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots 17$$

or: $$\frac{t - t_1}{V - V_1} = \frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}(V - V_1) + \frac{\mathbf{r}\mu v V_1}{A^2(-\Delta P)} \quad \ldots\ldots\ldots\ldots 18$$

Thus, there where is a linear relation between $V^2$ and $t$, and between $(t - t_1)/(V - V_1)$ and $(V - V_1)$,
Where:
$(t - t_1)$: represents the time of the constant pressure filtration.
$(V - V_1)$: the corresponding volume of filtrate obtained.

### 4.4.2. Flow of liquid through the cloth:

Experimental work on the flow of the liquid under streamline conditions has shown that the flow rate is directly proportional to the pressure difference. It is the resistance of the cloth plus initial layers of deposited particles that is important since the latter, not only form the true medium, but also tend to block the pores of the cloth thus increasing. It is therefore usual to combine the resistance of the cloth with that of the first few layers of particles and suppose that this corresponds to a thickness ***L*** of cake as deposited at a later stage. The resistance to flow through the cake and cloth combined is now considered.

### 4.4.3 Flow of filtrate through the cloth and cake combined:

If the filter cloth and the initial layers of cake are together equivalent to a thickness ***L*** of cake as deposited at a later stage in the process, and if **-Δ*P*** is the pressure drop across the cake and cloth combined, then:

$$\frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)}{r\mu(L + L_e)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 19$$

which may be compared with equation (2).

Thus:

$$\frac{dV}{dt} = \frac{A(-\Delta P)}{r\mu\left(\frac{V\upsilon}{A} + \frac{V_e\upsilon}{A}\right)} = \frac{A^2(-\Delta P)}{r\mu\upsilon(V + V_e)} \quad \ldots\ldots\ldots\ldots\ldots\ldots 20$$

For the period of *constant rate filtration*: equation(20) may be integrated between the limits $t = 0$, $V = 0$ and $t = t_1$, $V = V_1$

$$\frac{V_1}{t_1} = \frac{A^2(-\Delta P)}{r\mu\upsilon(V + V_e)}$$

or:

$$\frac{t_1}{V_1} = \frac{r\mu\upsilon}{A^2(-\Delta P)}V_1 + \frac{r\mu\upsilon}{A^2(-\Delta P)}V_e$$

or: $$V_1^2 + V_e V_1 = \frac{A^2(-\Delta P)}{r\mu v} t_1 \quad \text{.............................21}$$

For a subsequent *constant pressure filtration*:
Equation(20)may be integrated between the limits $t = 0$, $V = 0$ and $t = t_1$, $V = V_1$

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v(V + V_e)} \quad \text{...........................................20}$$

$$\int_0^{V_1} (V + V_e)\, dV = \frac{A^2(-\Delta P)}{r\mu v} \int_{0_1}^{t_1} dt$$

$$\frac{V_1^2}{2} + V V_e = \frac{A^2(-\Delta P)}{r\mu v} t \quad \text{....................................22a}$$

$$t = \frac{r\mu v}{A^2(-\Delta P)} \left( \frac{V_1^2}{2} + V V_e \right) \quad \text{...................................22b}$$

For a subsequent *constant pressure filtration*: stating at $V_1$ the equation (20) integrated between $t = t_1$, $V = V_1$ and $t = t$, $V = V$:

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v(V + V_e)}$$

$$\int_{V_1}^{V} (V + V_e)\, dV = \frac{A^2(-\Delta P)}{r\mu v} \int_{t_1}^{t} dt$$

$$\frac{1}{2}(V^2 - V_1^2) + V_e (V - V_1) = \frac{A^2(-\Delta P)}{r\mu v}(t - t_1) \quad \text{..............7.23}$$

or:

$$(V - V_1 + 2V_1)(V - V_1) + 2V_e (V - V_1) = \frac{2A^2(-\Delta P)}{r\mu v}(t - t_1)$$

or: $$\frac{t - t_1}{V - V_1} = \frac{r\mu v}{2A^2(-\Delta P)}(V - V_1) + \frac{r\mu v}{A^2(-\Delta P)}(V_1 + V_e) \quad \text{......................7.24}$$

Thus there is a linear relation between $(t - t_1)/(V - V_1)$ and $V - V_1$, and the slope is proportional to the specific resistance, as in the case of the flow of the filtrate through the filter cake alone given by equation 7.18, although the line does not now go through the origin.