Al-Mustaqbal University / College of Engineering & Technology
Class: first
Subject: integral Mathematics/Code: UOMU024024
Lecturer: M.Sc. Alaa Khalid
Lecture name: introduction of integration
Lecture: 1
2nd term

Introduction of Integration

In this course, we'll discuss the definition of integral calculus, indefinite and definite integration, key integration rules and some essential integration methods with some examples. What is Integral Calculus?

You are probably already familiar with differentiation, which is the process used to calculate the instantaneous rate of change of a function. What is the difference between integration and differentiation? Well, you can think about integration as the reverse operation of differentiation. Together, differentiation and integration make up the essential operations of calculus and are related by the fundamental theorems of calculus.

That is:

Integration is the opposite of differentiation, meaning finding the function if its derivative is known.

$$If \ {}^{d}/_{dx}\ (F(x)) = f(x) \ then \int f(x)dx = F(x).$$

Where $\int$ *is called the integral sign* .This symbol indicates that we're calculating the anti-derivative function of $f(x)$

The function $f(x)$ is called the integrand, and it's the function we're taking the integral of.

The letters dx in calculus represent the differential dx. The differential dx indicates that we're integrating $f(x)$ with respect to the variable x.

$F(x)$ is the anti-derivative function that gives back $f(x)$ when differentiated.

Email : alaa.khalid.abdalreda@uomus.edu.iq

Types of integration

1. Indefinite integral
2. Definite integral

1. Indefinite integral

$$\int f(x)dx = F(x) + C.$$

The capital letter *C* represents a constant value called the constant of integration.

Standard Integration Rule

1) $\int du = u(x) + c$
2) $\int a \cdot u(x)dx = a \int u(x)\, dx$
3) $\int \big(u(x) \mp v(x)\big)\, dx = \int u(x)\, dx \mp \int v(x)\, dx$
4) $\int u^n\, du = \frac{u^{n+1}}{n+1} + c \quad when\; n \neq -1 \;\; \& \int u^{-1}\, du = \int \frac{1}{u} du = \ln u + c$
5) $\int a^u\, du = \frac{a^u}{\ln a} + c \quad \Rightarrow \quad \int e^u\, du = e^u + c$

Email : alaa.khalid.abdalreda@uomus.edu.iq

Examples

[1] $\int 6\ dx = 6x + C$

[2] $\int -3\, dx = -3x + C$

[3] $\int \frac{1}{4}\, dy = \frac{1}{4}y + C$

[5] $\int x^4\ dx = \frac{x^{4+1}}{4+1} + C = \frac{x^5}{5} + C$

[6] $\int x^{-3}\, dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$

[7] $\int 4x^3\, dx = \frac{4x^4}{4} + C = x^4 + C$

[8] $\int -6x^{-5}\, dx = \frac{-6x^{-4}}{-4} + C = \frac{3}{2x^4} + C$

[9] $\int \sqrt{x}\ dx = \int x^{\frac{1}{2}}\, dx$

$$= \frac{2}{3}x^{\frac{3}{2}} + C$$

[10] $\int (3x^2 + 5)\, dx = \frac{3x^3}{3} + 5x + C = x^3 + 5x + C$

[11] $\int (6x^2 - 5x + 3)\, dx = \frac{6x^3}{3} - \frac{5x^2}{2} + 3x + C$

$$= 2x^3 - \frac{5}{2}x^2 + 3x + C$$

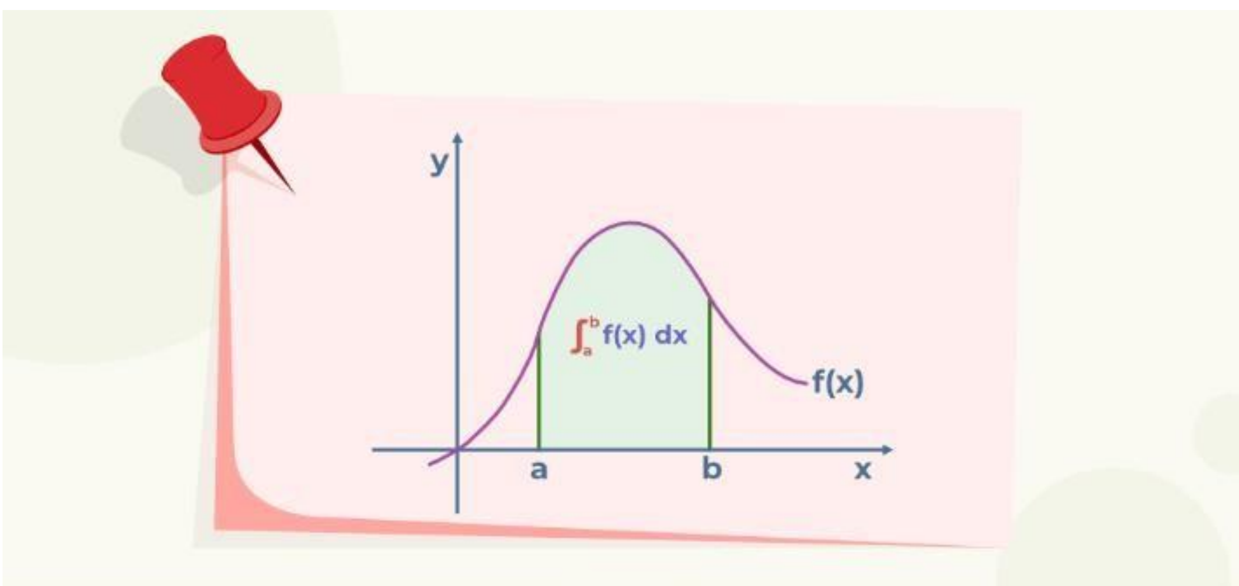
[12] $\int (x^4 - 2x^3 + x^2)\, dx = \frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} + C = \frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} + C$

