Cramer's Rule

Introduction

Cramer's rule is a method for solving linear simultaneous equations. It makes use of determinants and so a knowledge of these is necessary before proceeding.

1. Cramer's Rule - two equations

If we are given a pair of simultaneous equations

$$a_1 x + b_1 y = d_1$$
$$a_2 x + b_2 y = d_2$$

then x, and y can be found from

$$x = \frac{\begin{vmatrix} d_1 & b_1 \\ d_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 \\ a_2 & d_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Example
Solve the equations

$$3x + 4y = -14$$
$$-2x - 3y = 11$$

Solution
Using Cramer's rule we can write the solution as the ratio of two determinants.

$$x = \frac{\begin{vmatrix} -14 & 4 \\ 11 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ -2 & -3 \end{vmatrix}} = \frac{-2}{-1} = 2, \qquad y = \frac{\begin{vmatrix} 3 & -14 \\ -2 & 11 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ -2 & -3 \end{vmatrix}} = \frac{5}{-1} = -5$$

The solution of the simultaneous equations is then $x = 2, y = -5$.

2. Cramer's rule - three equations

For the case of three equations in three unknowns: If

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

then x, y and z can be found from

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

Exercises

Use Cramer's rule to solve the following sets of simultaneous equations.

a)

$$7x + 3y = 15$$
$$-2x + 5y = -16$$

b)

$$x + 2y + 3z = 17$$
$$3x + 2y + z = 11$$
$$x - 5y + z = -5$$

Answers

a) $x = 3$, $y = -2$.　　b) $x = 1$, $y = 2$, $z = 4$