Al-Mustaqbal University / College of Engineering & Technology
Class: first
Subject: intergral Mathematics/Code:UOMU024024
Lecturer:M.Sc. Alaa Khalid
Lecture name: integrals of hyperbolic functions
Lecture: 4
2nd term

Integrals of inverse trigonometric functions:

1) $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + c = -\cos^{-1}\frac{u}{a} + c \quad ; \; \forall u^2 < a^2$
2) $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c = -\frac{1}{a}\cot^{-1}\frac{u}{a} + c$
3) $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + c = -\frac{1}{a}\csc^{-1}\left|\frac{u}{a}\right| + c \quad ; \; \forall u^2 > a^2$

Ex 3: Evaluate the following integrals:

1) $\int \frac{x^2}{\sqrt{1-x^6}}dx$
2) $\int \frac{dx}{\sqrt{9-x^2}}$
3) $\int \frac{x}{1+x^4}dx$
4) $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$
5) $\int \frac{dx}{x\sqrt{4x^2-1}}$
6) $\int \frac{x}{\sqrt{1-x^4}}dx$
7) $\int \frac{dx}{1+3x^2}$
8) $\int \frac{2\cos x\,dx}{1+(\sin x)^2}$

Sol:

1) $\frac{1}{3}\int \frac{1}{\sqrt{1-(x^3)^2}}(3x^2\,dx) = \frac{1}{3}\sin^{-1}x^3 + c$
2) $\int \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\frac{x}{3} + c$
3) $\frac{1}{2}\int \frac{2x}{1+(x^2)^2}dx = \frac{1}{2}\tan^{-1}x^2 + c$
4) $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx = \sin^{-1}(\tan x) + c$
5) $\int \frac{2dx}{2x\sqrt{4x^2-1}} = \sec^{-1}(2x) + c$
6) $\int \frac{x}{\sqrt{1-x^4}}dx = \frac{1}{2}\sin^{-1}x^2 + c$
7) $\frac{1}{\sqrt{3}}\int \frac{\sqrt{3}\,dx}{1+(\sqrt{3}x)^2} = \frac{1}{\sqrt{3}}\tan^{-1}(\sqrt{3}\,x) + c$
8) $2\int \frac{\cos x\,dx}{1+(\sin x)^2} = 2\tan^{-1}(\sin x) + c$

$\sin^{-1} x$

$u = x^3 \rightarrow du = 3x^2$
$u = x \rightarrow du = dx$
$u = x^2 \rightarrow du = 2x$
$u = \tan x \rightarrow du = \sec^2 x$
$u = 2x \rightarrow du = 2$
$u = \sqrt{3}x \rightarrow du = \sqrt{3}$
$u = \sin x \rightarrow du = \cos x\,dx$

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Integrals of hyperbolic functions:

1) $\int \sinh u \, du = \cosh u + c$
2) $\int \cosh u \; du = \sinh u + c$
3) $\int \tanh u \, du = \ln(\cosh u) + c$
4) $\int \coth u \, du = \ln(\sinh u) + c$
5) $\int \operatorname{sech}^2 u \, du = \tanh u + c$
6) $\int \operatorname{csch}^2 u \, du = \coth u + c$
7) $\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + c$
8) $\int \operatorname{csch} u \coth u \; du = -\operatorname{csch} u + c$

$Ex\ 4: Evaluate\ the\ following\ integrals:$

1) $\int \frac{\cosh(\ln x)}{x} dx$
2) $\int \sinh(2x + 1) \; dx$
3) $\int \operatorname{sech}^2(2x - 3) \; dx$
4) $\int x \cosh(3x^2) \, dx$
5) $\int \sinh^4 x \cosh x \; dx$

$Sol:$

1) $\int \cosh(\ln x) \cdot \frac{dx}{x} = \sinh(\ln x) + c$
2) $\frac{1}{2} \int \sinh(2x + 1)\,(2dx) = \frac{1}{2} \cosh(2x + 1) + c$
3) $\int \operatorname{sech}^2(2x - 3) \; dx$ =1\2**tanh (2x-3) +c**
4) $\frac{1}{6} \int \cosh(3x^2)\,(6x\,dx) = \frac{1}{6} \sinh(3x^2) + c$
5) $\int \sinh^4 x\,(\cosh x \; dx) = \frac{\sinh^5 x}{5} + c$

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Integrals of inverse hyperbolic functions:

1) $\int \frac{du}{\sqrt{1+u^2}} = \sinh^{-1} u + c$

2) $\int \frac{du}{\sqrt{u^2-1}} = \cosh^{-1} u + c$

3) $\int \frac{du}{1-u^2} = \begin{Bmatrix} \tanh^{-1} u + c & |u| < 1 \\ \coth^{-1} u + c & |u| > 1 \end{Bmatrix} = \frac{1}{2} \ln \left|\frac{1+u}{1-u}\right| + c,$

4) $\int \frac{du}{u\sqrt{1-u^2}} = -\operatorname{sech}^{-1} |u| + c$

5) $\int \frac{du}{u\sqrt{1+u^2}} = -\operatorname{csch}^{-1} |u| + c$

Ex 5: Evaluate the following integrals:

1) $\int \frac{dx}{\sqrt{1+4x^2}}$ 2) $\int \frac{dx}{\sqrt{4+x^2}}$ 3) $\int \frac{dx}{1-x^2}$ 4) $\int \frac{dx}{x\sqrt{4+x^2}}$

5) $\int \frac{\sec^2 \theta}{\sqrt{\tanh^2 \theta - 1}} \, d\theta$

Sol:

1) $\frac{1}{2} \int \frac{2\,dx}{\sqrt{1+4x^2}} = \frac{1}{2} \sinh^{-1} 2x + c$

2) $\int \frac{\frac{1}{2}dx}{\sqrt{1+\left(\frac{x}{2}\right)^2}} = \sinh^{-1} \frac{x}{2} + c$

3) $\int \frac{dx}{1-x^2} = \tanh^{-1} x + c \quad if \quad |x| < 1$

4) $\int \frac{dx}{x\sqrt{4+x^2}} = \frac{1}{2} \int \frac{\frac{1}{2}dx}{\frac{x}{2}\sqrt{1+\left(\frac{x}{2}\right)^2}} = \frac{1}{2} \operatorname{csch}^{-1} \left|\frac{x}{2}\right| + c$

5) $\int \frac{1}{\sqrt{\tan^2 \theta - 1}} (\sec^2 \theta \;\, d\theta) = \cosh^{-1}(\tan \theta) + c$

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