$$\int \frac{e^x}{1+3e^x}\,dx = \frac{3}{3}\int e^x\,(1+3e^x)^{-1}\,dx = \frac{1}{3}\int \frac{3e^x}{1+3e^x}\,dx = \frac{1}{3}\ln(1+3e^x) + c$$

$$\int 2^{-4x}\ dx = \quad \Rightarrow \quad = \int -4\,2^{-4x}\ln 2\ dx = \frac{1}{-4}\frac{1}{\ln 2}2^{-4x} + c$$

2- Definite integral

These types of integrals have different outputs. The definite integral outputs a unique number that represents the area enclosed by a function's curve and the x-axis over some interval [*a*,*b*]. The indefinite integral outputs a function's antiderivative function, accompanied by the constant of integration C.



Definite Integral

Email : alaa.khalid.abdalreda@uomus.edu.iq

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

The letters *a* and *b* are called integral bounds or limits. The letter *a* represents the lower bound, while *b* represents the upper bound.

Examples

$$[1]\ \int_1^3 x^3\,dx = \left[\frac{x^4}{4}\right]_1^3 = \frac{(3)^4}{4} - \frac{(1)^4}{4} = \frac{81}{4} - \frac{1}{4} = \frac{80}{4} = 20$$

$$[2]\ \int_{-1}^0 (3x^2 + 2x + 1)\,dx = \left[\frac{3x^3}{3} + \frac{2x^2}{2} + x\right]_{-1}^0 = [x^3 + x^2 + x]_{-1}^0$$
$$= (0) - (-1 + 1 - 1) = -(-1) = 1$$

$$[3]\ \int_1^2 (x^{-2} + 2x + 1)dx = \left[\frac{x^{-1}}{-1} + \frac{2x^2}{2} + x\right]_1^2 = \left[\frac{-1}{x} + x^2 + x\right]_1^2$$
$$= \left[\frac{-1}{2} + 4 + 2\right] - [-1 + 1 + 1]$$
$$= \frac{-1}{2} + 6 - 1 = \frac{-1}{2} + 5 = \frac{-1 + 10}{2} = \frac{9}{2}$$

Email : alaa.khalid.abdalreda@uomus.edu.iq

[4] $$\int_1^3 (x^4 + 4x)dx = \left[\frac{x^5}{5} + \frac{4x^2}{2}\right]_1^3 = \left[\frac{x^5}{5} + 2x^2\right]_1^3$$

$$= \left[\frac{(3)^5}{5} + 2(3)^2\right] - \left[\frac{(1)^5}{5} + 2(1)\right]$$

$$= \left[\frac{243}{5} + 18\right] - \left[\frac{1}{5} + 2\right]$$

$$= \left[\frac{243 + 90}{5}\right] - \left[\frac{1 + 10}{5}\right] = \frac{333}{5} - \frac{11}{5} = \frac{322}{5}$$

[5] $$\int_1^3 \frac{2x^3 - 4x^2 + 5}{x^2} dx = \int_1^2 \left(\frac{2x^3}{x^2} - \frac{4x^2}{x^2} + \frac{5}{x^2}\right) dx$$

$$= \int_1^3 (2x - 4 + 5x^{-2})dx \qquad = \left[\frac{2x^2}{2} - 4x + \frac{5x^{-1}}{-1}\right]_1^3$$

$$= \left[x^2 - 4x - \frac{5}{x}\right]_1^3$$

$$= \left[9 - 12 - \frac{5}{3}\right] - [1 - 4 - 5] = \left[-3 - \frac{5}{3}\right] - [-8]$$

$$= \frac{-9 - 5}{3} + 8 = \frac{-14}{3} + 8 = \frac{-14 + 24}{3} = \frac{10}{3}$$