جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

قسم الامن السيبراني

Department of Cyber Security

Subject:

Predicates and Quantifiers

Class:

First

Lecturer:

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Lecture: (8)

Introduction

- Propositional logic, studied in previous lectures cannot adequately express the meaning of all statements in mathematics and in natural language. For example, suppose that we know that

  "Every computer connected to the university network is functioning properly"

- No rules of propositional logic allow us to conclude the truth of the statement.

- In this lecture we will introduce a more powerful type of logic called ***predicate logic.***

Predicate:

$$x \text{ is greater than } 3$$

Example1:

Let $P(x)$ denote the statement "$x > 3$."

What are the truth values of $P(4)$ and $P(2)$?

Solution

We obtain the statement *P(4)* by setting $x = 4$ in the statement "$x > 3$." Hence, *P(4),* which is the statement "$4 > 3$," is true. However, *P(2),* which is the statement "$2 > 3$," is false.

Let P(x) denote the statement "x > 3."

What are the truth values of P (4) and P (2)?

P (4) = T

P (2) = F

Example2:

Let $Q(x, y)$ denote the statement "$x = y + 3$."
What are the truth values of the propositions
$Q(1, 2)$ and $Q(3, 0)$?
F T

Example3:

1. Let $P(x)$ denote the statement "$x \leq 4$." What are the truth values?

a) $P(0)$ **b)** $P(4)$ **c)** $P(6)$

2. Let $P(x)$ be the statement "the word x contains the letter a." What are the truth values?

a) $P(\text{orange})$ **b)** $P(\text{lemon})$
c) $P(\text{true})$ **d)** $P(\text{false})$

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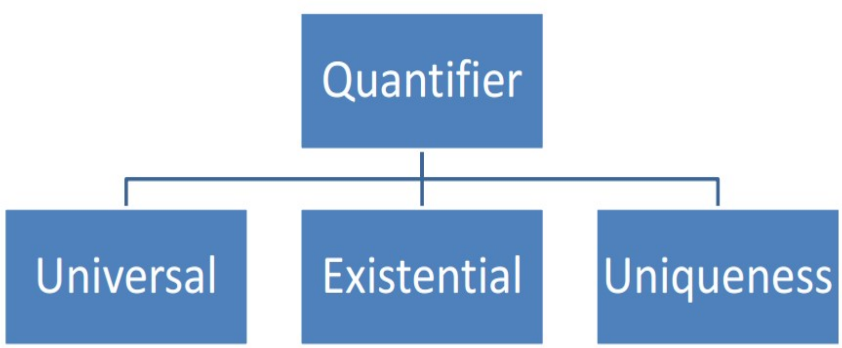
Quantifiers:

Expresses the extent to which a predicate is true over a **range** of elements.

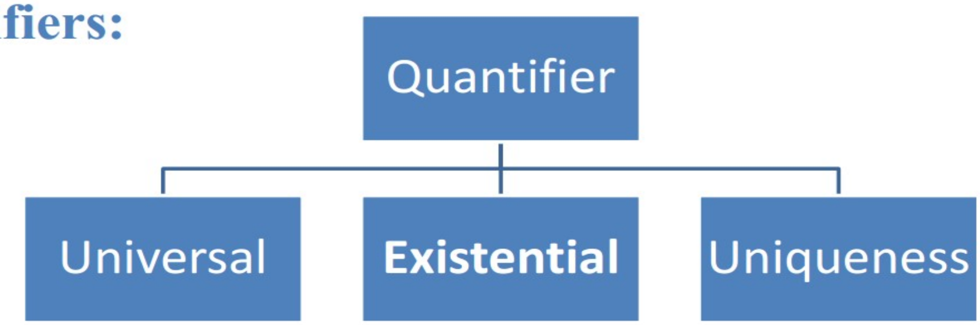


Vx P(x)

The universal quantification of P(x) is the statement

"P(x) for all values of x in the domain."

Quantifiers:

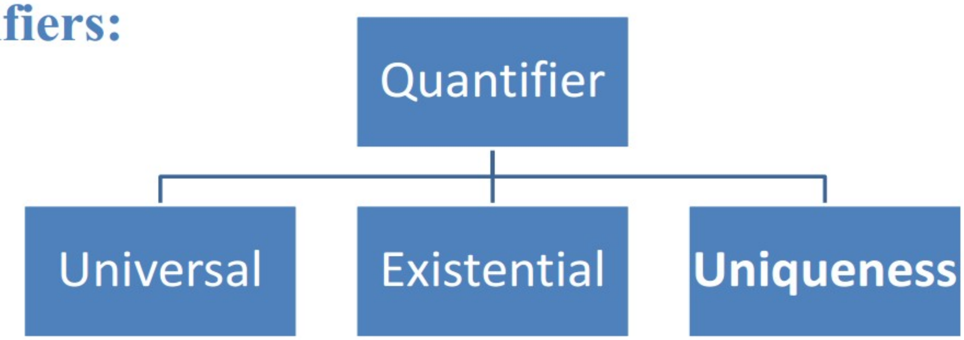


$$\exists x\, P(x)$$

The *existential quantification* of $P(x)$ is the proposition

"There exists an element x in the domain such that $P(x)$."

Quantifiers:



$$\exists!\, xP(x)$$

$$\exists_1 xP(x)$$

"There exists a unique x such that $P(x)$ is true."

TABLE 1 Quantifiers.

Statement	*When True?*	*When False?*
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Example1:

Let $P(x)$ be the statement "$x + 1 > x$."

What is the truth value of the quantification $\forall x P(x)$,

where the domain consists of all real numbers?

Solution: Because $P(x)$ is true for all real numbers x, the quantification

$$\forall x P(x)$$

is true.

Example2:

Let $Q(x)$ be the statement "$x < 2$."
What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers?

Solution: $Q(x)$ is not true for every real number x, because, for instance, $Q(3)$ is false. That is, $x = 3$ is a counterexample for the statement $\forall x Q(x)$. Thus $\forall x Q(x)$ is false.

Example3:

Let $P(x)$ denote the statement "$x > 3$."
What is the truth value of the quantification $\exists x P(x)$, where the domain consists of all real numbers?

Solution: Because "$x > 3$" is sometimes true—for instance, when $x = 4$—the existential quantification of $P(x)$, which is $\exists x P(x)$, is true.

Example 4:

What is the truth value of ∀x P(x), where P(x) is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?

***Solution*:**

The statement ∀x P(x) is the same as the conjunction

$$P(1) \wedge P(2) \wedge P(3) \wedge P(4)$$

Because the domain consists of the integers 1, 2, 3, and 4. Because P(4), which is the statement "$4^2 < 10$," is false, it follows that ∀x P(x)is false.

Example5:

What is the truth value of $\exists x P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4?

Solution: Because the domain is $\{1, 2, 3, 4\}$, the proposition $\exists x P(x)$ is the same as the disjunction $P(1) \vee P(2) \vee P(3) \vee P(4)$. Because $P(4)$, which is the statement "$4^2 > 10$," is true, it follows that $\exists x P(x)$ is true.

Example6:

Let $P(x)$ be the statement "$x = x^2$." If the domain consists of the integers, what are the truth values?

a) $P(0)$ T
b) $P(1)$ T
c) $P(2)$ F
d) $P(-1)$ F
e) $\exists x P(x)$ T
f) $\forall x P(x)$ F