Al-Mustaqbal University / College of Engineering & Technology
Class: first
Subject: integral Mathematics/Code: UOMU024024
Lecturer: M.Sc. Alaa Khalid
Lecture name: Integral of trigonometric functions
Lecture: 3
2nd term

Odd and even powers of sine and cosine:

To integrate an odd positive power of $\sin x$ $(say\ \sin^{2n+1} x)$ $we\ split\ off$ $a\ factor\ of\ \sin x\ \ and\ rewrite\ the\ remaining\ even\ power\ in\ terms\ of\ the\ cosine. We\ write:-$

$$\int \sin^{2n+1} x\ \ dx = \int (1-\cos^2 x)^n \sin x\ \ dx$$

$$\int \cos^{2n+1} x\ \ dx = \int (1-\sin^2 x)^n \cos x\ \ dx$$

$Ex\ 2: Evaluate:-$ 1) $\int \sin^3 x\ \ dx$ 2) $\int \cos^5 c\ dx$

Sol:

1) $\int \sin^3 x\ \ dx = \int \sin^2 x \sin x\ \ dx = \int (1-\cos^2 x) \sin x\ \ dx$

$$= \int \sin x\ \ dx + \int \cos^2 x\,(-\sin x) dx = -\cos x + \frac{1}{3}\cos^3 x + c$$

2) $\int \cos^5 c\ dx = \int \cos^4 x \cos x\ \ dx = \int (1-\sin^2 x)^2 \cos x\ \ dx$

$$= \int \cos x\ \ dx - 2\int \sin^2 x \cos x\ \ dx + \int \sin^4 x \cos x\ \ dx$$

$$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$$

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$$\int \sin^4 x \cos^3 x \; dx = \int \sin^4 x \cos^2 x \cos x \; dx =$$

$$\int \sin^4 x \,(1-\sin^2 x) \cos x \; dx = \int \sin^4 x \cos x \; dx - \int \sin^6 x \cos x \; dx$$

$$= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$$

To integrate an even positive power of sine (say $\sin^{2n} x$) *we use the relations*: –

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad or \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Then we can write:-

$$\int \sin^{2n} x \; dx = \int \left(\frac{1-\cos 2\theta}{2}\right)^n dx \quad and \quad \int \sin^{2n} x \; dx = \int \left(\frac{1-\cos 2\theta}{2}\right)^n dx$$

$$Ex\ 3: Evaluate:\ \ 1) \int \cos^2\theta \; d\theta \qquad 2) \int \sin^4\theta \; d\theta$$

Sol:

1) $\int \cos^2\theta \; d\theta = \int \frac{1+\cos 2\theta}{2} \; d\theta = \frac{1}{2}\left[\int d\theta + \frac{1}{2}\int 2\cos 2\theta \; d\theta\right]$

$$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right] + c$$

2) $\int \sin^4\theta \; d\theta = \int \left(\frac{1-\cos 2\theta}{2}\right)^2 d\theta = \frac{1}{4}\left[\int d\theta - \int \cos 2\theta\,(2d\theta) + \int \cos^2 2\theta \, d\theta\right]$

$$= \frac{1}{4}\left[\theta - \sin 2\theta + \int \frac{1+\cos 4\theta}{2} \; d\theta\right] = \frac{1}{4}\left[\theta - \sin 2\theta + \frac{1}{2}\left(\theta + \frac{1}{4}\sin 4\theta\right)\right] + c$$

$$= \frac{3}{8}\theta - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta + c$$